Deciphering the Archaeological Record: Cosmological Imprints of Non-Minimal Dark Sectors

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University of Wisconsin

Wednesday, November 13th, 2019

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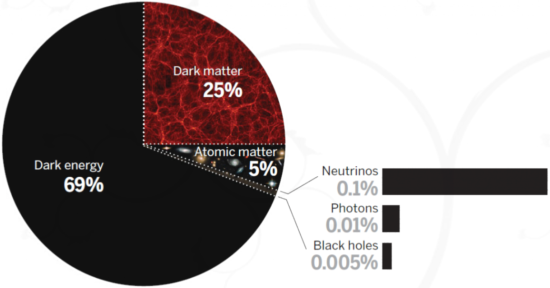

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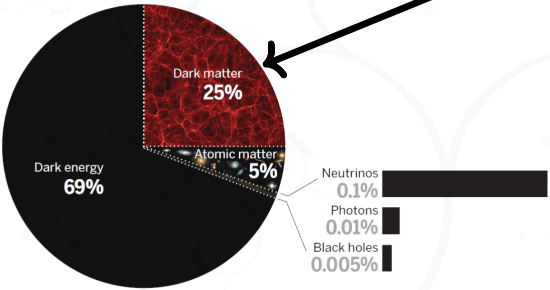

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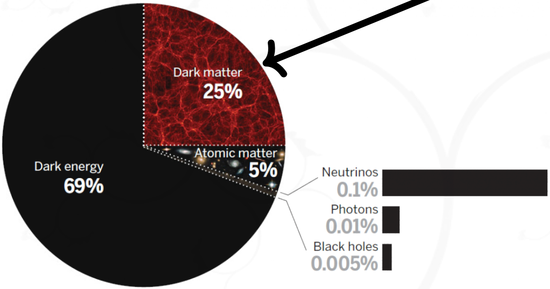

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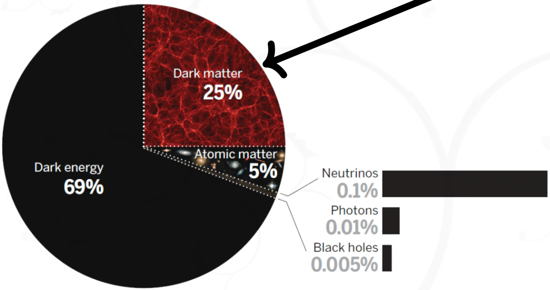

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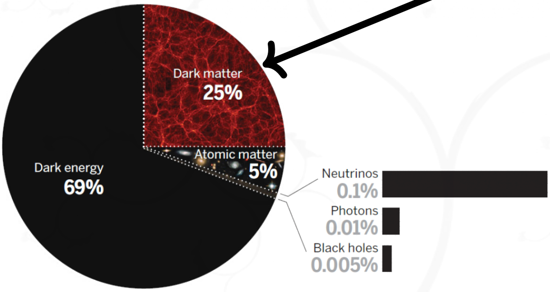

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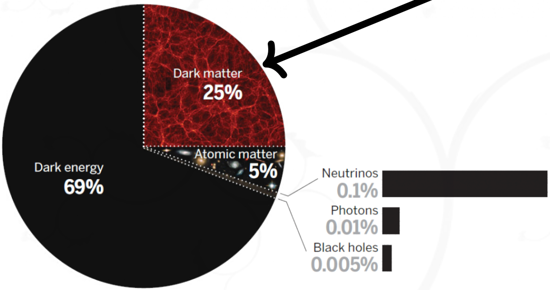

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- one species? or many components?
- interactions with SM? within dark sector itself?
- what dynamics is involved in establishing DM today?

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We are interested in how dark matter drives **cosmological structure**.

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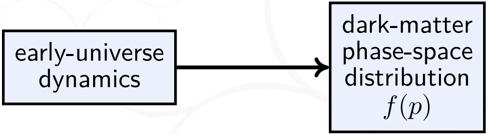

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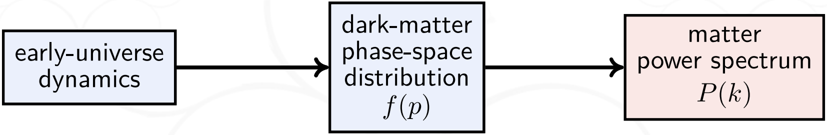

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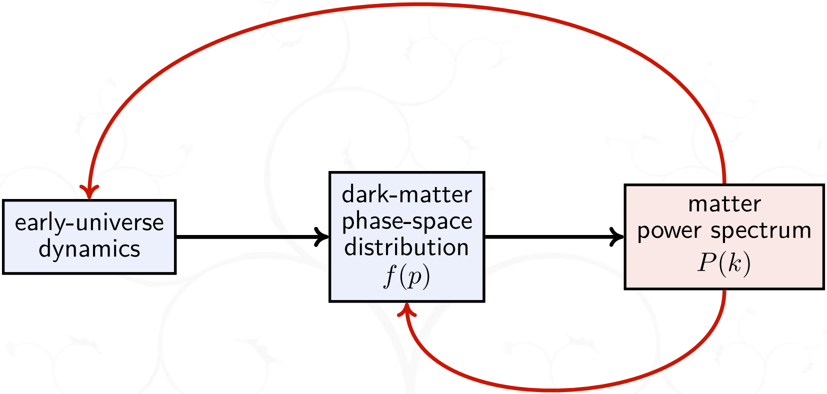

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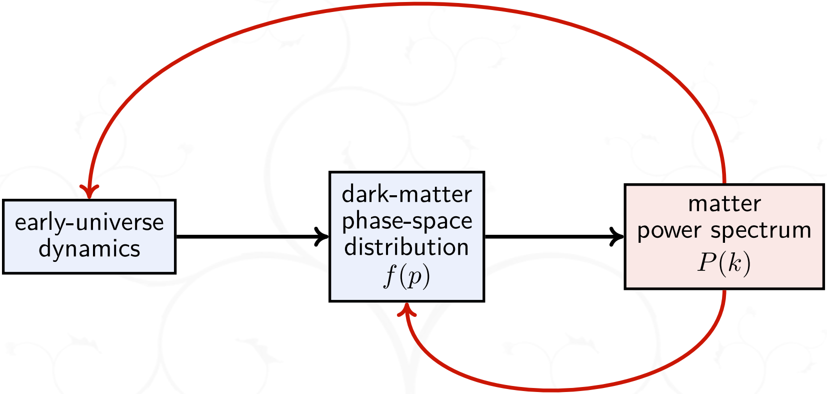

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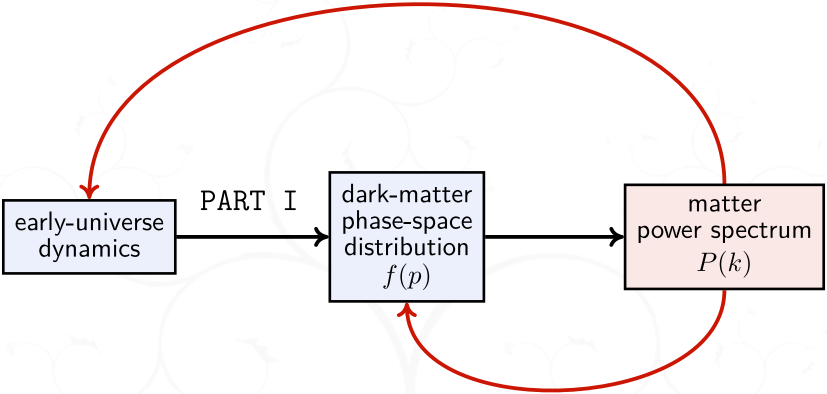

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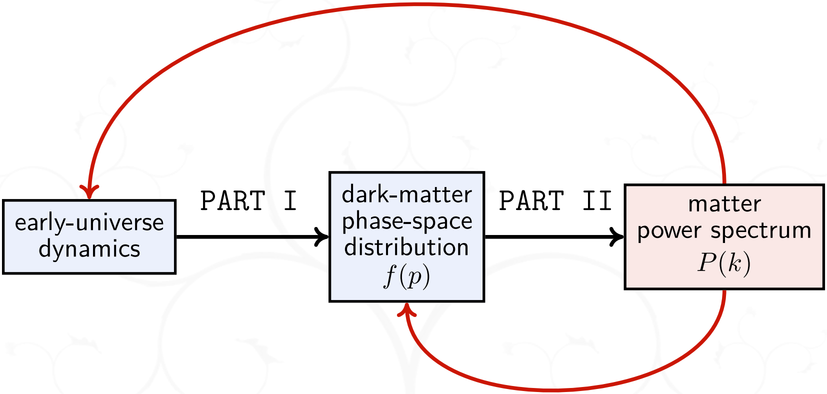

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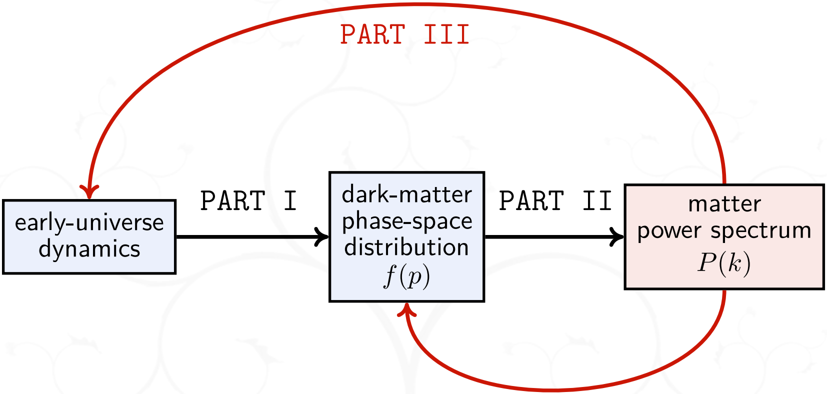

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PART I

early-universe dynamics $\longrightarrow$ DM phase-space distribution

Ⓘ Early Dynamics $\longrightarrow$ DM Momentum Distributions

• In general, once the dark matter is produced in the early universe its properties are described by its phase-space distribution $f(\vec{x},\vec{p},t) \approx f(p,t)$:

homogeneity/isotropy

number density:

$$n(t) = g_{\rm int} \int \frac{d^3p}{(2\pi)^3} f(p,t)$$

energy density:

$$\rho(t) = g_{\rm int} \int \frac{d^3p}{(2\pi)^3} E f(p,t)$$

pressure:

$$P(t) = g_{\rm int} \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E} f(p,t)$$

equation of state

$$w(t) = \frac{P(t)}{\rho(t)}$$

$\Rightarrow$ the distribution $f(p,t)$ is the **central quantity** in understanding cosmological properties of the dark sector

(I) Early Dynamics ⟶ DM Momentum Distributions

- It is important to understand how $f(p)$ evolves in an FRW background:

$$p(t) = p(t')\underbrace{\frac{a(t')}{a(t)}}_{\text{redshifting}} \quad \text{gives} \quad \frac{d\log p}{dt} = -\underbrace{H(t)}_{\text{Hubble parameter}}$$

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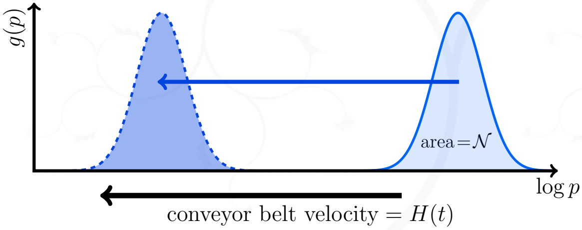

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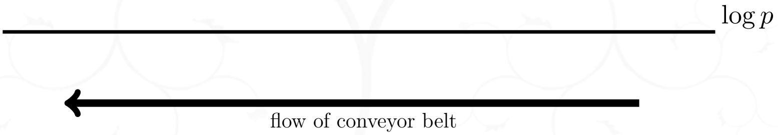

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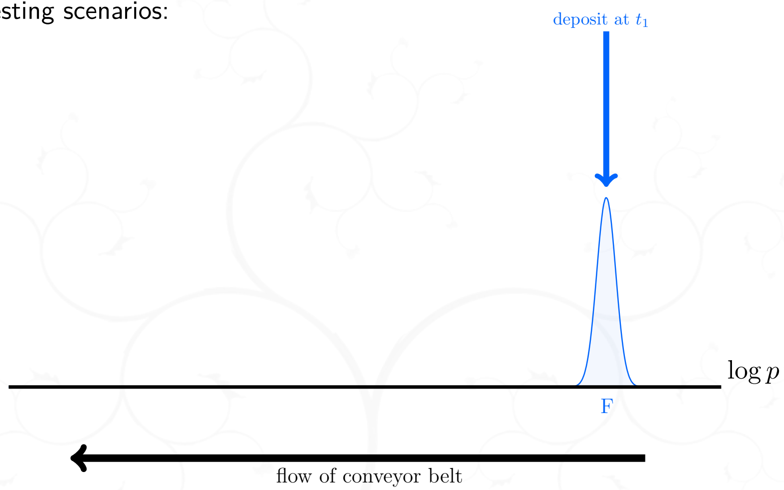

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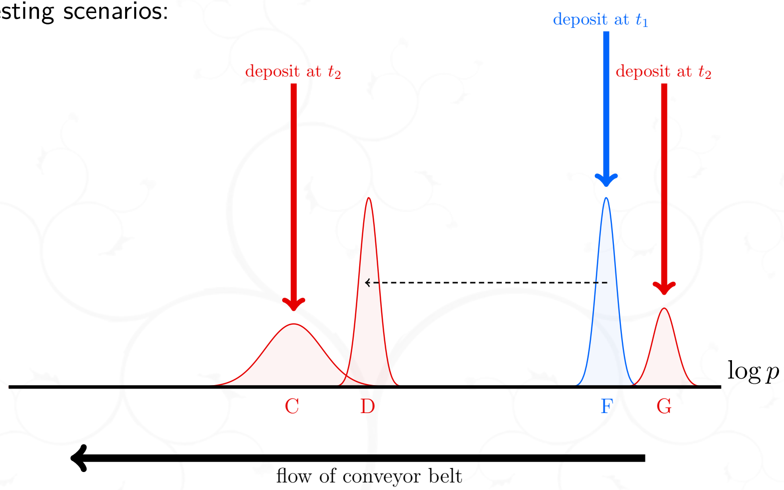

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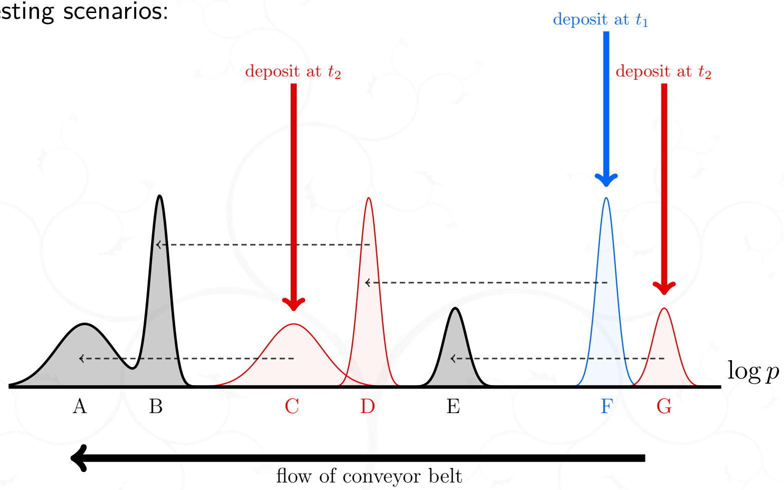

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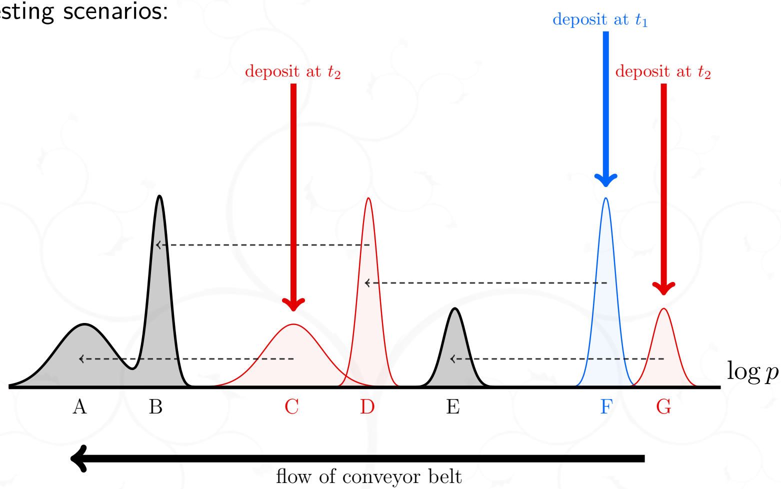

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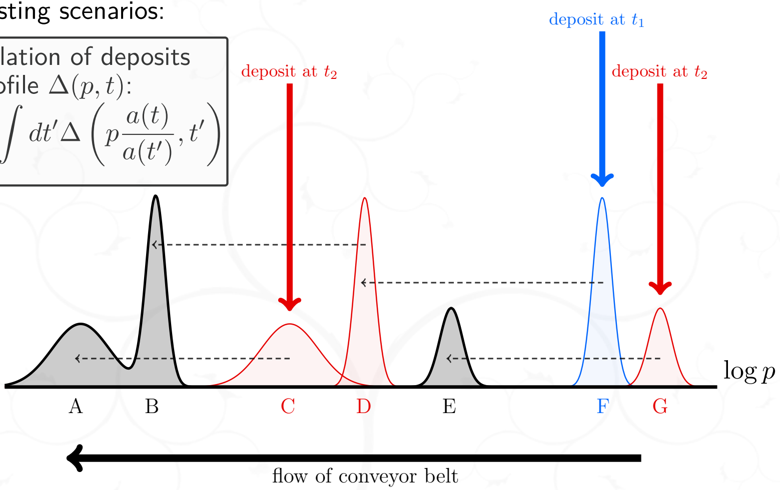
accumulation of deposits with profile $\Delta(p,t)$:

$$g(p) = \int dt' \Delta\left(p\frac{a(t)}{a(t')}, t'\right)$$



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what ***properties*** naturally give rise to such deposits?

If the dark sector contains an ensemble of states with different masses, then these deposits arise naturally from **intra-ensemble decays** (decays ***within*** dark sector)

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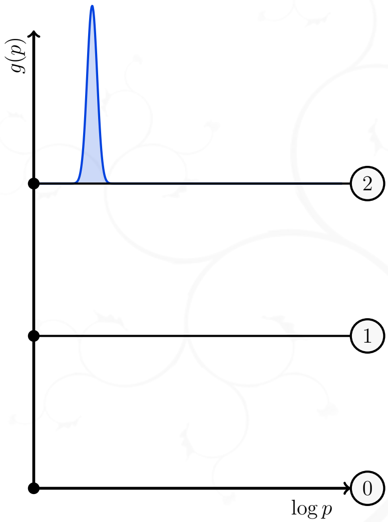

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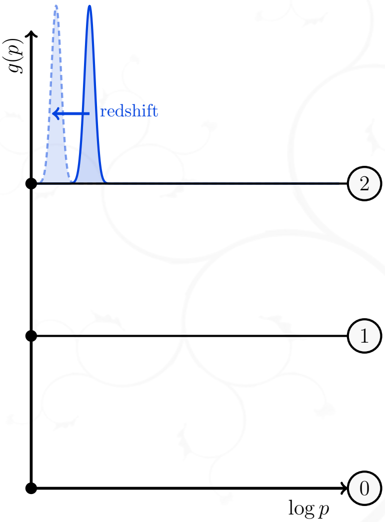

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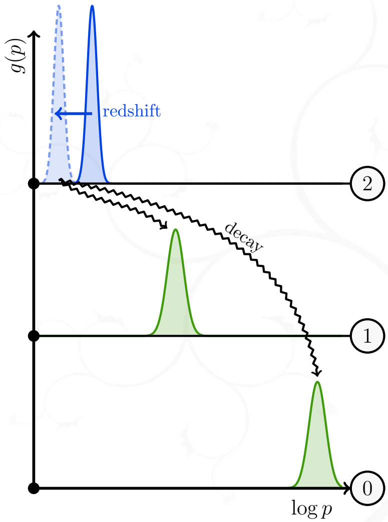

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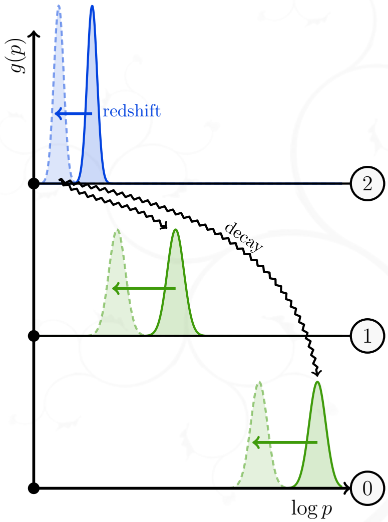

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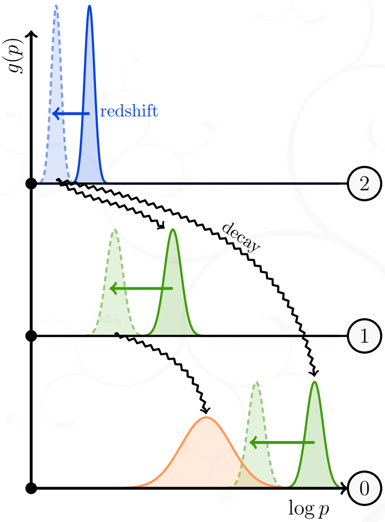

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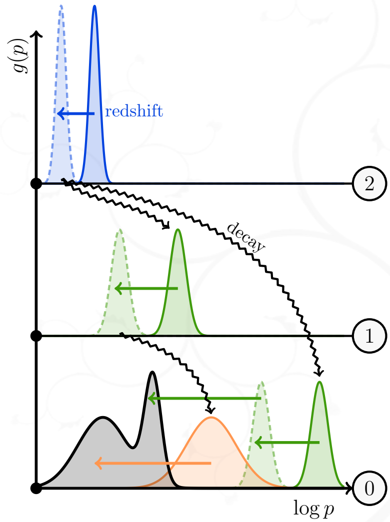

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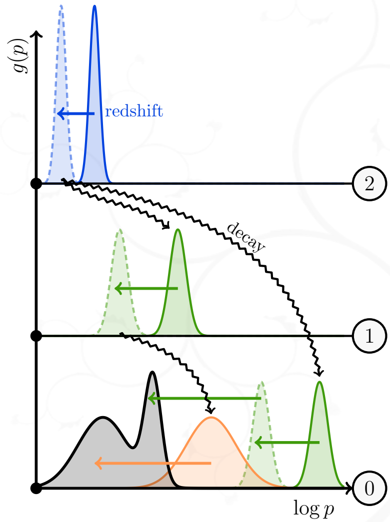

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but what *precisely* sets the detailed **shape** of each packet?

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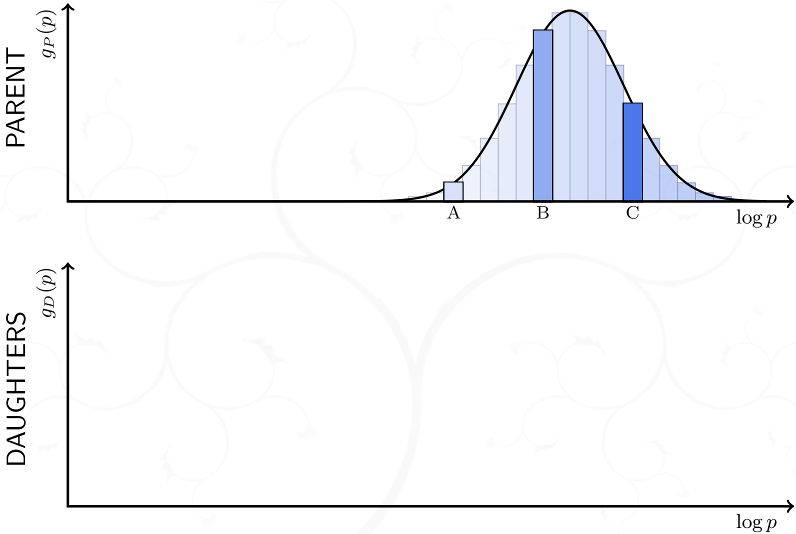

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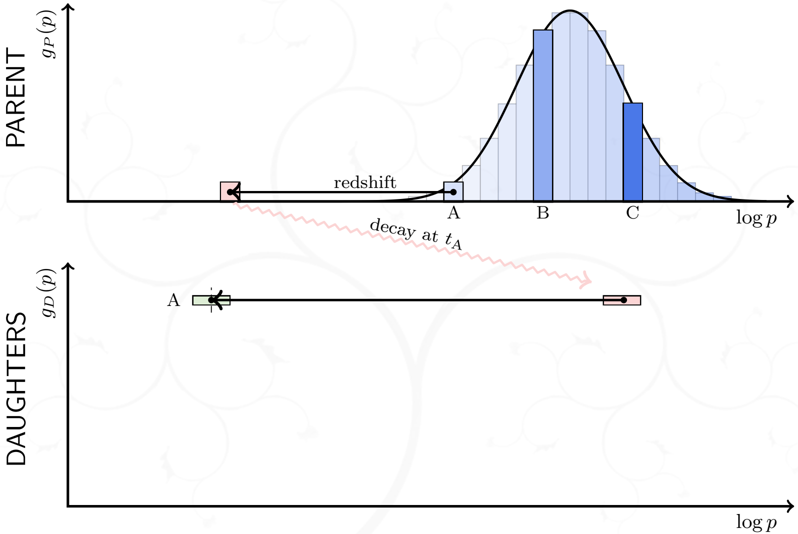

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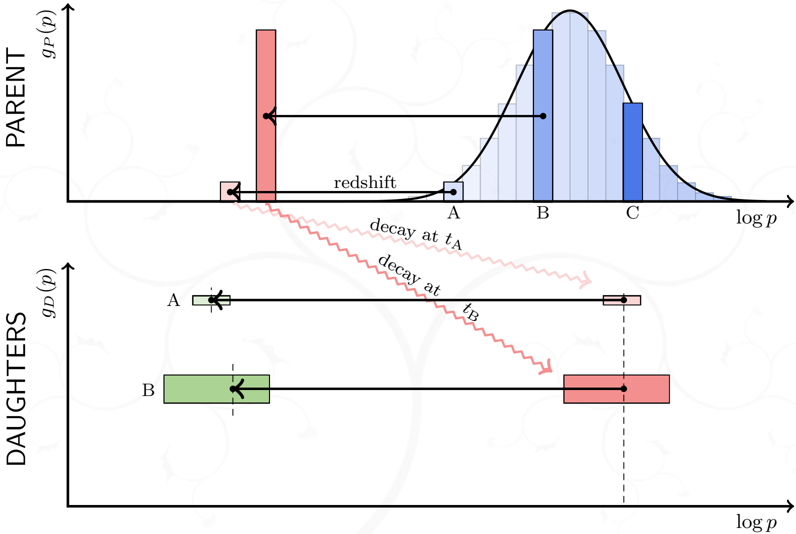

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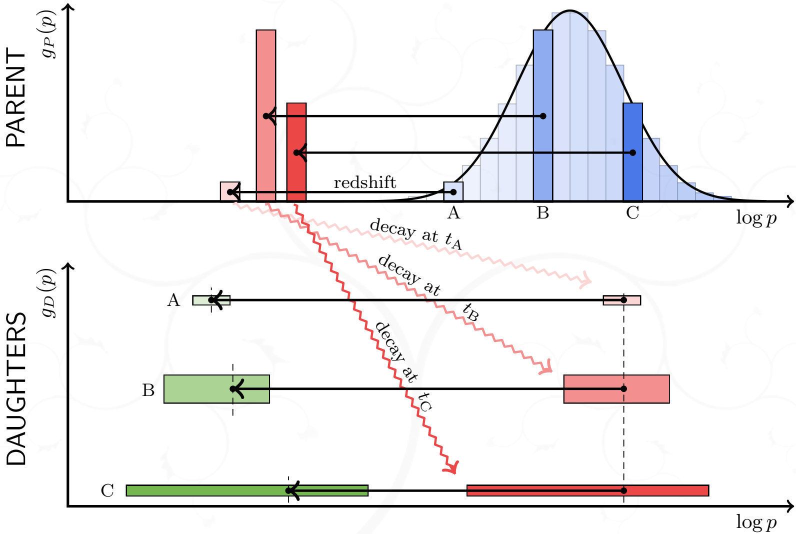

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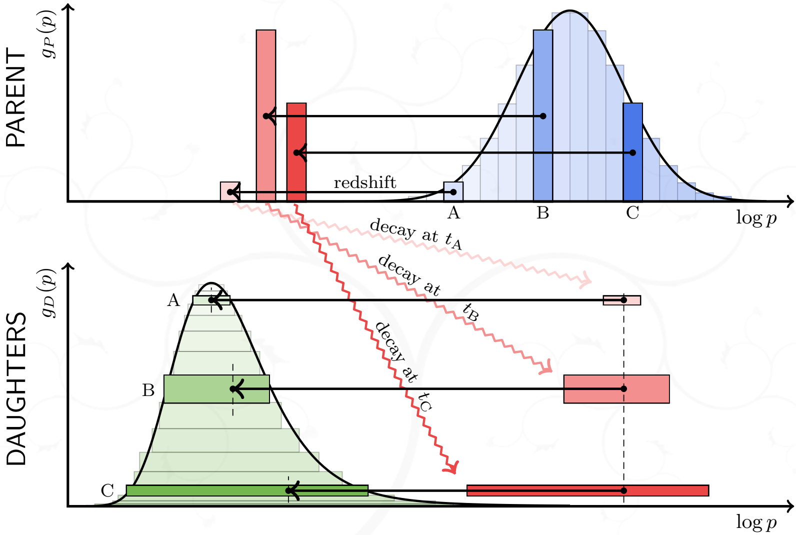

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***leftward* tilt** (positive skew) ⟶ **relativistic** at production

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We could have a narrow daughter packet (*i.e.*, $\Delta p \ll m$ and $\Delta p \ll \langle p \rangle$) with a parent packet that is either

- relativistic with a close-to-marginal decay
- *non*-relativistic with a far-from-marginal decay

but the **tilt/skewness allows us to to distinguish**

Ⓘ Early Dynamics $\longrightarrow$ DM Momentum Distributions

- We can go even further and map out all of the correlations:

<table>
<tr><th colspan="4">Daughter packet</th><th colspan="2">Parent packet</th><th>Decay</th><th>Decay near</th></tr>
<tr><th>rel?
(max p)</th><th>tilt</th><th>width
$\Delta p/m$</th><th>relative width
$\Delta p/\langle p\rangle$</th><th>rel at
production?</th><th>rel at
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</table>

and even apply these to the **constituent parts** of multi-modal distributions.

Ⓘ Early Dynamics ⟶ DM Momentum Distributions

• To verify that these features appear we need to (numerically) solve the Boltzmann system:

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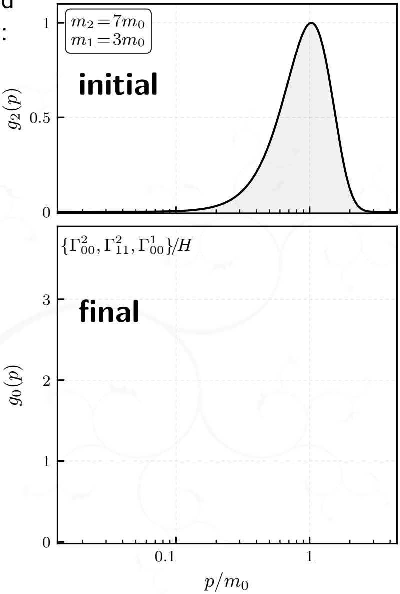

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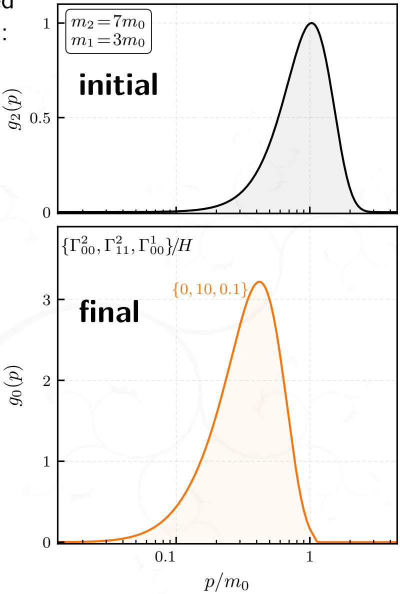

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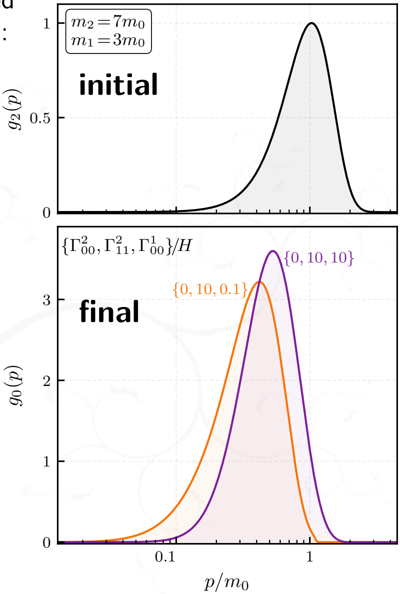

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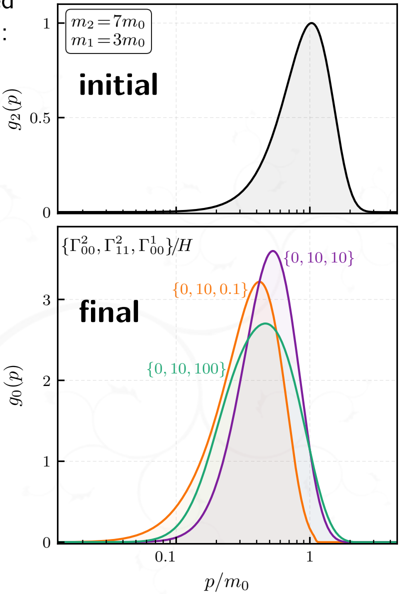

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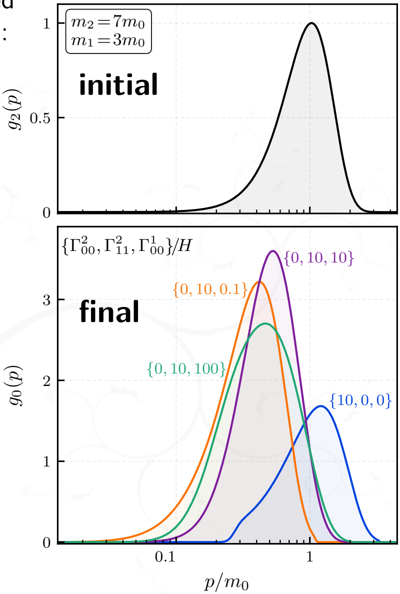

(I) Early Dynamics $\longrightarrow$ DM Momentum Distributions

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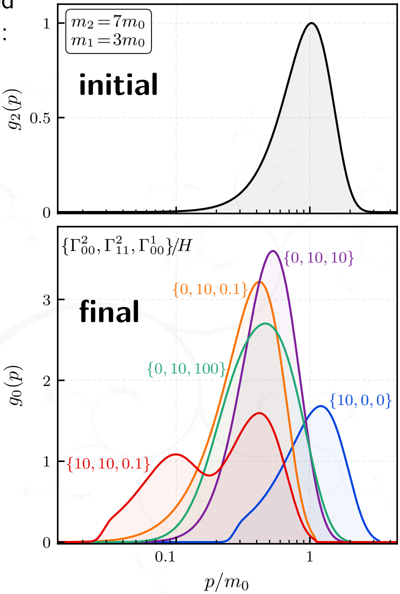

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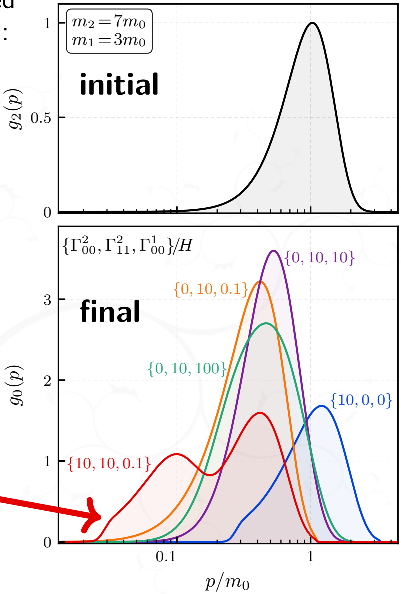
In the only case with ***competing*** decay chains we find **multi-modal distributions are produced**.



PART II

DM phase-space distribution $\longrightarrow$ matter power spectrum

INITIAL CONDITIONS
(primordial perturbations)

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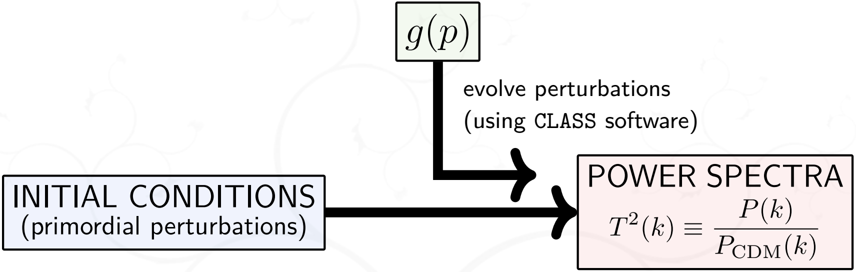
→

POWER SPECTRA

$$T^2(k) \equiv \frac{P(k)}{P_{\rm CDM}(k)}$$


$g(p)$
evolve perturbations
(using CLASS software)
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II Momentum Distributions ⟶ Matter Power Spectra

II Momentum Distributions $\longrightarrow$ Matter Power Spectra

- (Cold) **dark matter** drives the growth of **structure**

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$$k_{\rm FSH}^{-1} \equiv \int_{t_{\rm prod}}^{t_{\rm now}} dt \frac{\langle v(t)\rangle}{a(t)}$$

as a benchmark for the scale below which structure is suppressed.

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We'll consider a different approach...

Our Approach:

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- We begin by considering *momentum slices* through the distribution:

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which retains $\mathcal{N} = \int d\log p\, g(p) = \int d\log k\, \widetilde{g}(k)$.

II Momentum Distributions $\longrightarrow$ Matter Power Spectra

we are finally equipped to ask:

Can we conjecture the ***relationship***

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let's do a bit of exploring...

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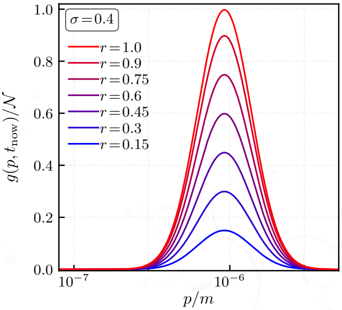
- We **vary the fraction** of dark matter abundance $r \equiv \Omega/\Omega_{\rm DM}$ carried by $g(p)$ and assume that the rest is pure CDM.



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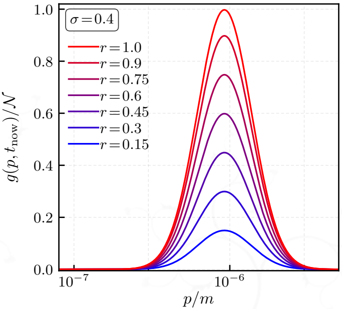

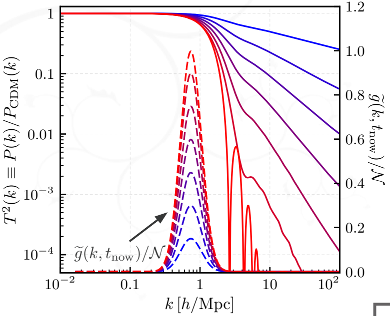

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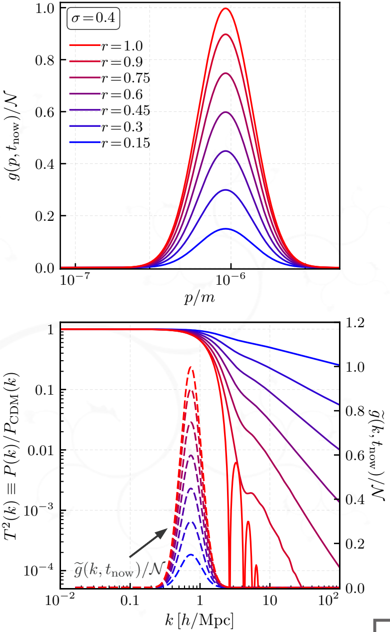

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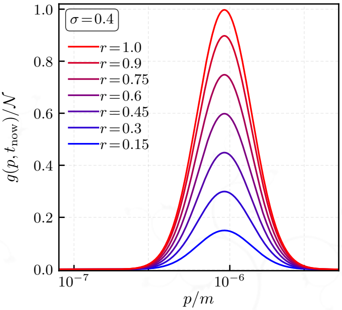

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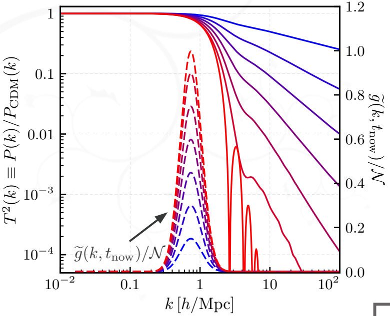

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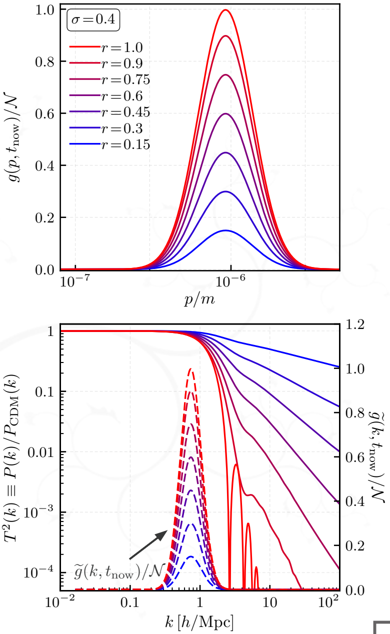

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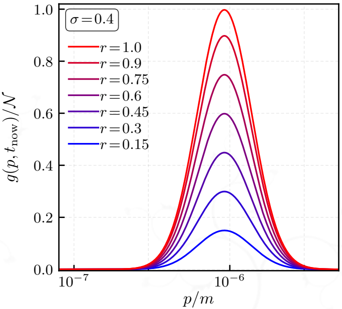

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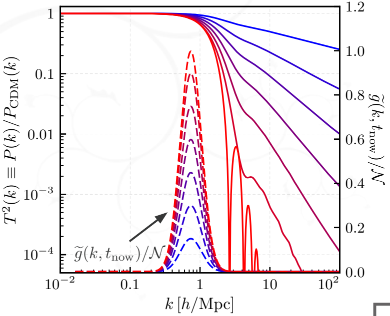
- acoustic oscillations begin to show as $\widetilde{g}(k)$ carries close to full DM abundance



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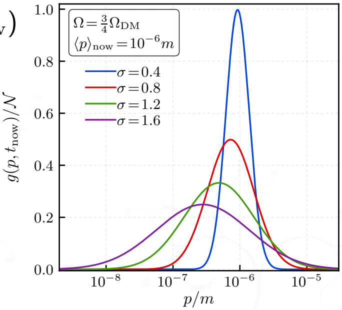

(II) Momentum Distributions $\longrightarrow$ Matter Power Spectra

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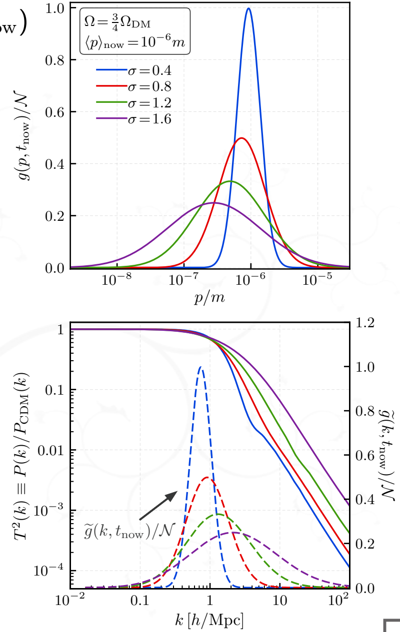

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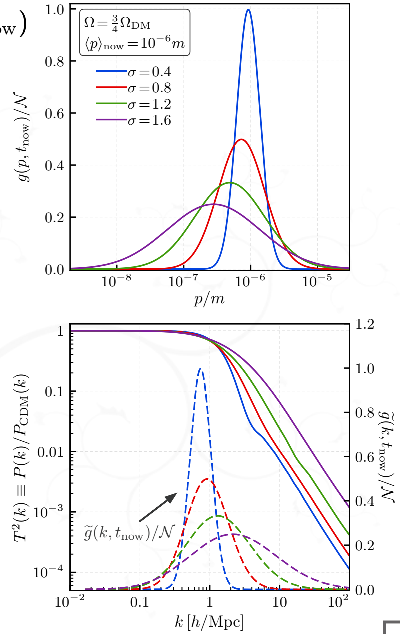

II Momentum Distributions $\longrightarrow$ Matter Power Spectra

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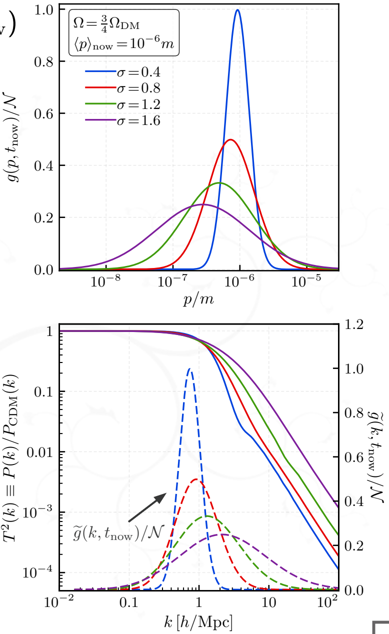
- as we widen the distribution:
  - slope of $T^2(k)$ changes more slowly



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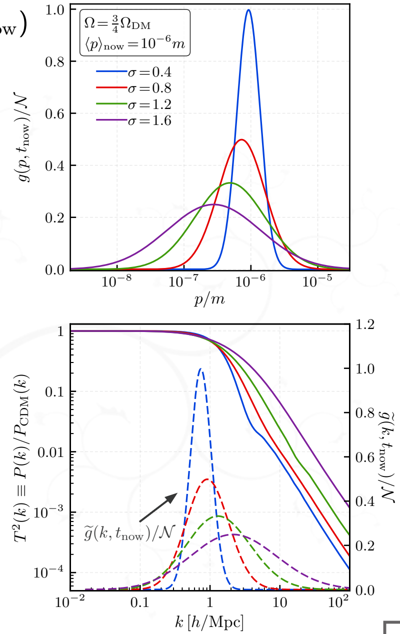

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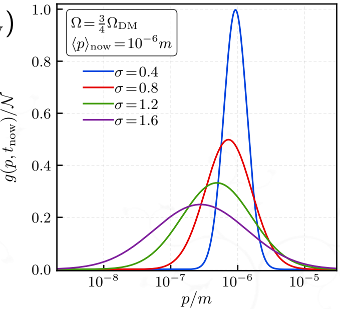

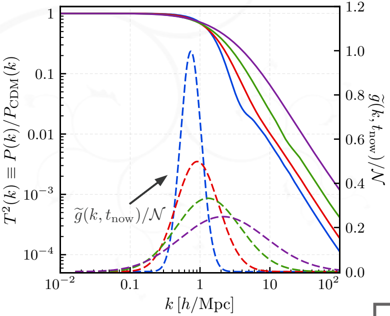

II Momentum Distributions $\longrightarrow$ Matter Power Spectra

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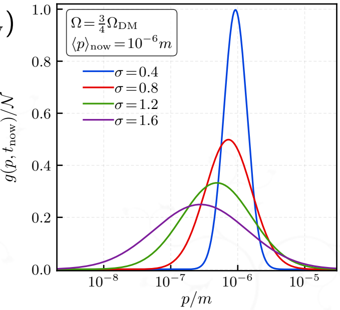

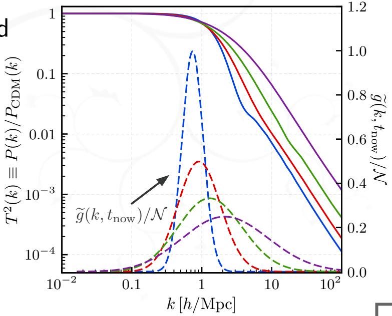

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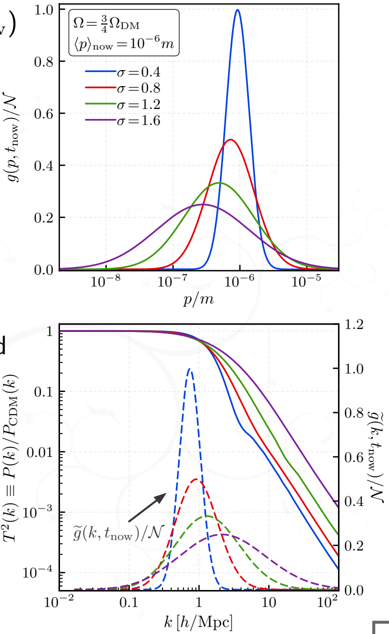
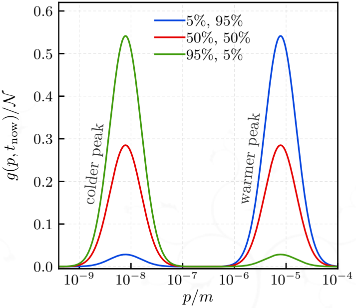
> $\widetilde{g}(k)$ abundance correlates **not** with suppression of $T^2(k)$ but with its ***slope***.



II Momentum Distributions $\longrightarrow$ Matter Power Spectra

- Do these observations survive for a more complicated $g(p)$ distribution?
- Let's examine two peaks and vary their *relative* abundances.



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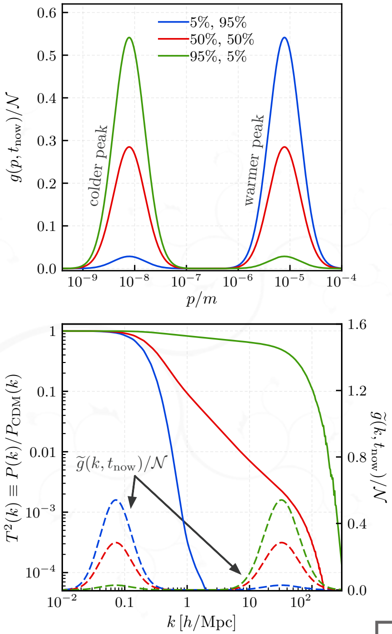

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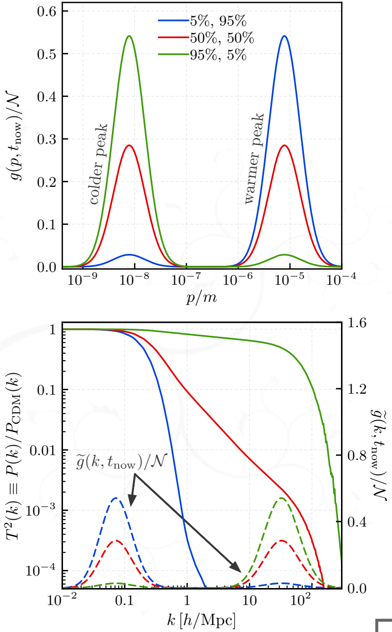

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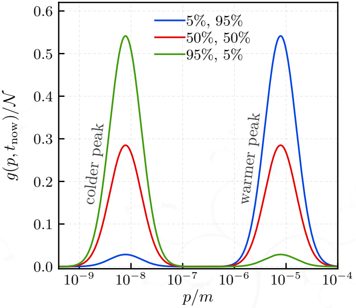

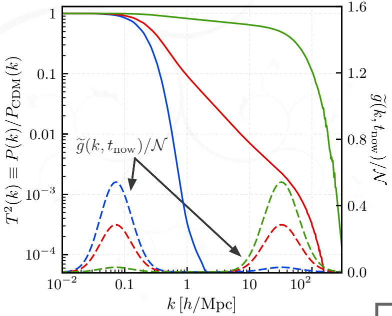

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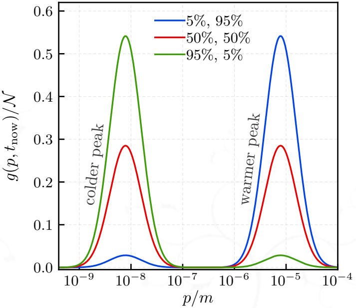

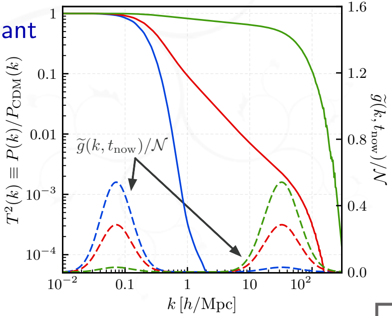

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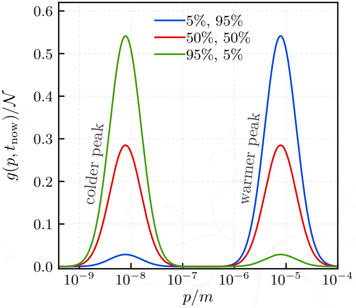

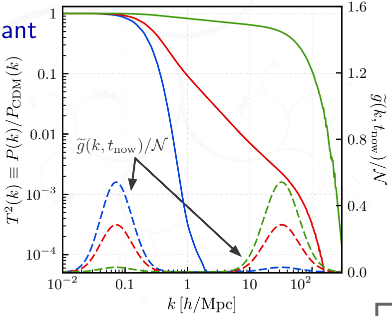

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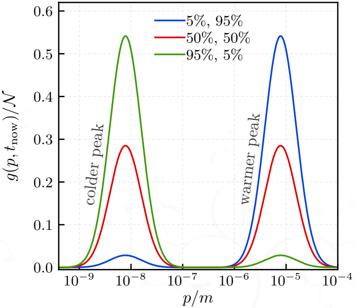
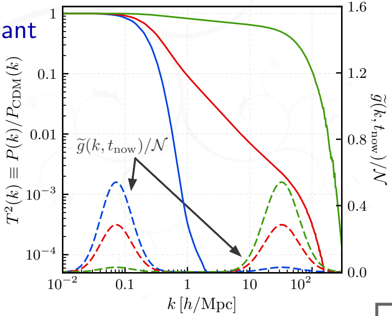
Next, let's quantify these observations....





PART III

The “Archaeological” Inverse Problem

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- At any particular k the accumulated abundance is

$$F(k) \equiv \frac{\int_{-\infty}^{\log k} \widetilde{g}(k')d\log k'}{\int_{-\infty}^{+\infty} \widetilde{g}(k')d\log k'} ,$$

or equivalently the fraction of our DM which is effectively "hot" (*i.e.*, free-streaming).

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phase-space distribution

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$\widetilde{g}(k)/\mathcal{N}$: phase-space distribution

$\left|\frac{d\log T^2}{d\log k}\right|$: transfer function **slope**

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$\widetilde{g}(k)/\mathcal{N}$: phase-space distribution; $\frac{d\log T^2}{d\log k}$: transfer function **slope**; $\frac{d^2\log T^2}{(d\log k)^2}$: transfer function **curvature**

III The "Archaeological" Inverse Problem

- Using our earlier results we can implicitly determine the function η:

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A technical aside:

Our conjecture has a built-in assumption that $d^2\log T^2(k)/(d\log k)^2$ is negative-semidefinite. This tends to cover cases in which $\widetilde{g}(k)$ is relatively **"clustered,"** regardless of the complexity of its shape.

An Illustrative Model of Multi-Component Decay Chains

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- Consider a model with $N+1$ real scalars $\{\phi_0, \phi_1, \ldots \phi_N\}$

with a mass spectrum

$$m_\ell \ = \ m_0 + \ell^\delta \Delta m$$

and Lagrangian

$$\mathcal{L} \ = \ \sum_{\ell=0}^{N} \left(\frac{1}{2}\partial_\mu \phi_\ell \partial^\mu \phi_\ell - \frac{1}{2} m_\ell^2 \phi_\ell^2 - \sum_{i=0}^{\ell} \sum_{j=0}^{i} c_{\ell i j} \phi_\ell \phi_i \phi_j \right) + \cdots$$

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$r > 0$: *maximally* exothermic decays

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fix parameters:

$N = 9$
$\delta = 1$
$\Delta m = 2m_0$
$\mu = 0.1 m_0$

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Illustrative Model of Multi-Component Decay Chains

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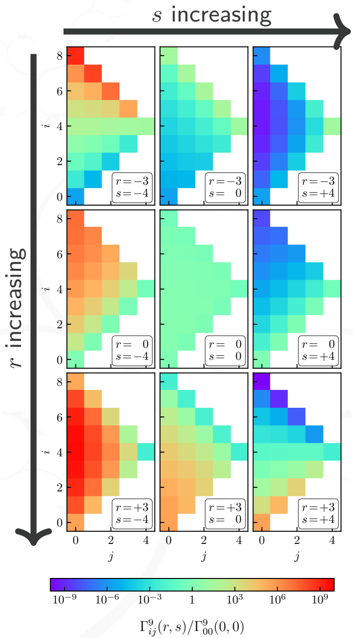
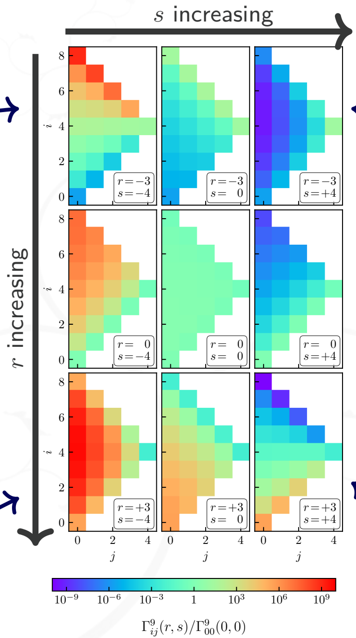


Illustrative Model of Multi-Component Decay Chains

tend to **minimize kinetic** energy and **favor asymmetry**

partial widths for $\phi_9 \longrightarrow \phi_i\phi_j \ (i \geq j)$

tend to produce **light states** and **favor asymmetry**



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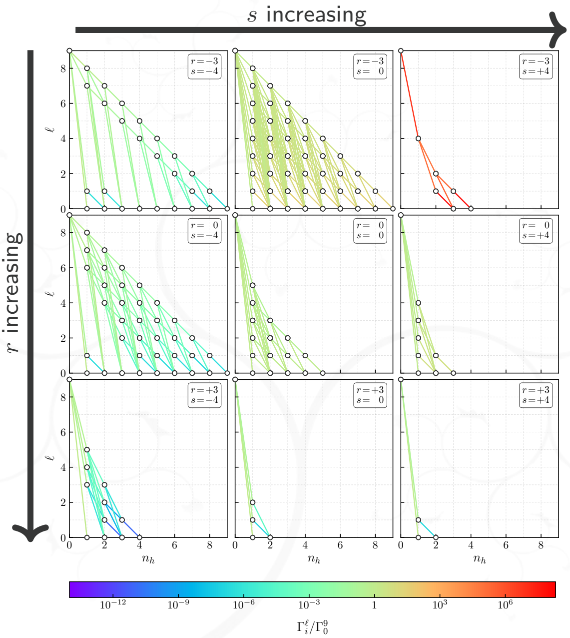

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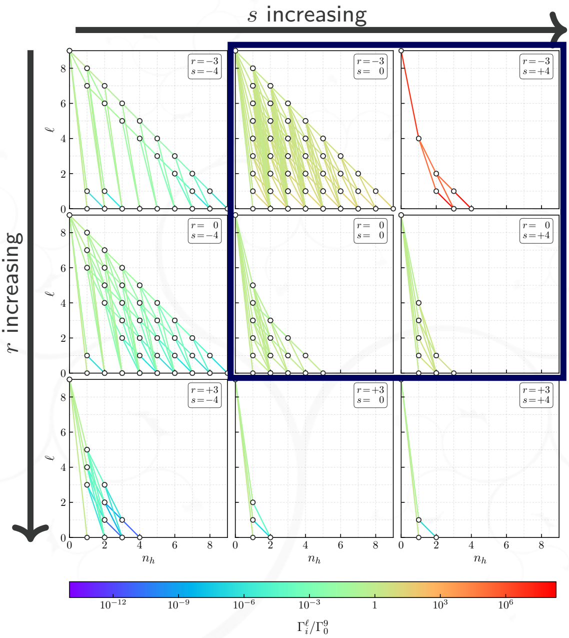

decays to lightest state occur over **similar times**

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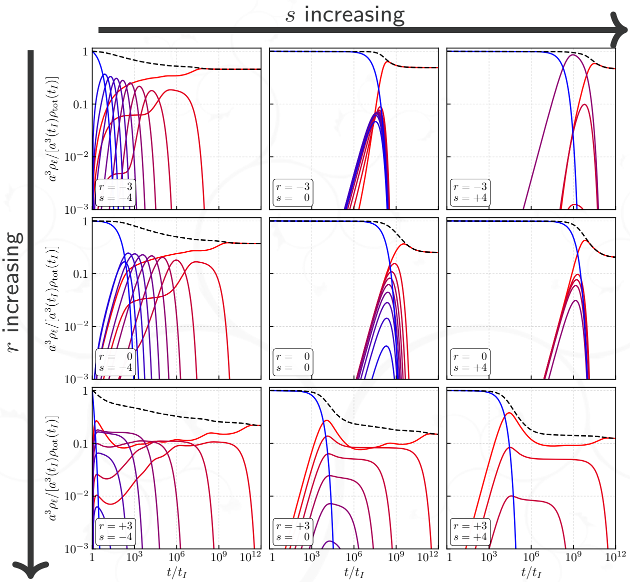

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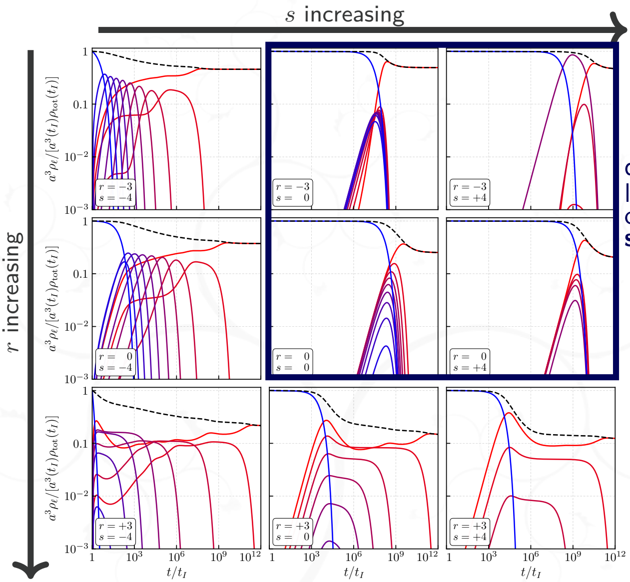

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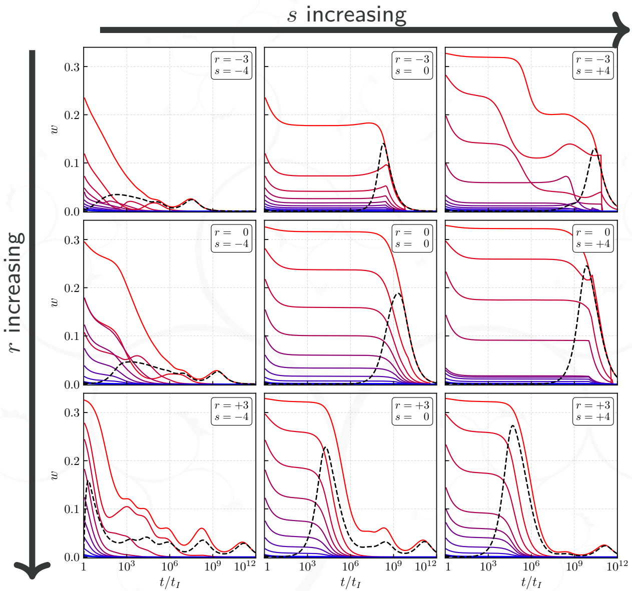

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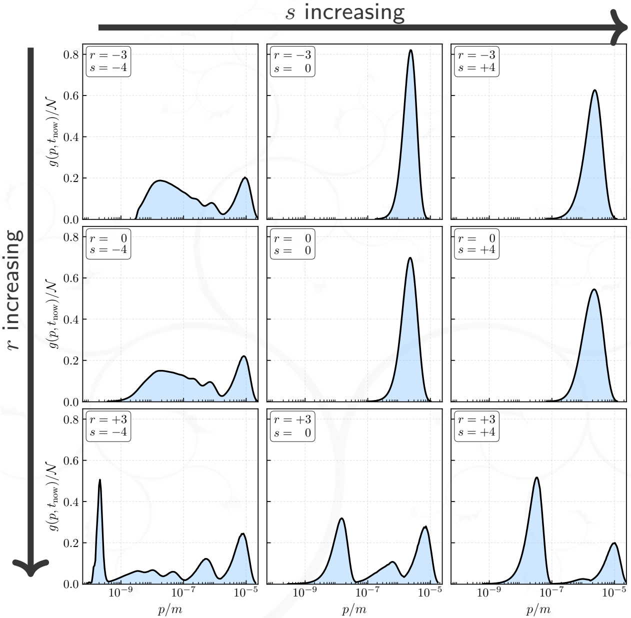

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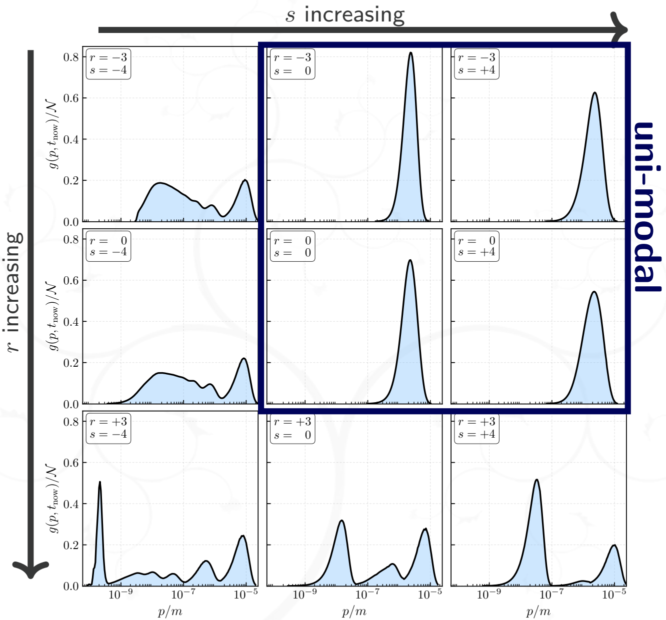

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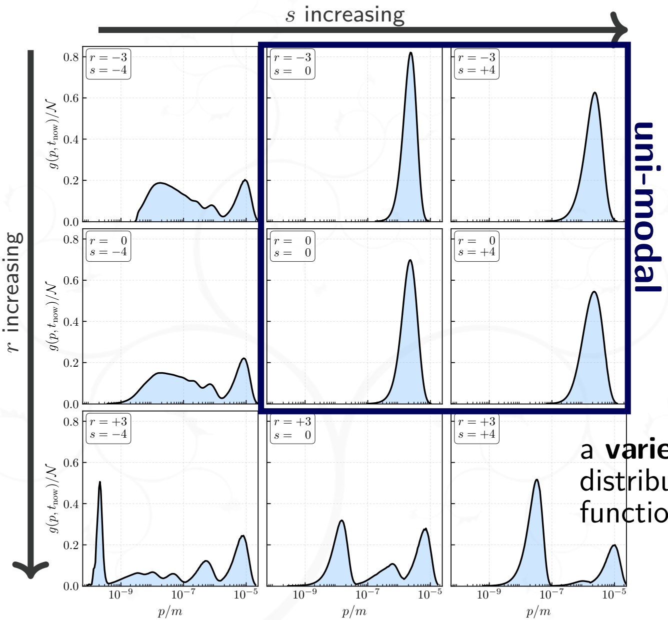
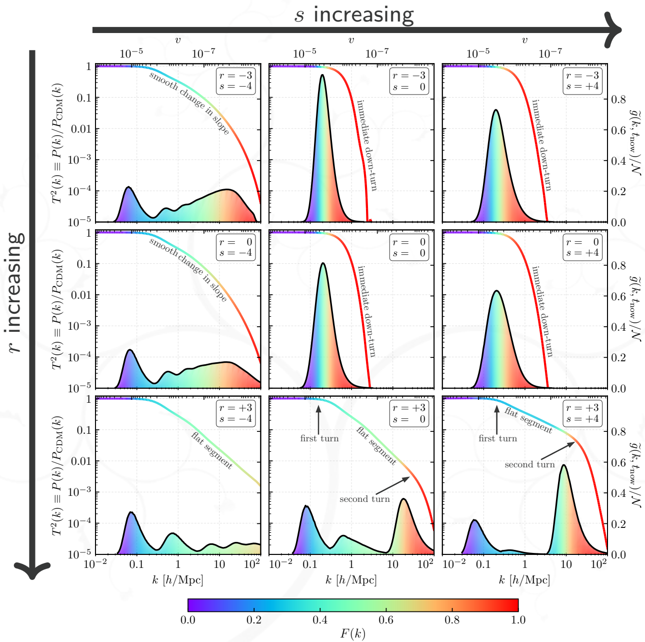


a **variety** of distribution functions emerge!

Illustrative Model of Multi-Component Decay Chains



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Recall our conjecture:

$$\frac{\widetilde{g}(k)}{\mathcal{N}} \approx \frac{1}{2}\left(\frac{9}{16}+\left|\frac{d\log T^2}{d\log k}\right|\right)^{-1/2}\left|\frac{d^2\log T^2}{(d\log k)^2}\right|$$

What features can we “resurrect” from this relation?

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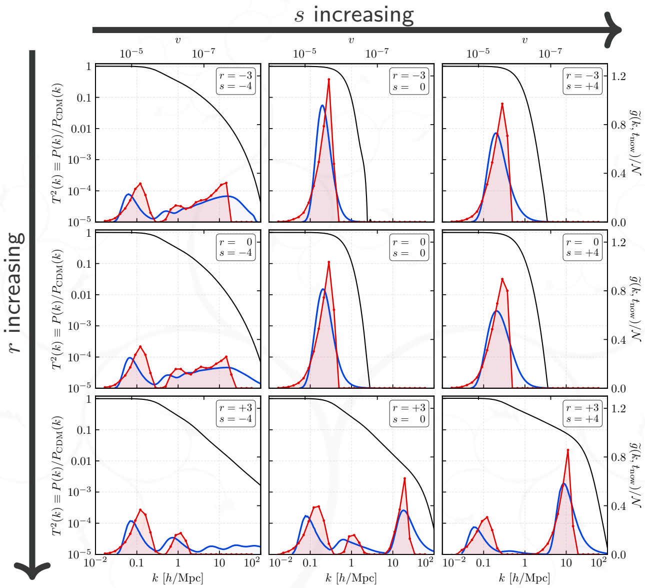

CONCLUSIONS

- Early-universe processes such as decays within the dark sector can leave identifiable imprints in $f(p)$ and $P(k)$; certain features may allow us to go backwards and archaeologically reconstruct the dark-matter distribution.
  - We found useful analytical tools, such as hot-fraction function $F(k)$.
  - Conjectured relation that can "resurrect" $f(p)$ features from $P(k)$.
- The dark sectors of string theory generically include unstable KK towers similar to the form we have discussed here, leading to multi-modal $f(p)$ distributions and non-trivial $P(k)$ spectra.
- Such approaches may be *only* probes for dark sector decoupled from SM.

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FUTURE WORK/DIRECTIONS:

- How to incorporate effects that come from SM couplings? Could affect evolution of phase-space distributions in some additional subtle ways.
- Incorporation of observational bounds/constraints (Lyman-α, *etc.*)
- How do these $T^2(k)$ fall within effective theories of structure formation?
- Addressing the non-linear regime...

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THANK YOU FOR YOUR ATTENTION!