Fermilab U.S. DEPARTMENT OF ENERGY Office of Science

Aspects of neutrino masses

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Madison seminar

20 November 2019

Outline

- Neutrino masses and mixing
- The baryon asymmetry of our Universe and leptogenesis
- Relating neutrino masses to the BAU and EW scale
- 1905.12642
- Neutrino masses from gravity: the Schwinger Dyson approach
- Two ways of solving the SDEs

1909.04675

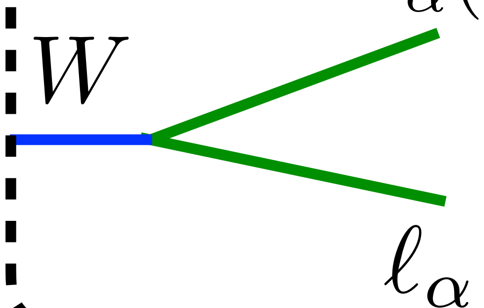
Neutrino Oscillations

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

flavour states — PMNS matrix — mass states

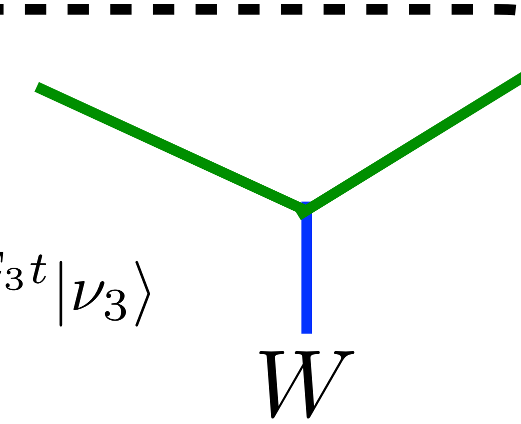
$$U m_\nu U^\dagger = m_{\nu \mathrm{diag}}$$

$$U_{PMNS} = U_e^\dagger U_\nu$$



$\nu_\alpha(t_0) \longrightarrow \nu_\beta(t)$

$$|\nu(t)\rangle = e^{-i\mathcal{H}t}|\nu(0)\rangle = \alpha e^{-iE_1 t}|\nu_1\rangle + \beta e^{-iE_2 t}|\nu_2\rangle + \gamma e^{-iE_3 t}|\nu_3\rangle$$



$$E_i \simeq E + \frac{m_i^2}{E} \implies E_i - E_j \simeq \frac{\Delta m_{ij}^2}{2E}$$

In the simplified two neutrino case:

$$P(\nu_\alpha \to \nu_\beta) = \sin^2(2\theta)\sin^2\frac{(\Delta m_{ij}^2 L)}{2E}$$

mass splitting

baseline distance between production and detection

mixing angle parametrises misalignment of bases

Energy of neutrino

- Neutrinos have (non-degenerate) masses
- Neutrinos mix i.e. PMNS matrix is a non-identity matrix

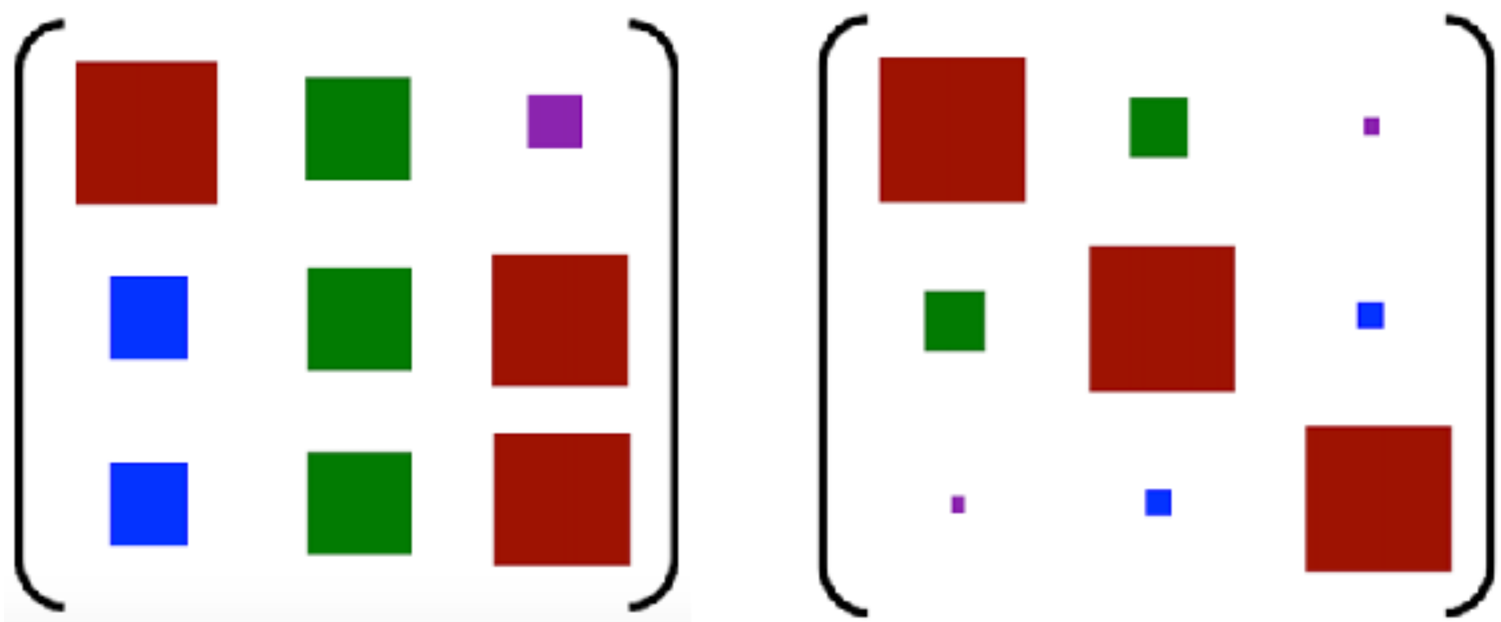
- If neutrinos Dirac fermions, PMNS: 3 mixing angles + 1 phase

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$41.1 \leq \theta_{23}(^\circ) \leq 51.3 \qquad 8.22 \leq \theta_{13}(^\circ) \leq 8.98 \qquad 31.61 \leq \theta_{12}(^\circ) \leq 36.27$$

$$144 \leq \delta(^\circ) \leq 357$$

nu-fit data 4.1



leptonic mixing **quark mixing**

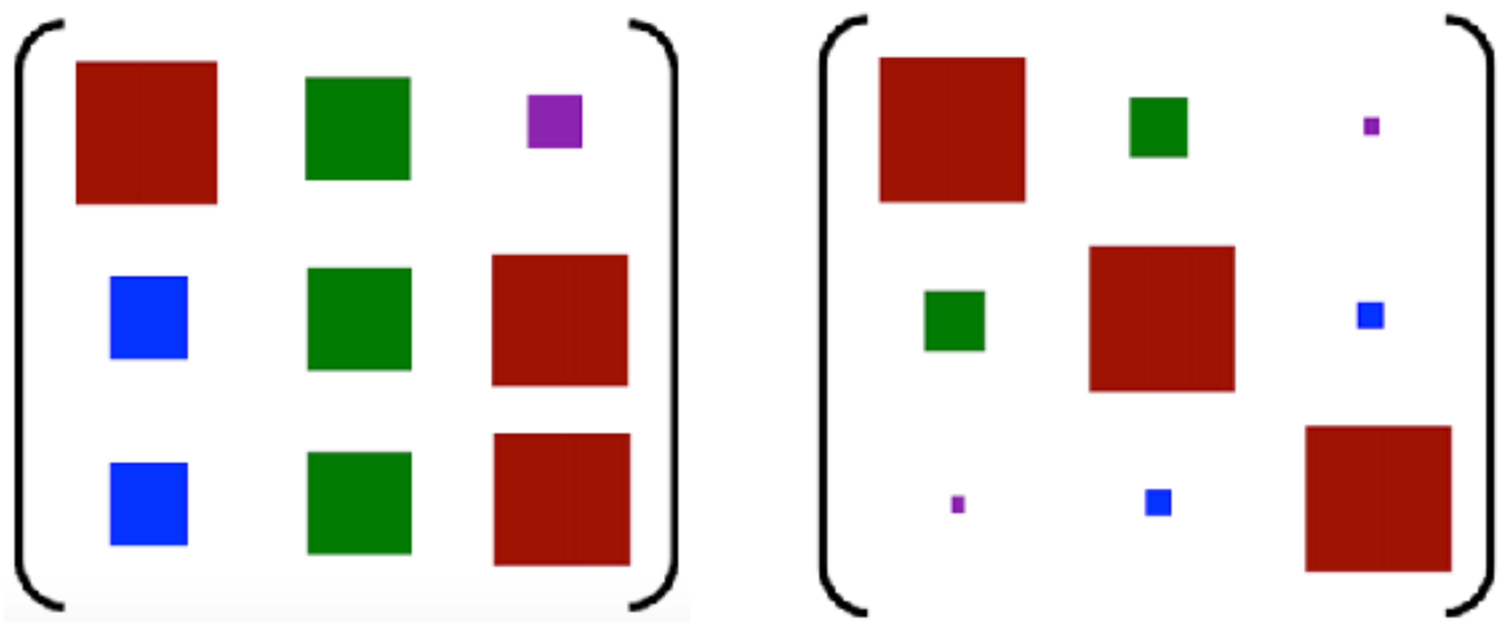
- Neutrinos have (non-degenerate) masses
- Neutrinos mix i.e. PMNS matrix is a non-identity matrix

- If neutrinos Majorana fermions, PMNS: 3 mixing angles + 3 phase

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

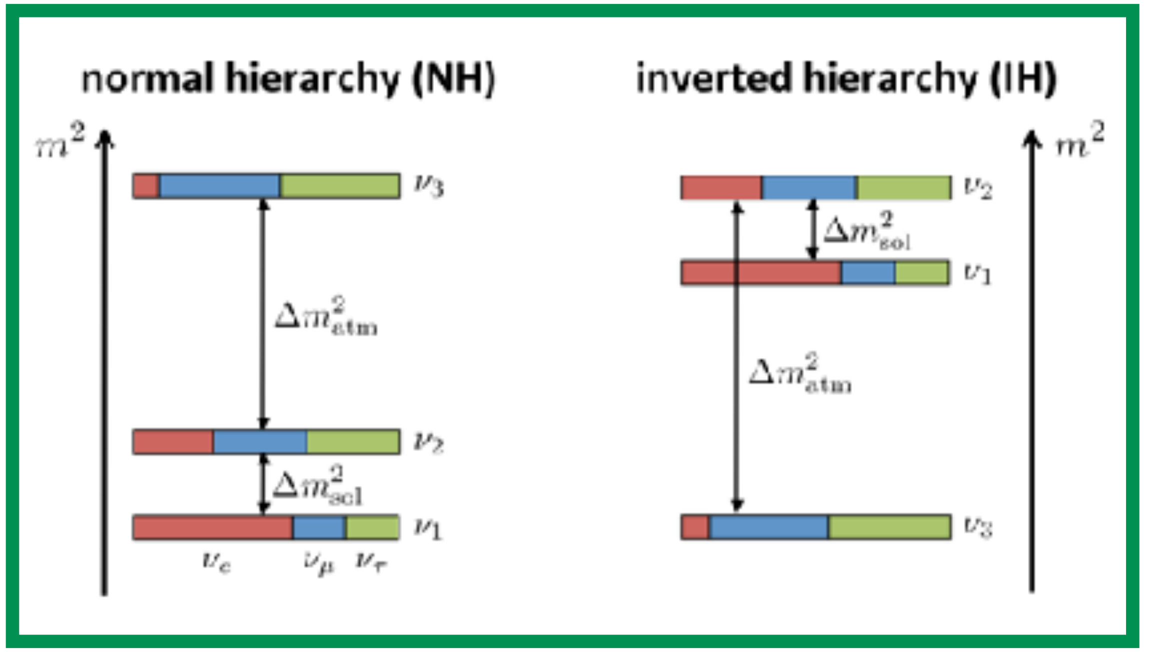
- Majorana nature of neutrinos not observable at oscillation experiments

nu-fit data 4.1



leptonic mixing **quark mixing**

- **How are the masses ordered?**
- **What are the precise values of the mixing angles?**
- **Is leptonic CP maximally violated?**



- **Are neutrinos Dirac or Majorana fermions?**

- **What is the mass of the lightest neutrino?**

$\sum m_\nu \lesssim 0.2\,\mathrm{eV}$ **cosmology, many groups see** PDG

$\sum m_\nu \leq 1.1\,\mathrm{eV}$ **recent measurement by KATRIN** 1909.06048

We do know neutrinos are significantly lighter than other SM fermions

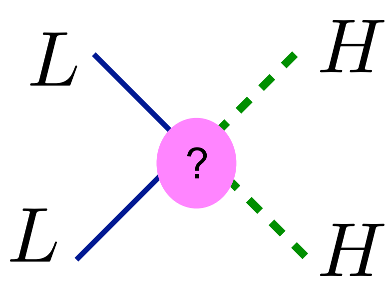
Relating neutrino masses to the baryon asymmetry of the Universe

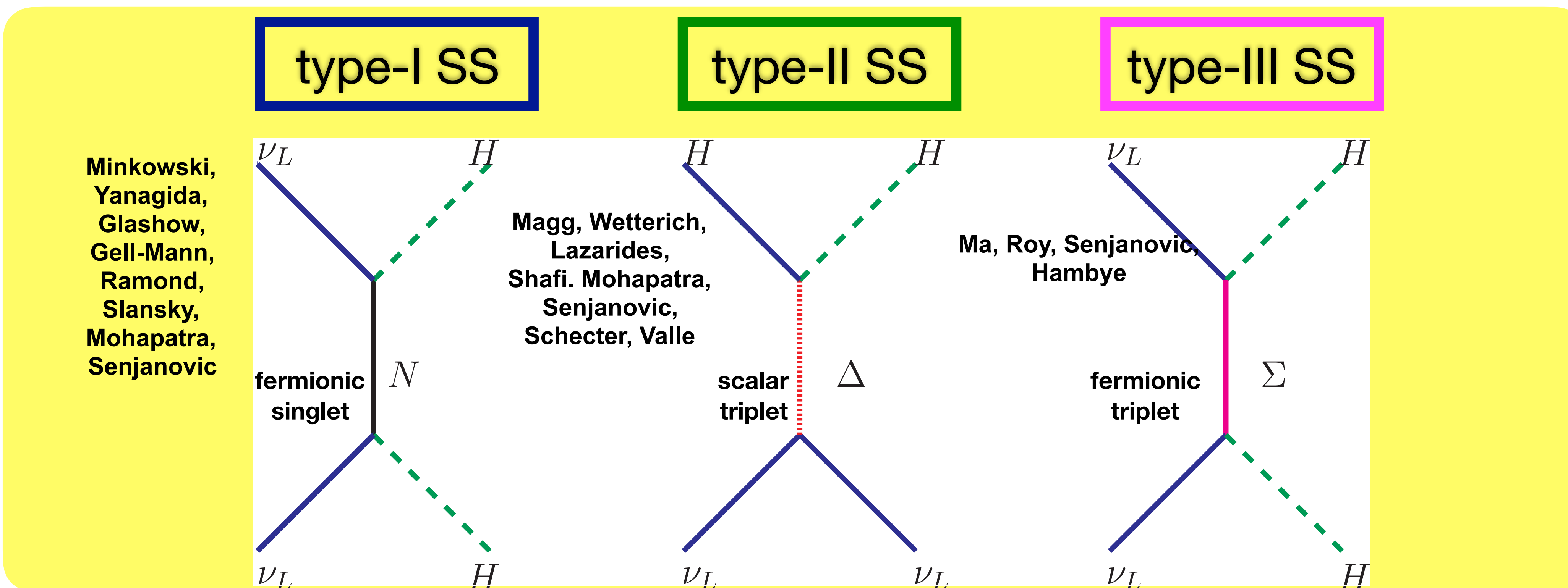
Leptogenesis: Motivation

Most theories of leptogenesis assume neutrinos are Majorana fermions*

$$-\mathcal{L}_{D=5} = \lambda \frac{L.HL.H}{M} = \frac{\lambda v^2}{M} \nu_L^T C^\dagger \nu_L$$

lepton number violating





$$\mathcal{L} = -Y_\nu \overline{N} L H - \frac{1}{2} \overline{N^C} M_N N$$

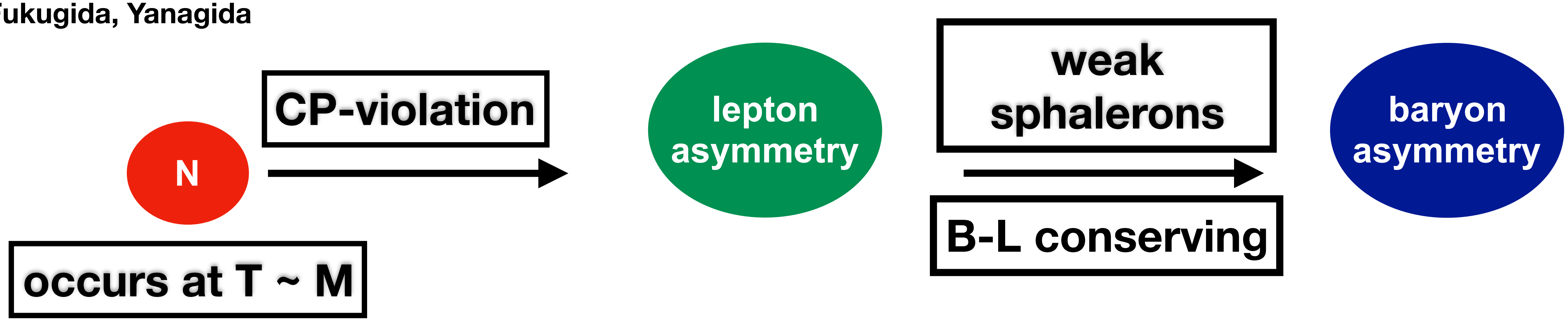
$$\begin{pmatrix} 0 & m_D \\ m_D^T & M_N \end{pmatrix}$$

$$m_\nu = \frac{Y_\nu^2 v^2}{M_N} \sim 0.1\,\mathrm{eV}$$

*there are exceptions (Dirac leptogenesis) Dick, Lindner, Wright, Ratz

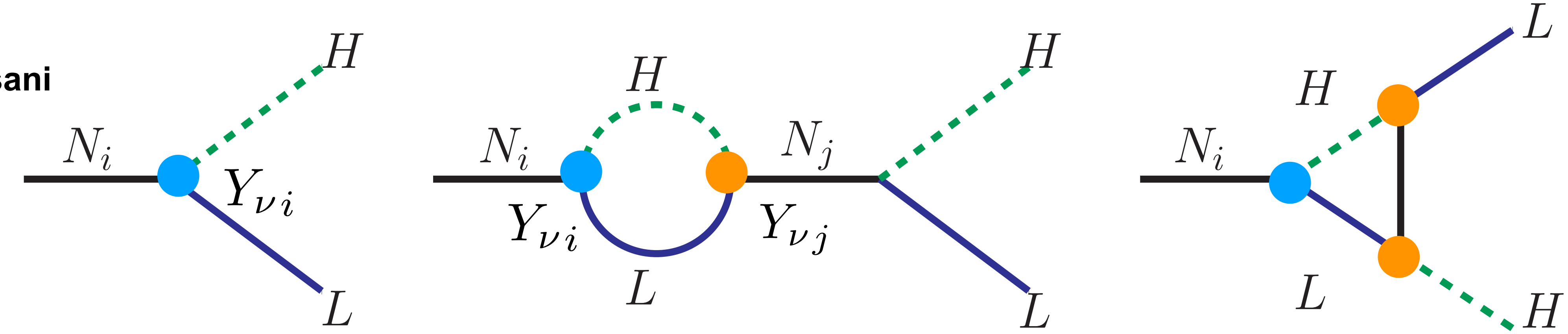
Leptogenesis via Decays

Fukugida, Yanagida



Decay asymmetry from interference between tree and loop level diagrams

Covi, Roulet, Vissani



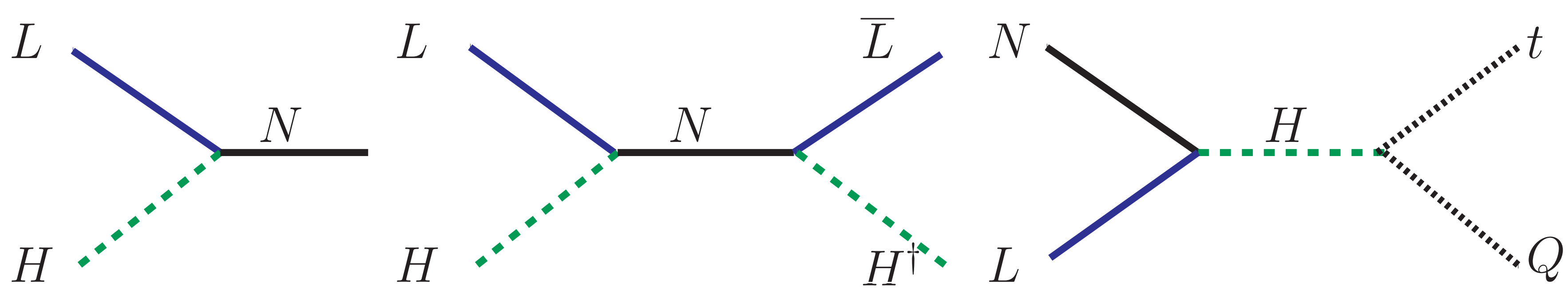
Decay Asymmetry

$$\epsilon_i = \frac{\Gamma_i - \overline{\Gamma}_i}{\Gamma_i + \overline{\Gamma}_i}$$

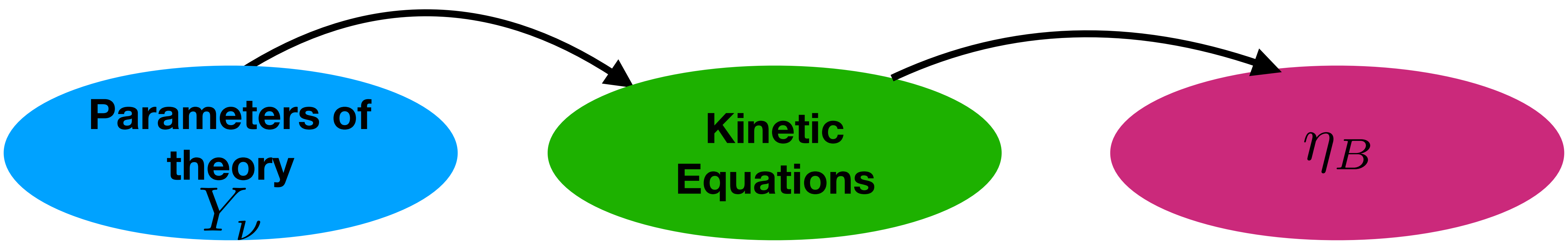
Calculation does not usually include finite density temperature effects

Basic Mechanism

Washout and Scattering processes



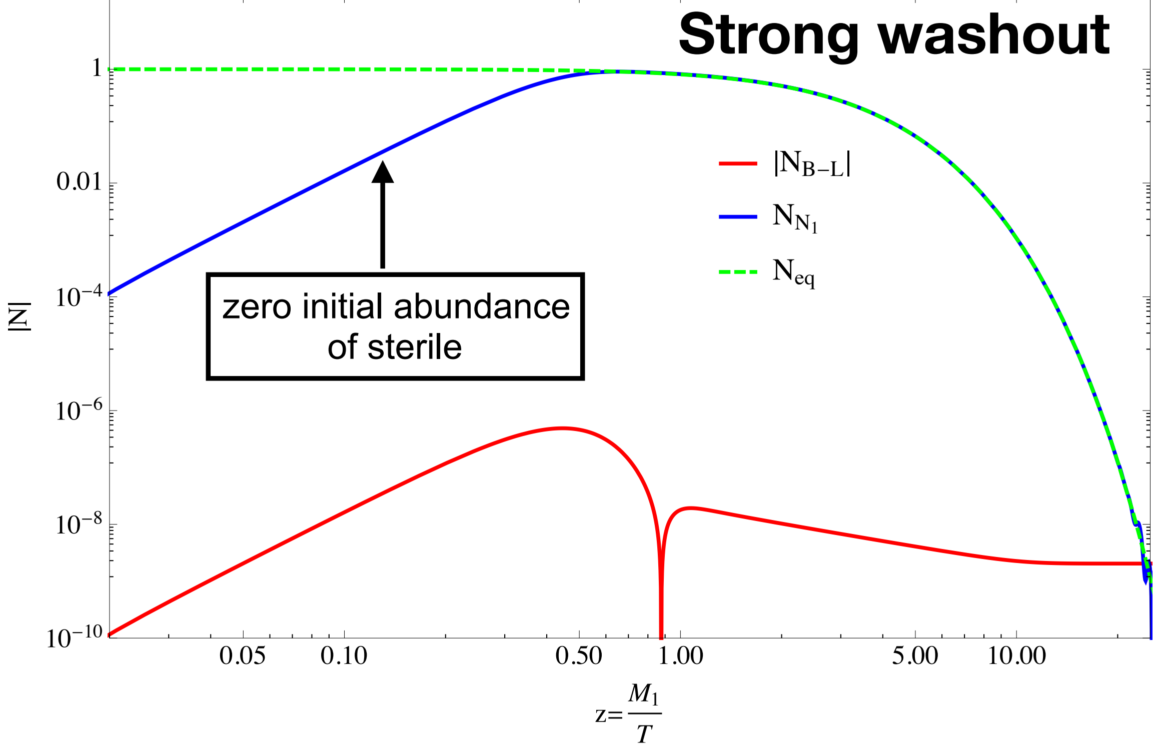
inverse Decays **ΔL=2** **ΔL=1**

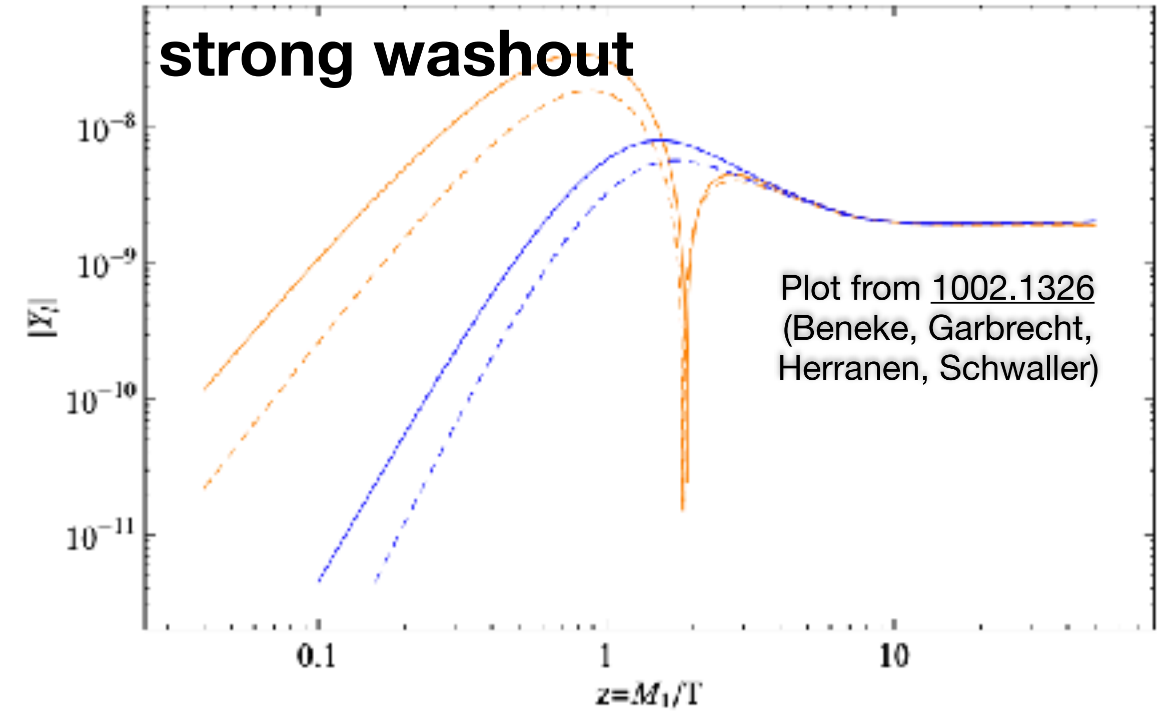


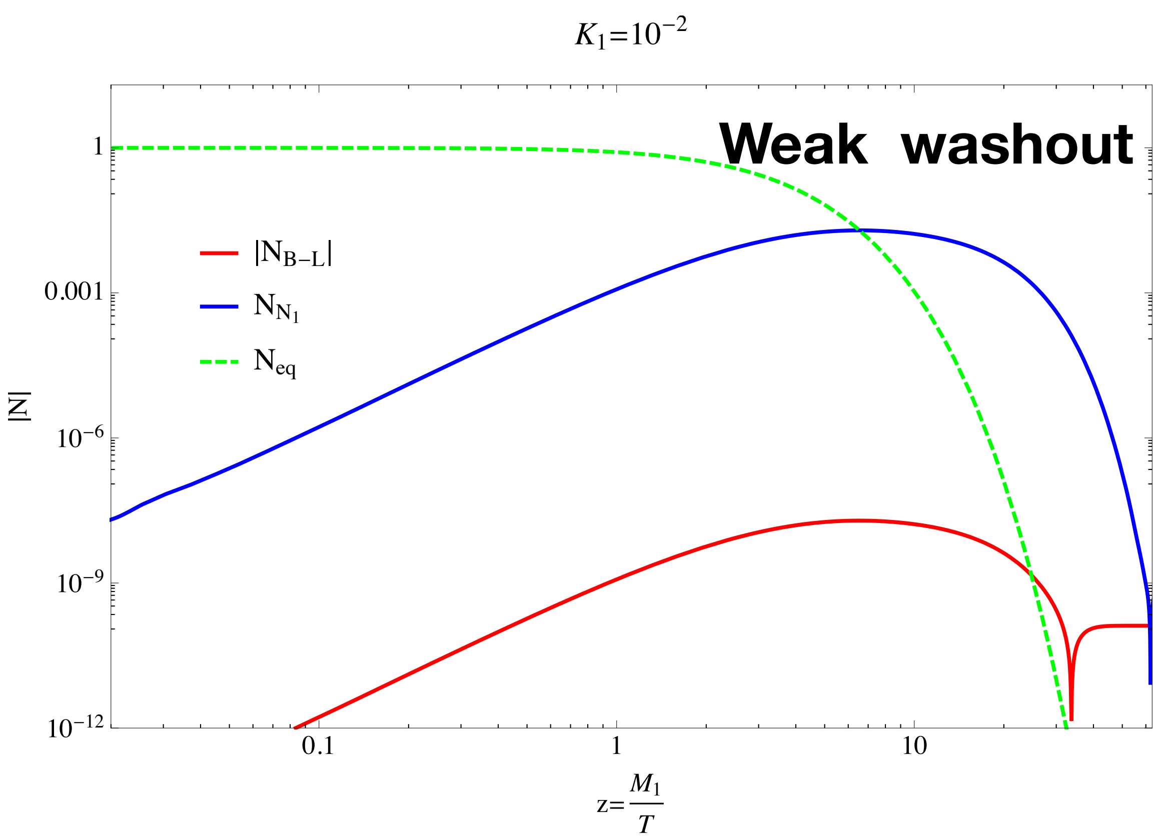
$$\frac{dn_{N_i}}{dz} = - D_i (n_{N_i} - n_{N_i}^{\rm eq}),$$

source **sink**

$$\frac{dn_{B-L}}{dz} = \sum_{i=1}^{3} \left(\epsilon^{(i)} D_i (n_{N_i} - n_{N_i}^{\rm eq}) - W_i n_{B-L} \right) .$$

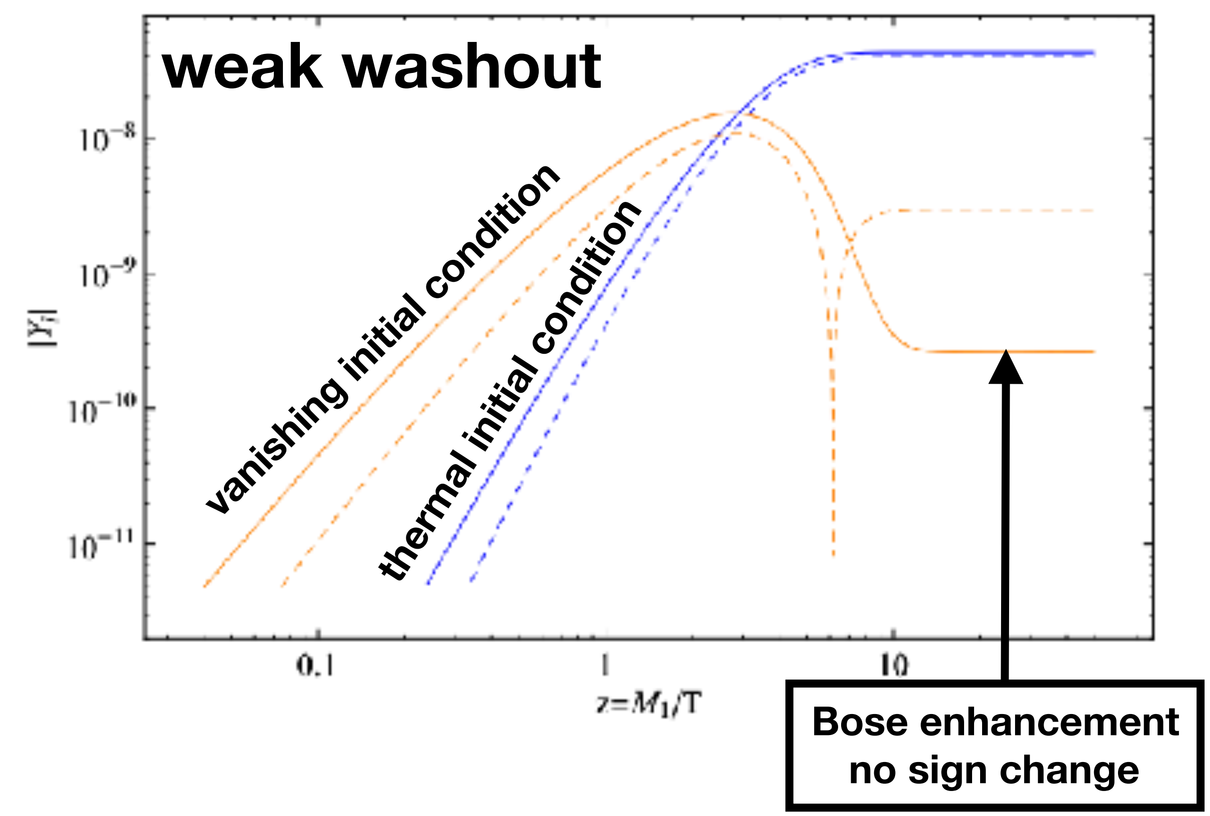






solid = finite density solution

dashed = no finite density effects



Much work has been done on improving the theoretical accuracy of leptogenesis calculations: CTP approach (Garbrecht etal, Drewes, Buchmuller, Anisimov, Mendizabal, Millington), thermal effects (Bodeker, Besak, Laine, Biondini, Brambilla, Vairo..) see 1711.02864

A Model Parameter Space

Casas, Ibarra

$$Y_\nu = \frac{1}{v} U_{\text{PMNS}} \sqrt{m} R^T \sqrt{M}$$

Neutrino Oscillation constraints
3 mixing angles
3 phases

sum of neutrino masses and neutrino oscillation
3 parameters

Complex, orthogonal matrix
6 real parameters

Unconstrained
3 masses
ensure at least mild hierarchy

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\omega_1} & s_{\omega_1} \\ 0 & -s_{\omega_1} & c_{\omega_1} \end{pmatrix} \begin{pmatrix} c_{\omega_2} & 0 & s_{\omega_2} \\ 0 & 1 & 0 \\ -s_{\omega_2} & 0 & c_{\omega_2} \end{pmatrix} \begin{pmatrix} c_{\omega_3} & s_{\omega_3} & 0 \\ -s_{\omega_3} & c_{\omega_3} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$c_{\omega_i} = \cos\omega_i \,, \; s_{\omega_i} = \sin\omega_i \,, \; \omega_i = x_i + i y_i$$

η_B is a function of up to 18 parameters.

Akmedov, Rubakov, Smirnov, Hernandez, Kekic, Lopez-Pavon, Racker, Salvado, Drewes, Garbcreht, Klaric, Gueter

Leptogenesis via oscillations

Flavour effects can lower the scale

Minimal Leptogenesis

~ eV   ~ 0.1 GeV   ~50 GeV   ~10^5 GeV   ~10^7 GeV   ~10^{14} GeV

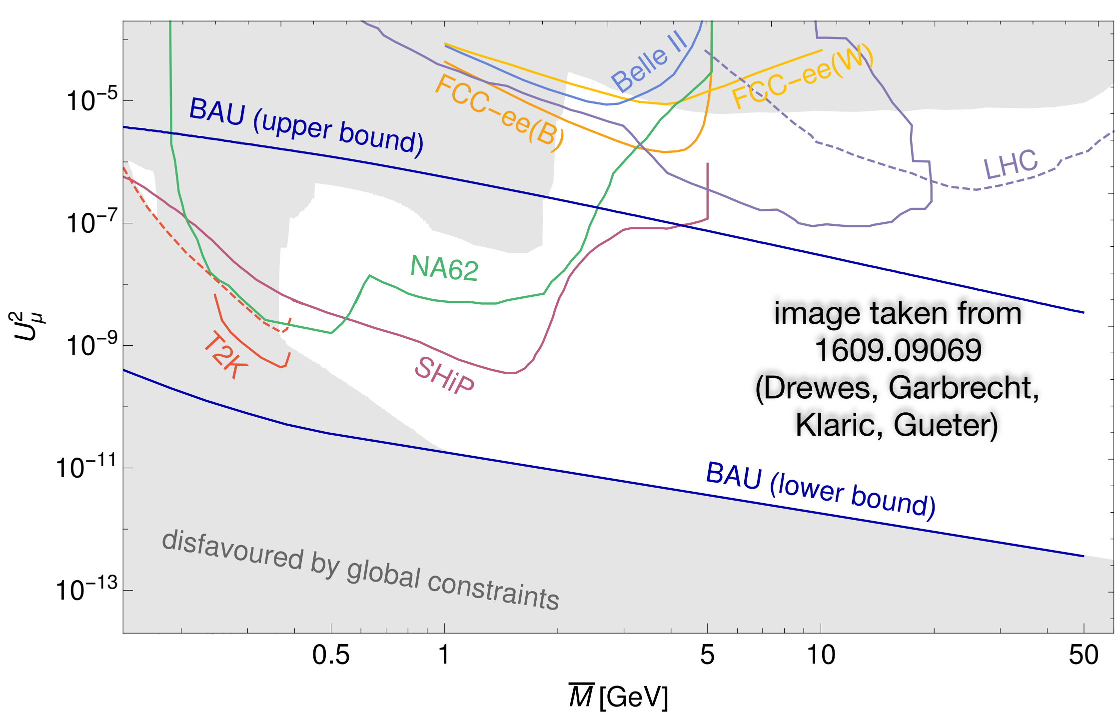
Pilaftsis, Underwood, Millington, Teresi

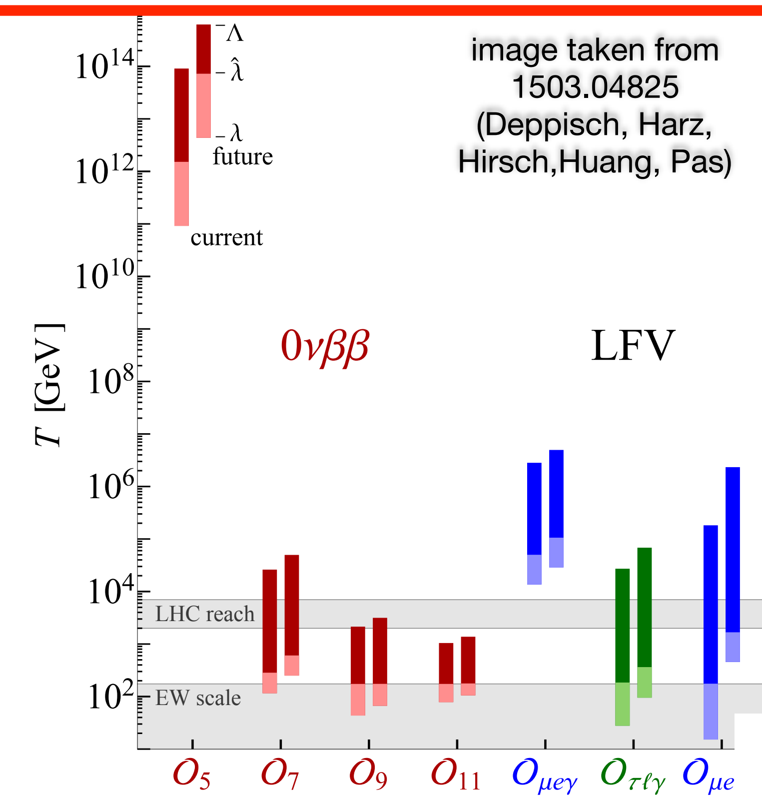
Resonant Leptogenesis

- **small neutrino masses ⟺ BAU**
- **minimal: 2RHN**
- **neutrino data, NDBD, LFV and LNV, cosmology in meson decays, collider searches**

- **small neutrino masses ⟺ BAU**
- **minimal: 2RHN**
- **Easily embedded in GUT models**
- **falsifiable**
- **Can induce the EW scale**

- **Scale too high can exacerbate**
- **Higgs fine tuning**
- **RHNs too heavy to produce**





Neutrino Option in Leptogenesis

1905.12642

Brivio, Moffat, Pascoli, Petcov, J.T

1905.12634

Brdar, Helmboldt, Iwamoto, Schmitz

UV completion of Neutrino Option

Brdar, Emonds, Helmboldt, Lindner 1807.11490

GW signature

Brdar, Helmboldt, Kubo 1810.12306

Neutrino Option in Leptogenesis

Type-I Seesaw as the Common Origin of Neutrino Mass, Baryon Asymmetry, and the Electroweak Scale

Motivation and idea

Tension between leptogenesis with heavy RHN and naturalness of the electroweak scale which is parametrised by μ

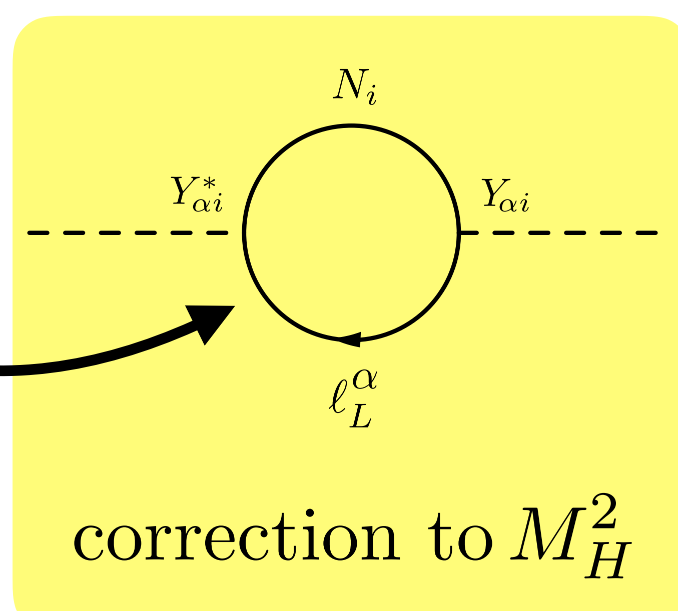
$$V = -\mu^2|\phi|^2 + \lambda|\phi|^4 \qquad \mu = \frac{m_h}{\sqrt{2}}$$

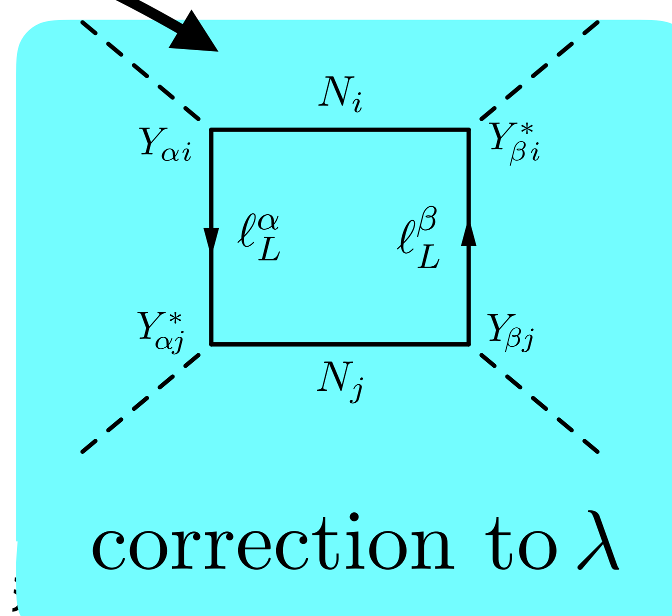
Vissani, Clarke, Foot, Volkas

If corrections to the Higgs mass should < 1 TeV $\Rightarrow$ M $\lesssim 10^7$ GeV

The Neutrino Option (Brivio, Trott) offers a different perspective:
Higgs potential is generated by radiative corrections from RHN

Scale invariance broken at quantum level





related results by Davoudiasl, Lewis, Casas, Clemente, Quiros, Ibarra, Bambhaniya, Dev, Goswami, Khan, Rodejohann

Scale for Neutrino Option: $M_i \lesssim 10^7\,\mathrm{GeV} \quad |Y_{\alpha i}| \sim 1\mathrm{TeV}/M_i$

Mass of RHN is only dimensional parameter of theory & controls breaking of conformal and lepton number symmetry.

Consider minimal case compatible with oscillation data (2RHN)

Parameter Space:

$$Y = f(m_2, m_3, \theta_{12}, \theta_{13}, \theta_{23}, \delta, \alpha_{21}, \alpha_{32}, \theta, M_1, M_2)$$

12 dimensional

$$R = \begin{pmatrix} 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \\ 1 & 0 & 0 \end{pmatrix}$$

$$\theta = x + iy$$

From seesaw scale RG run to EW scale:

$$M_H^2(\mu = v) \sim \frac{M_i^2 |Y_i|^2}{8\pi^2} \qquad m_\nu(\mu = v) \simeq \frac{v^2 |Y_i|^2}{2M_i}$$

Type-I ss Leptogenesis consistent with Neutrino Option

1. Non-resonant leptogenesis with R-matrix enhancement

R-matrix entries very large i.e. enlarge the imaginary parts which allows for larger Yukawa.

Problematic as this leads to fine-tuned cancellation between the tree and one-loop light neutrino mass matrices and give too large a Higgs mass.

2. Resonant leptogenesis

Go to resonantly enhanced regime, $\Delta M \sim \Gamma_N \ll M$, of leptogenesis. CP-asymmetry receives enhancement from oscillations and mixing of RHNs see 0309342, 1711.02863, 1404.1003, 1410.6434

Boltzmann equations

Dev, Millington, Pilaftsis and Teresi 1404.1003

formulae from 1611.03827 for review see 1711.02863

$$\frac{dn_{N_i}}{dz} = -D_i(n_{N_i} - n_{N_i}^{\rm eq}),$$

$$\frac{dn_{\alpha\alpha}}{dz} = \sum_i \left(\epsilon_{\alpha\alpha}^{(i)} D_i(n_{N_i} - n_{N_i}^{\rm eq}) - p_{i\alpha} W_i n_{\alpha\alpha} \right) , \quad i = 1, 2 \, , \;\; \alpha = e, \mu, \tau \, .$$

oscillations of RHN due to interaction with thermal BG (T≠ 0 effect)

mixing effect comes from off diagonal transitions in self-energy (T = 0 effect)

Boltzmann equations

Dev, Millington, Pilaftsis and Teresi 1404.1003

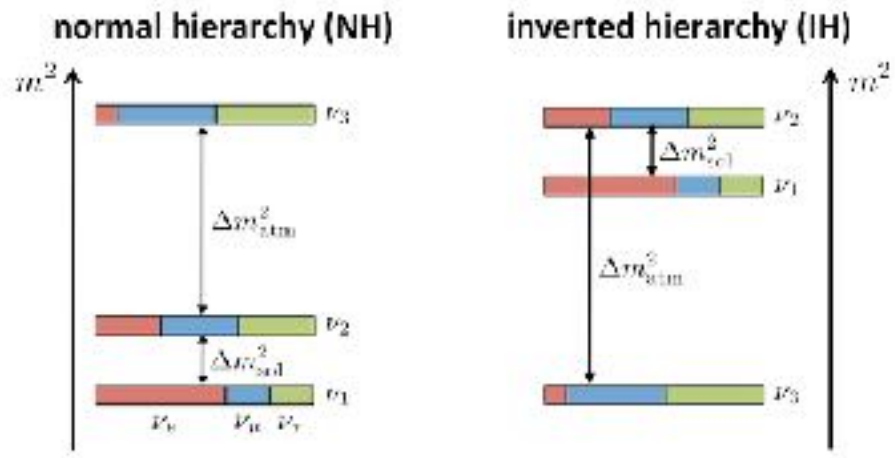
formulae from 1611.03827 for review see 1711.02863

$$\frac{dn_{N_i}}{dz} = -D_i(n_{N_i} - n_{N_i}^{\rm eq}),$$

$$\frac{dn_{\alpha\alpha}}{dz} = \sum_i \left(\epsilon_{\alpha\alpha}^{(i)} D_i(n_{N_i} - n_{N_i}^{\rm eq}) - p_{i\alpha} W_i n_{\alpha\alpha}\right) , \quad i = 1, 2\,, \;\; \alpha = e, \mu, \tau\,.$$

Asymmetry estimate

$$n_{B-L} \approx \frac{\pi^2}{6 z_d} n^{\rm eq}(0) \sum_{\alpha=e}^{\tau} \frac{\epsilon_{\alpha\alpha}^{(1)}}{K_1 p_{1\alpha}}$$



$$\frac{\epsilon_{\alpha\alpha}^{(1)}}{K_1 p_{1\alpha}} \approx 16 m_* \left(f_{\rm osc} + f_{\rm mix}\right) \frac{m_2 - m_3}{(m_2 + m_3)^2} e^{-4y} \sin 2x$$

Normal Ordering

m_*: $10^{-3}\,{\rm eV}$

$$\frac{\epsilon_{\alpha\alpha}^{(1)}}{K_1 p_{1\alpha}} \approx 16 m_* \left(f_{\rm osc} + f_{\rm mix}\right) \frac{m_1 - m_2}{(m_1 + m_2)^2} e^{-4y} \sin 2x$$

Inverted Ordering

Higgs mass from Neutrino Option

$$\Delta M_H^2 = \frac{1}{8\pi^2 v^2} \cosh(2y) M^3 (m_1 + m_2 + m_3) \qquad M = \frac{M_1 + M_2}{2}$$

$$\Delta M_H^2 = \frac{1}{8\pi^2 v^2} \cosh(2y) M^3 (m_1 + m_2 + m_3)$$

$$\frac{\epsilon_{\alpha\alpha}^{(1)}}{K_1 p_{1\alpha}} \approx 16 m_* (f_{\rm osc} + f_{\rm mix}) \frac{m_2 - m_3}{(m_2 + m_3)^2} e^{-4y} \sin 2x$$

Normal Ordering

$$\frac{\epsilon_{\alpha\alpha}^{(1)}}{K_1 p_{1\alpha}} \approx 16 m_* (f_{\rm osc} + f_{\rm mix}) \frac{m_1 - m_2}{(m_1 + m_2)^2} e^{-4y} \sin 2x$$

Inverted Ordering

Maximise of all terms in the above expressions except y and then find largest y for which leptogenesis is successful ⇒ lowest M

Upper M scale ⇒ y=0° comes from the Neutrino Option

$$1.2 \times 10^6 < M \ (\text{GeV}) < 8.8 \times 10^6$$

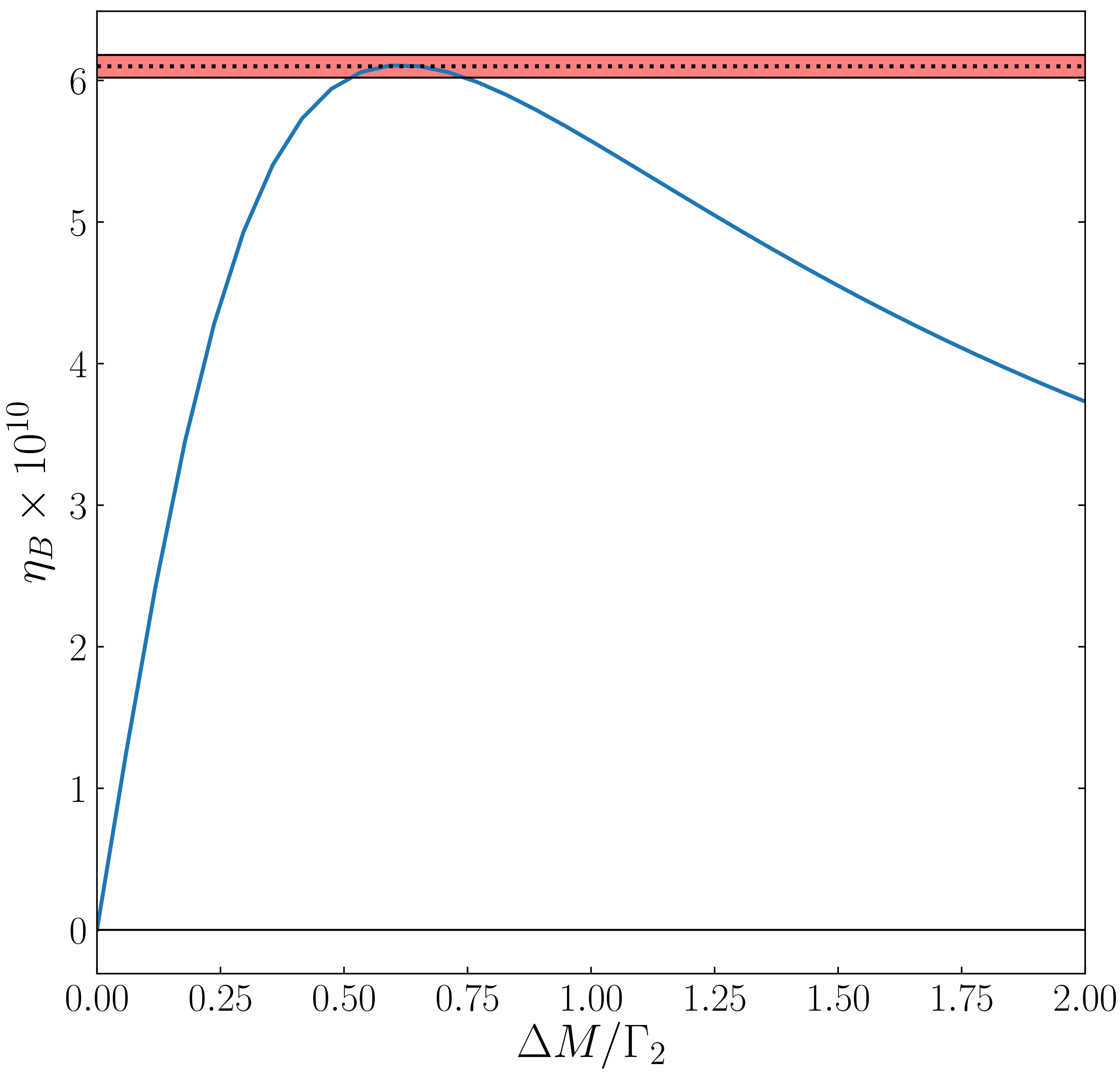
Normal Ordering

$$2.4 \times 10^6 < M \ (\text{GeV}) < 7.4 \times 10^6$$

Inverted Ordering

$$\frac{m_2 - m_3}{(m_2 + m_3)^2} > \frac{m_1 - m_2}{(m_1 + m_2)^2}$$

We need M to compensate for this effect which means M is larger for IO than NO

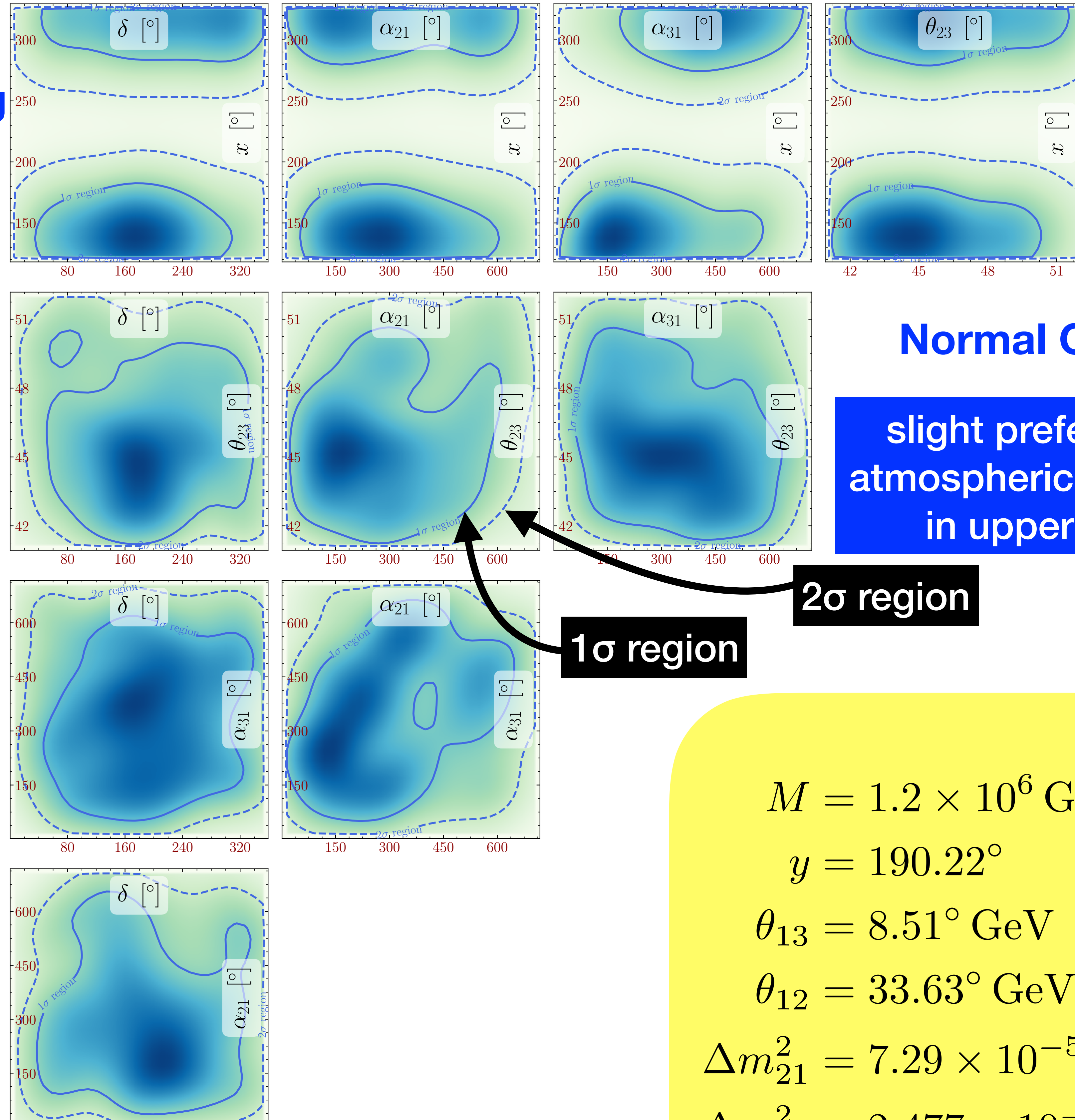


$\eta_B \times 10^{10}$
6
5
4
3
2
1
0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
$\Delta M/\Gamma_2$

Numerically confirmed: fix M and y s.t NO works and explore remaining PS

PS exploration using Bayesian inference tool MultiNest and visualisation SUPERPLOT

$$\frac{\Delta M}{M} \sim 10^{-8}$$



Normal Ordering

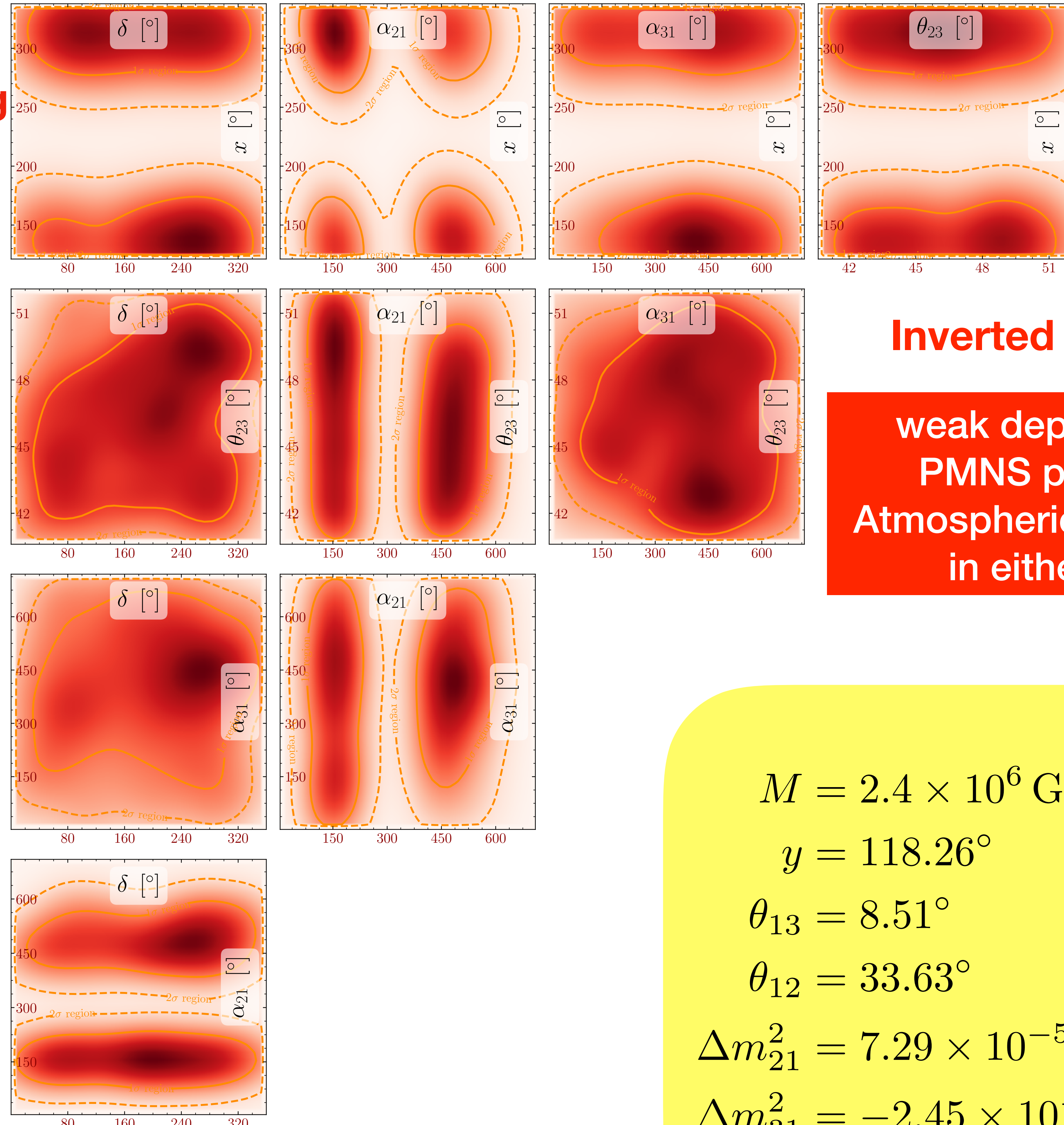
slight preference for atmospheric angle to be in upper octant.

$$M = 1.2 \times 10^6\,\text{GeV}$$
$$y = 190.22^\circ$$
$$\theta_{13} = 8.51^\circ\,\text{GeV}$$
$$\theta_{12} = 33.63^\circ\,\text{GeV}$$
$$\Delta m^2_{21} = 7.29 \times 10^{-5}\,\text{eV}^2$$
$$\Delta m^2_{31} = 2.477 \times 10^{-3}\,\text{eV}$$

For Lower bound of IO:

PS exploration using Bayesian inference tool MultiNest and visualisation SUPERPLOT

$$\frac{\Delta M}{M} \sim 10^{-8}$$



Inverted Ordering

weak dependence on PMNS parameters. Atmospheric angle can be in either octant.

$$\begin{aligned} M &= 2.4 \times 10^{6}\,\mathrm{GeV} \\ y &= 118.26^\circ \\ \theta_{13} &= 8.51^\circ \\ \theta_{12} &= 33.63^\circ \\ \Delta m^2_{21} &= 7.29 \times 10^{-5}\,\mathrm{eV}^2 \\ \Delta m^2_{31} &= -2.45 \times 10^{-3}\,\mathrm{eV}^2 \end{aligned}$$

Half Time Summary

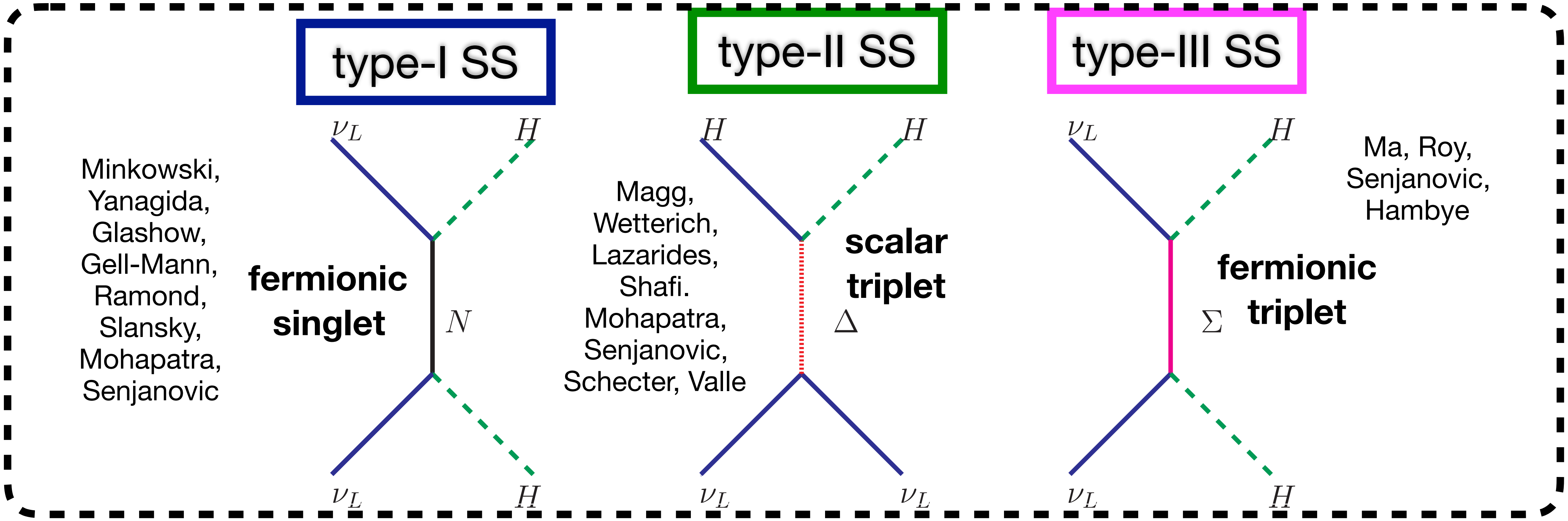
- The Neutrino Option demonstrates that the Type-I seesaw can generate the Higgs potential and therefore the EW scale.
- If neutrino masses are explained by a type-I seesaw, leptogenesis is a plausible cosmological consequence and provides an explanation for the BAU.
- We tie these two concepts together such that neutrino masses, the generation of the BAU and EW scale have a common origin.
- The mass range for the RHN is fairly constrained with quasi-degenerate RHN with $M \sim 10^6$ GeV for both mass orderings with $\Delta M/M \sim 10^{-8}$.
- There is little dependence on low energy parameters but if neutrinos are normally ordered slight preference for upper octant.

Neutrino masses and gravity

Neutrino Masses

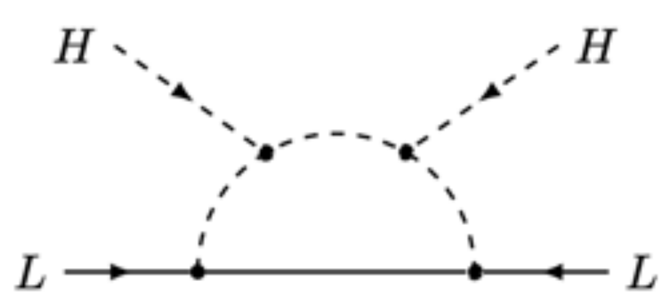
- Write a Dirac mass term analogous to other SM fermions

$$-\mathcal{L}_{d=5} = \lambda \frac{L.H L.H}{M}$$



- radiative models

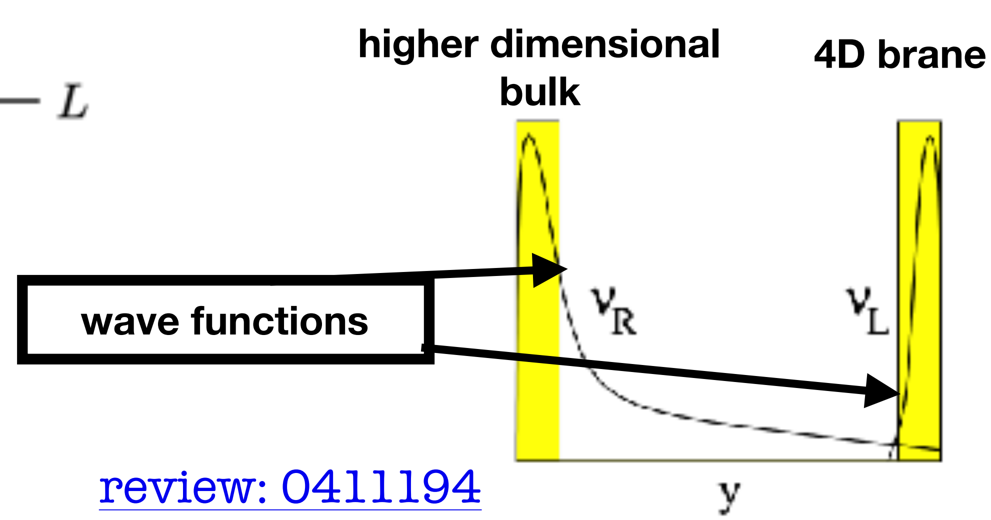
Babu, Leung, Bhattacharya, Wudka, Gouvea, Jenkins, Kobach, Ma



review: 1706.08524

- Extra dimensions

Arkani-Hamed, Dimopoulos, Dvali, March-Russell



review: 0411194

Neutrino Masses from gravity

Dvali & Funcke (1602.03191)

Logic: make an analogue of gravity with QCD

u, d, s are light relative to c, b and t so **approximate** flavour symmetry

$$U(3)_A \times U(3)_V$$

At energies below $\Lambda_{\rm QCD} \sim 300\,{\rm MeV}$ quarks confine into hadrons

Once confinement occurs, relevant d.o.f baryons and mesons $\langle \overline{q} q \rangle$

Ground state break symmetry

$$U(3)_A \times U(3)_V \xrightarrow{\langle \overline{q} q \rangle} U(3)_V$$

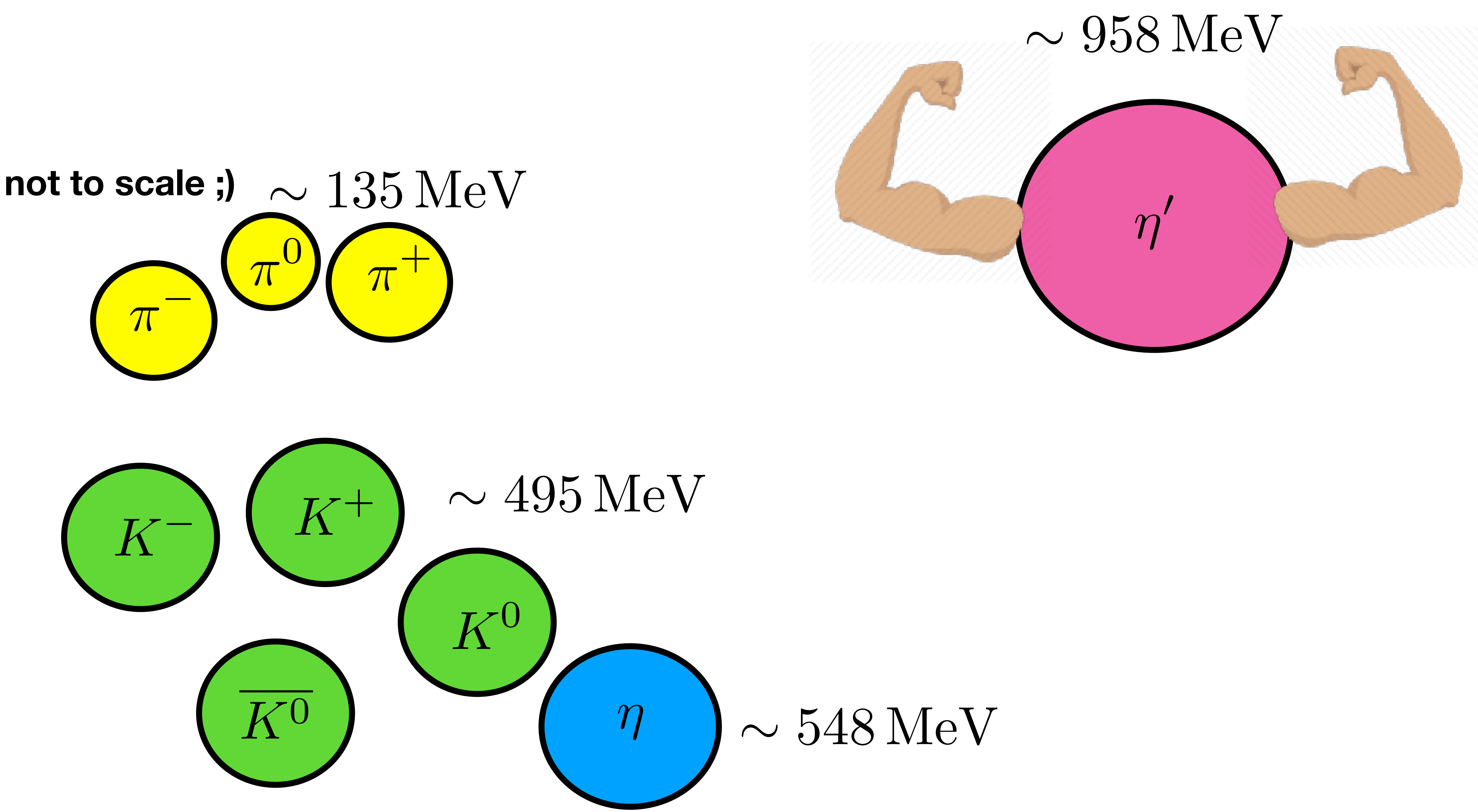
quark condensate spontaneously breaks symmetry

Broken symmetry contains $U(1)_A$ and an SU(3) part which are broken and via Goldstone's theorem 9 pseudo GBs:

$$1(\eta') + 8(\pi, \eta, K)$$

Neutrino Masses from gravity (1602.03191)

η' is heavy relative to the other mesons and its mass gets raised due to non-perturbative QCD effects.



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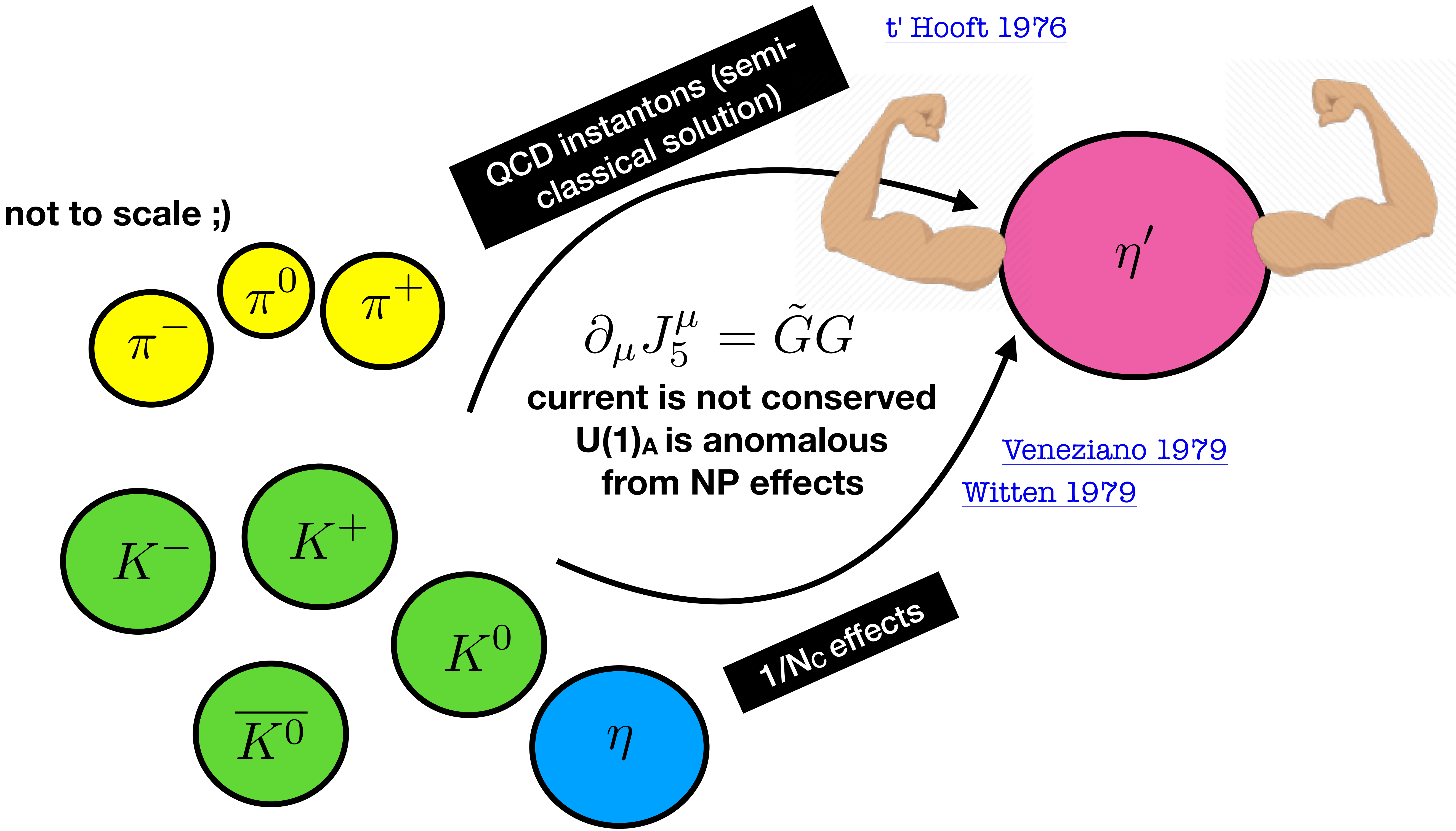


Neutrino Masses from Gravity (1602.03191)

Assume gravity has a theta term: $\mathcal{L}_G \supset \theta_G \tilde{R}R$ ← **Riemann tensor**

They postulate neutrinos have zero bare mass. They can condense via NP gravitational effects and use SDA in analogue with QCD:

$$\nu\overline{\nu} = \langle \nu\overline{\nu} \rangle = v e^{i\phi} \implies \Lambda_G \sim v \sim m_\nu \sim \nu\overline{\nu}$$

analogous to Λ_{QCD} free parameter

$$U(3)_V \times U(3)_A \to U(1)^3$$

$1(\eta_\nu) + 14(\phi)$ **pseudo Goldstone bosons**

small neutrino masses from this gravitational θ term triggers neutrino condensate and introduces an **infrared gravitational scale**.

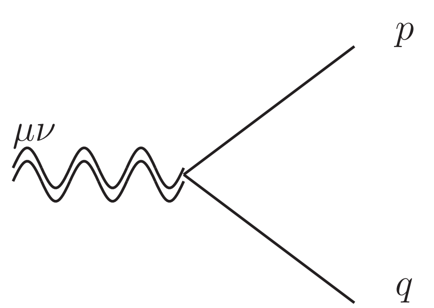
One massive GB analogous to eta prime and remainder massless

Neutrinos acquire mass from their NP coupling to neutrino condensate.

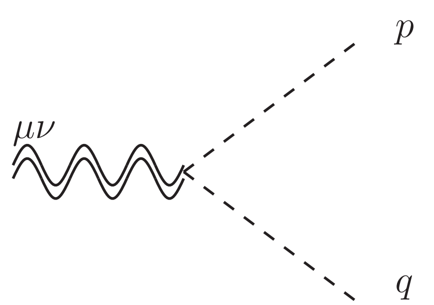
Neutrino Masses from Gravity 1909.04675

With Gabriela Barenboim (Valencia) and Ye-Ling Zhou (Southampton)

- We treat gravity as an EFT similar to Donoghue see 9405057v1 for a review. Start with flat metric and perturb around it, gravity non-Abelian gauge theory with spin-2 gauge boson.

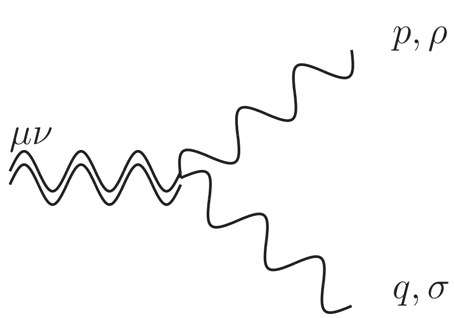


$$\tau_1^{\mu\nu}(p,q) = \frac{i\kappa}{8}\left[(q-p)^\mu \gamma^\nu + (q-p)^\nu \gamma^\mu - 2\eta^{\mu\nu}(\not{q} - \not{p})\right]$$



$$\tau_2^{\mu\nu}(p,q) = \frac{i\kappa}{2}\left[p^\mu q^\nu + p^\nu q^\mu - \eta^{\mu\nu} p\cdot q\right]$$

$$\kappa = \sqrt{32\pi G}$$
$$G = \frac{1}{M_{Pl}^2}$$



$$\tau_3^{\mu\nu\rho\sigma}(p,q) = i\kappa\left[-\mathcal{P}^{\mu\nu\rho\sigma} - \frac{1}{2}\eta^{\mu\nu}p^\sigma q^\rho + \eta^{\sigma\rho}(p^\mu q^\nu + p^\nu q^\mu) + \frac{1}{2}(\eta^{\mu\sigma}p^\nu q^\rho + \eta^{\nu\sigma}p^\rho q^\mu + \eta^{\nu\rho}p^\sigma q^\mu + \eta^{\mu\rho}p^\nu q^\sigma)\right]$$

$$\mathcal{P}^{\mu\nu\rho\sigma} = \frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma})$$

Neutrino Masses from Gravity

1909.04675

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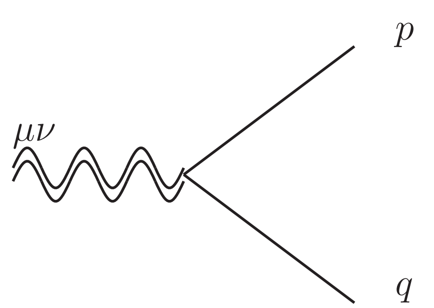

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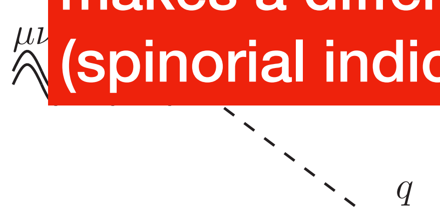
This 2 is often missing in the literature, it should be there! It makes a difference! Problem comes from embedding fermions (spinorial indices) into GR.



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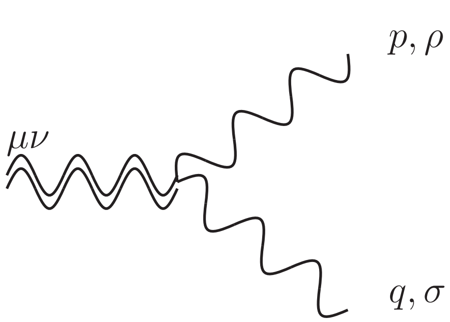


$$\tau_3^{\mu\nu\rho\sigma}(p,q) = i\kappa[-\mathcal{P}^{\mu\nu\rho\sigma} - \frac{1}{2}\eta^{\mu\nu}p^\sigma q^\rho + \eta^{\sigma\rho}(p^\mu q^\nu + p^\nu q^\mu)$$
$$+ \frac{1}{2}(\eta^{\mu\sigma}p^\nu q^\rho + \eta^{\nu\sigma}p^\rho q^\mu + \eta^{\nu\rho}p^\sigma q^\mu + \eta^{\mu\rho}p^\nu q^\sigma)]$$

$$\mathcal{P}^{\mu\nu\rho\sigma} = \frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma})$$

- “Gravity triggered neutrino condensates” (1009.2504) used SDEs as a means of studying RH neutrino condensation. We revisit this paper and calculation techniques for light neutrinos.

- Here RHN are heavy and light neutrinos get mass from type-I ss. RHN condensate can drive inflation (0811.2998)

- SDE are an infinite tower of integral coupled equations which relate the Green’s functions of a theory to each other.

- We have to choose some truncation skim. For us this is one loop improved.

- This allows us to derive a neutrino gap equation.

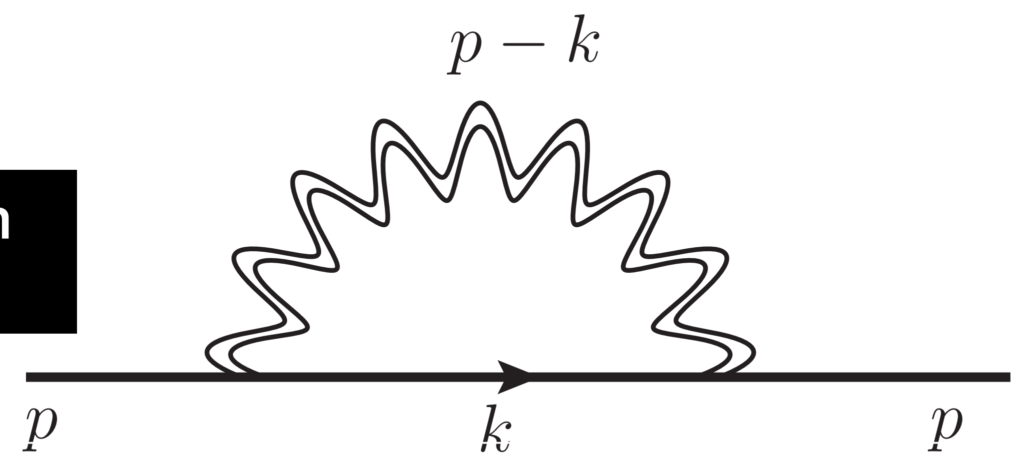
Neutrino Masses from Gravity

Apply Schwinger-Dyson equation to find non-trivial vacuum.

$$S'_F(p) = \frac{i}{\not{p} - \Sigma(p)} = \frac{i}{\alpha(p^2)\not{p} - \beta(p^2)} \qquad m_F = \beta(p^2)/\alpha(p^2)$$

Assume neutrino has zero valued bare mass and Dirac fermion.

graviton-fermion-fermion vertex



undressed graviton propagator

Leading self-energy correction

$$-i\Sigma(p) = \int \frac{d^4k}{(2\pi)^4} \tau_1^{\mu\nu}(p,-k) S'_F(k) G_{\mu\nu\rho\sigma}(p-k) \tau_1^{\rho\sigma}(k,-p)$$

Neutrino Masses from Gravity

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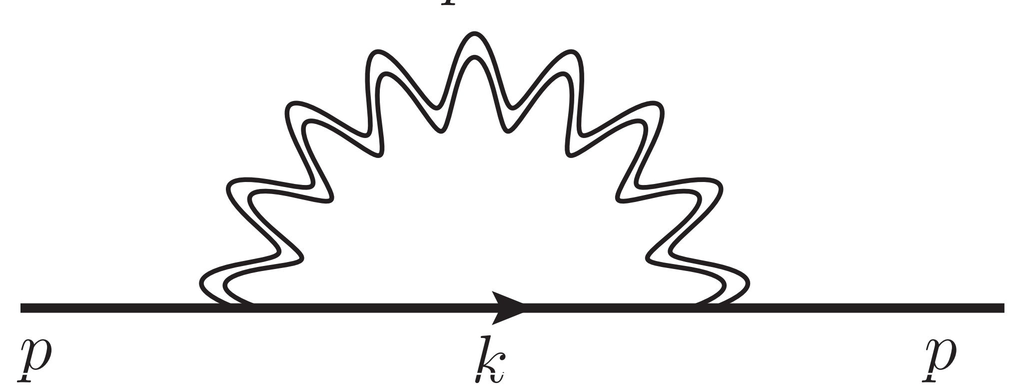

Taking the appropriate Dirac trace

$$\alpha(p^2) = 1 - \frac{1}{4p^2}\mathrm{tr}(\not{p}\Sigma(p)) \quad \beta(p^2) = \frac{1}{4}\mathrm{tr}(\Sigma(p))\,.$$

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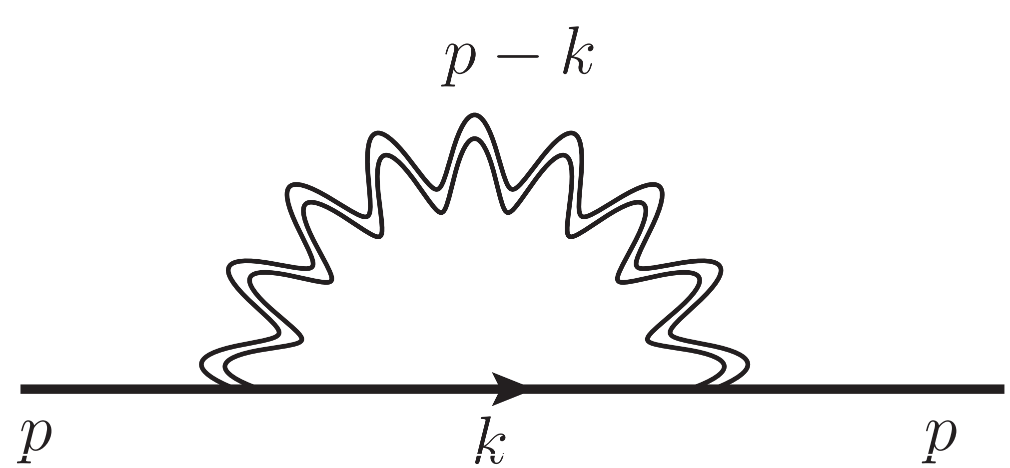

$$\beta(p^2) = 0 \ \forall p$$

$$\alpha(p^2) = 1 - i2\pi G \int \frac{d^4k}{(2\pi)^4} \frac{\alpha(k^2)}{\alpha^2(k^2)k^2 - \beta^2(k^2)} \frac{\left[2(k\cdot p)^2 + 4k^2p^2 + 3k\cdot p(k^2+p^2)\right]}{p^2(p-k)^2}$$

Neutrino Masses from Gravity

Apply Schwinger-Dyson equation to find non-trivial vacuum.

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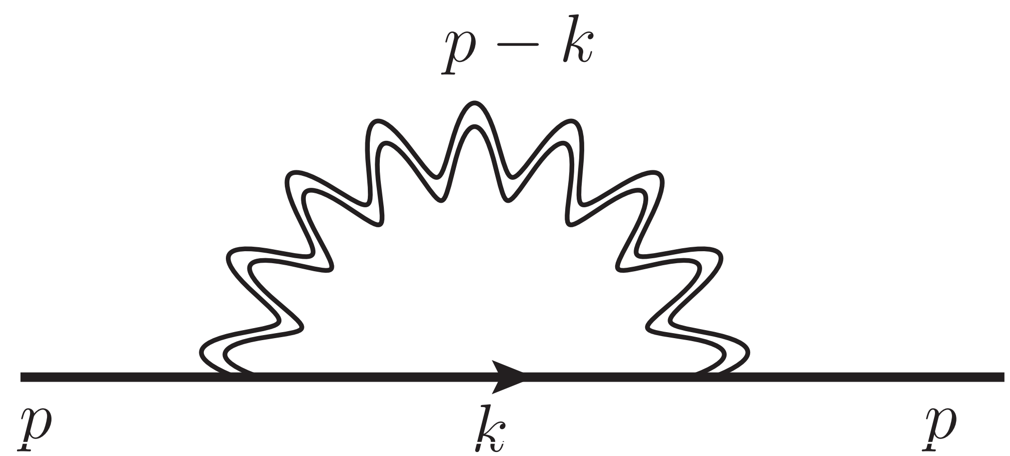
Assume neutrino has zero valued bare mass. Leading contribution from undressed graviton propagator is vanishing. Need to dress graviton.

no mass dynamically induced if graviton is undressed



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Neutrino Masses from Gravity

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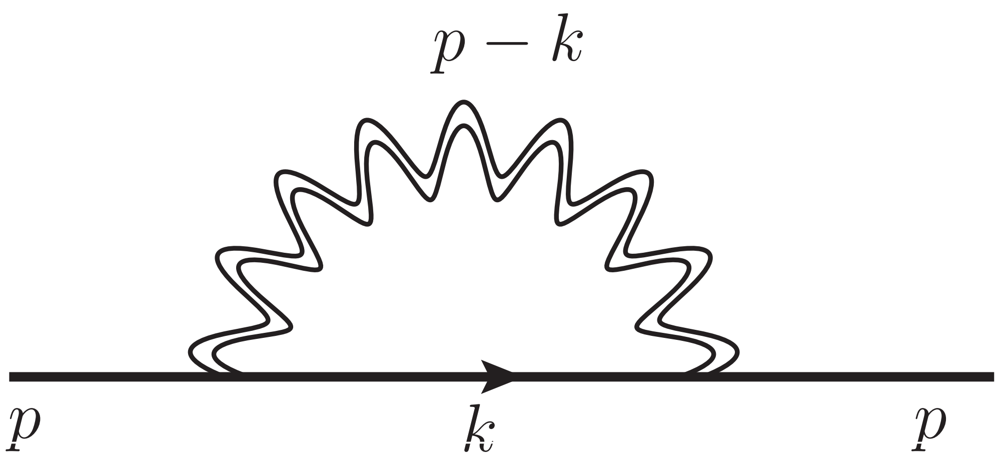

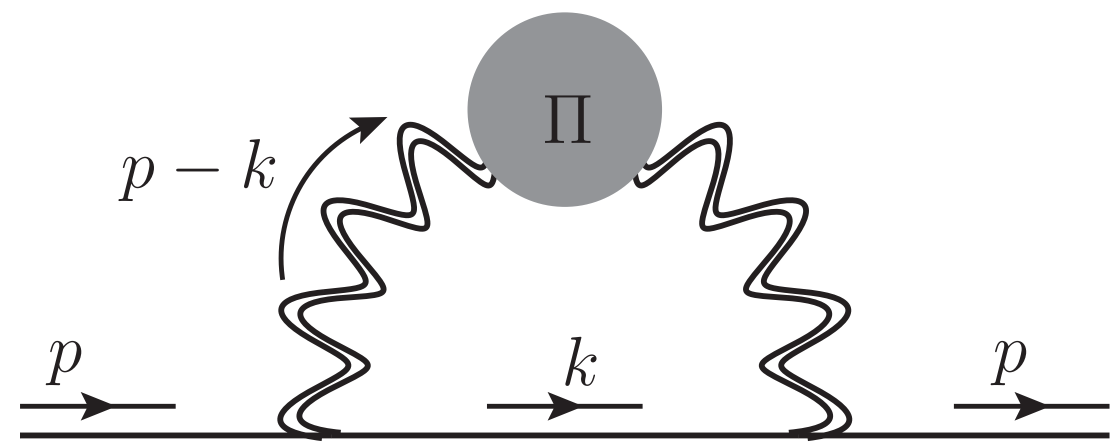

$$G'_{\mu\nu\rho\sigma}(p-k) \to G_{\mu\nu\rho\sigma}(p-k) + G_{\mu\nu\alpha\beta}(p-k)\Pi^{\alpha\beta,\gamma\delta}(p-k)G_{\rho\sigma\gamma\delta}(p-k)$$

Neutrino Masses from Gravity

Apply Schwinger-Dyson equation to find non-trivial vacuum.

$$S'_F(p) = \frac{i}{\not{p} - \Sigma(p)} = \frac{i}{\alpha(p^2)\not{p} - \beta(p^2)} \qquad m_F = \beta(p^2)/\alpha(p^2)$$

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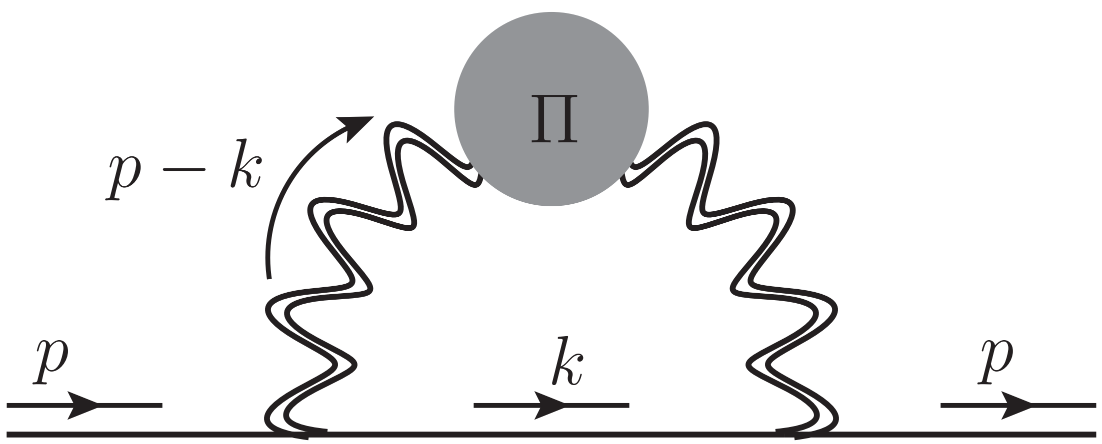
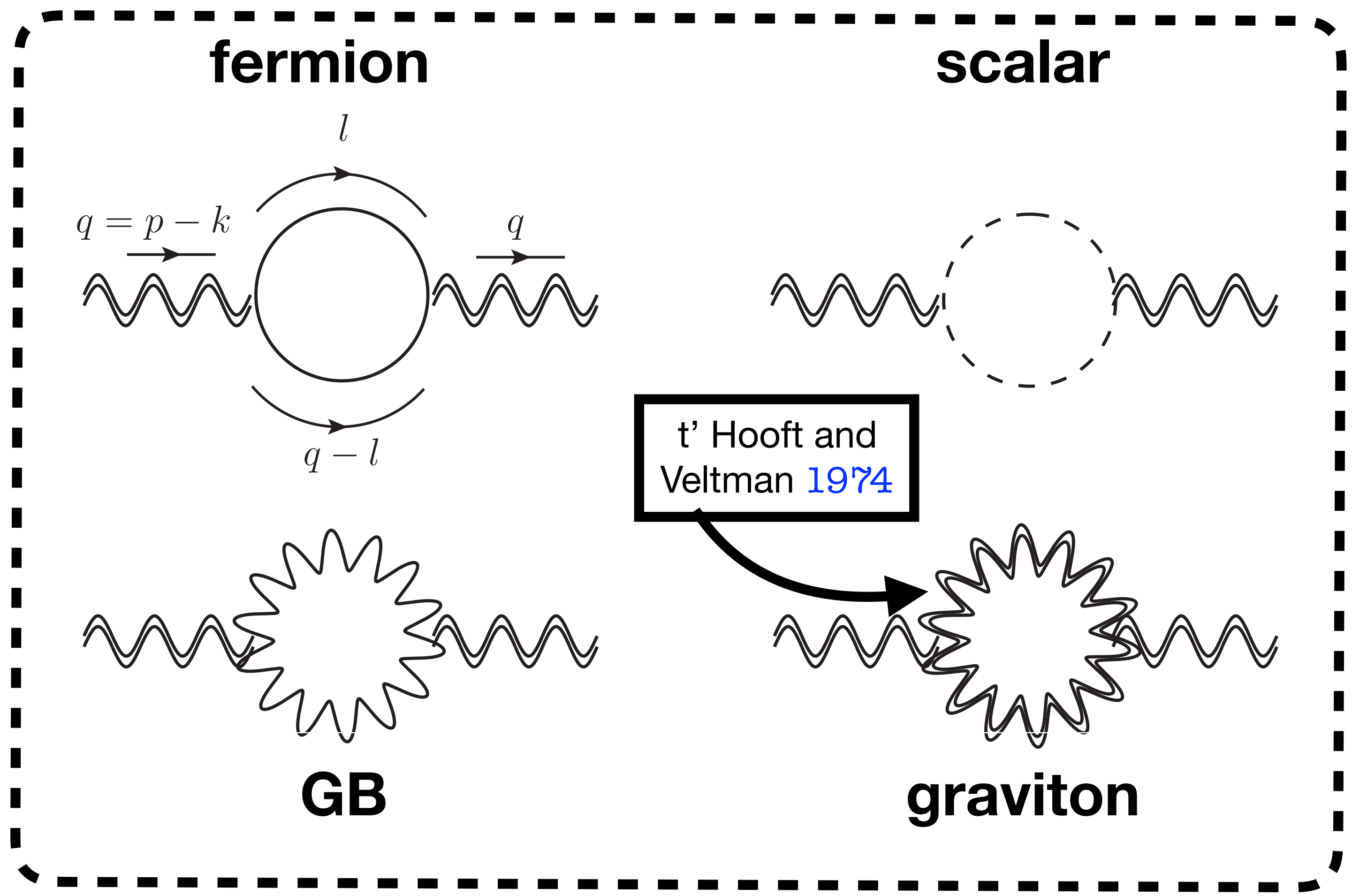


Vacuum polarisations Π



Neutrino Masses from Gravity

From vacuum polarisations find expression

$$\beta(p^2) = i8G^2 \int \frac{d^4k}{(2\pi)^4} \frac{\beta(k^2)}{\alpha^2(k^2)k^2 - \beta^2(k^2)} \left[A(k+p)^2 - B\frac{\left(p^2-k^2\right)^2}{8(p-k)^2} \right] \log\left[\frac{\mu^2}{-(p-k)^2}\right]$$

$$\alpha(p^2) = 1 - i2\pi G \int \frac{d^4k}{(2\pi)^4} \frac{\alpha(k^2)}{\alpha^2(k^2)k^2 - \beta^2(k^2)} \frac{\left[2(k\cdot p)^2 + 4k^2p^2 + 3k\cdot p(k^2+p^2)\right]}{p^2(p-k)^2}$$

A and B function of matter running in the loops

$$A = \frac{27/2N_{\rm ms} + 6N_{\rm df} + 12N_{\rm gb} + N_{\rm cs} + 267N_{\rm gr}}{288}$$

$$B = \frac{9N_{\rm ms} + 6N_{\rm df} + 12N_{\rm gb} + N_{\rm cs} + 186N_{\rm gr}}{288}.$$

$A = 2.61 \quad B = 2.27$ SM particle content + 1 graviton

Neutrino Masses from Gravity

Rotate to Euclidean space and rescale momenta: $x = \frac{p_E^2}{\Lambda^2} \quad y = \frac{k_E^2}{\Lambda^2}$

$$\alpha(p_E^2) = 1 - 2\pi G \int \frac{d^4 k_E}{(2\pi)^4} \frac{\alpha(k_E^2)}{\alpha^2(k_E^2) k_E^2 + \beta^2(k_E^2)} \frac{\left[2(k_E \cdot p_E)^2 + 4k_E^2 p_E^2 + 3k_E \cdot p_E(k_E^2 + p_E^2)\right]}{p_E^2 (p_E - k_E)^2}$$

$$\beta(p_E^2) = -8G^2 \int \frac{d^4 k_E}{(2\pi)^4} \frac{\beta(k_E^2)}{\alpha^2(k_E^2) k_E^2 + \beta^2(k_E^2)} \left[A(k_E + p_E)^2 - B\frac{\left(p_E^2 - k_E^2\right)^2}{8(p_E - k_E)^2}\right] \log\left[\frac{\mu^2}{(p_E - k_E)^2}\right]$$

UV cutoff

$$\alpha(x) = 1 - \frac{G\Lambda^2}{(2\pi)^2} \int_0^1 dy \frac{y\alpha(y)}{y\alpha^2(x) + \beta^2(y)} K(x, y)$$

$$\beta(x) = \frac{8G^2\Lambda^4}{(2\pi)^3} \int_0^1 dy \frac{y\beta(y)}{y\alpha^2(y) + \beta^2(y)} L(x, y)$$

Kernels

Kernel structure

Rotate to Euclidean space, rescale momentum

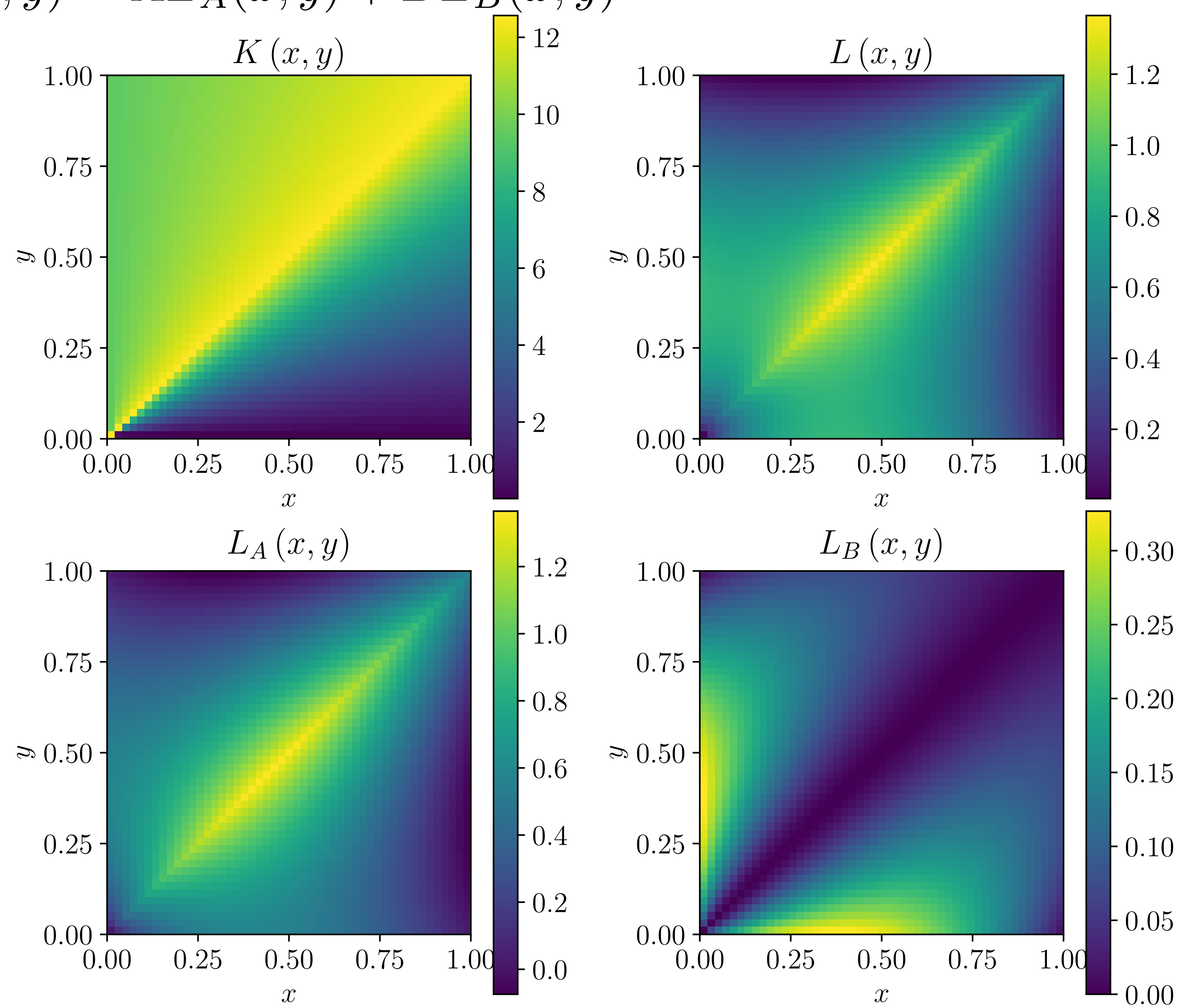
rescaled p momentum

$$\alpha(x) = 1 - \frac{G\Lambda^2}{(2\pi)^2}\int_0^1 dy \frac{y\alpha(y)}{y\alpha^2(x) + \beta^2(y)} K(x, y)\,,$$

rescaled k momentum

$$\beta(x) = \frac{8G^2\Lambda^4}{(2\pi)^3}\int_0^1 dy \frac{y\beta(y)}{y\alpha^2(y) + \beta^2(y)} L(x, y)\,.$$

$L(x, y) = AL_A(x, y) + BL_B(x, y)$



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Kernel structure

Rotate to Euclidean space, rescale momentum

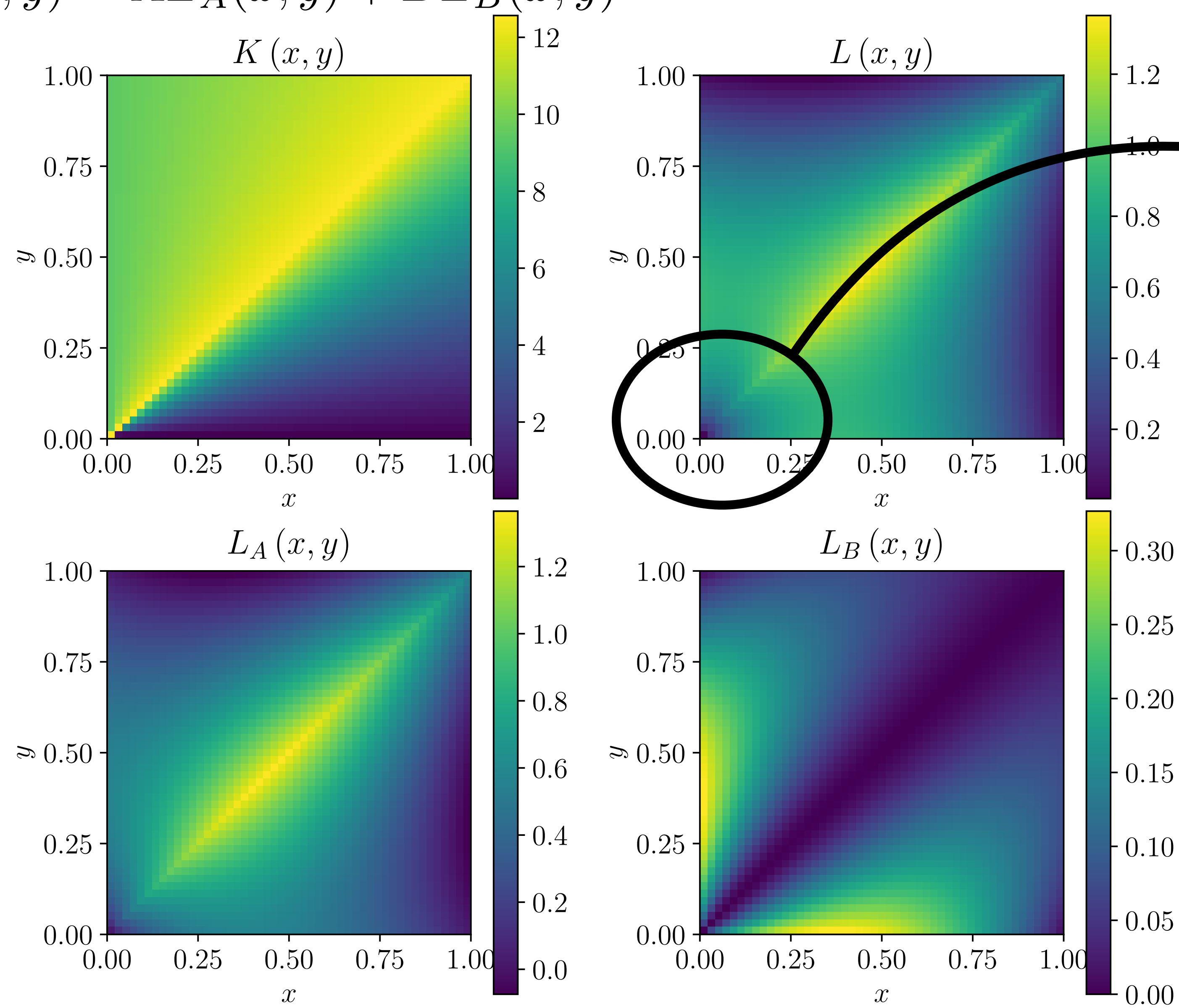
$$\alpha(x) = 1 - \frac{G\Lambda^2}{(2\pi)^2}\int_0^1 dy \frac{y\alpha(y)}{y\alpha^2(x) + \beta^2(y)} K(x, y)\,,$$

$$\beta(x) = \frac{8G^2\Lambda^4}{(2\pi)^3}\int_0^1 dy \frac{y\beta(y)}{y\alpha^2(y) + \beta^2(y)} L(x, y)\,.$$

rescaled p momentum

rescaled k momentum

$L(x,y) = AL_A(x,y) + BL_B(x,y)$



gravity $L(0,0) = 0$
$x = y = 0$

QCD $L(0,0) \neq 0$
$x = y = 0$

d-wave interaction due to spin 2 nature of graviton

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Use two methods to solve SDEs using two well known methods

1. **Solve equations iteratively and apply extrapolation**

2. **Make informed Ansatz of the kernel and check for self consistency of non-trivial vacuum.**

Solving SD Equation - Extrapolation

$$\alpha^{(i+1)}(x) = 1 - \frac{G\Lambda^2}{(2\pi)^2}\int_0^1 dy \frac{y\alpha^{(i)}(y)}{y{\alpha^{(i)}}^2(x) + {\beta^{(i)}}^2(y)} K(x,y)$$

$$\beta^{(i+1)}(x) = \frac{8(G\Lambda^2)^2}{(2\pi)^3}\int_0^1 dy \frac{y\beta^{(i)}(y)}{y{\alpha^{(i)}}^2(y) + {\beta^{(i)}}^2(y)} L(x,y)\,.$$

Start with two trial functions

$$\alpha^{(0)}(x) = c_1\,, \quad \beta^{(0)}(x) = c_2$$

$$\text{tolerance} \equiv \frac{\beta^{(i+1)}(x)}{\beta^{(i)}(x)} - 1$$

This method allows us to find the NP non-trivial vacuum.

Solution (true non-trivial vacuum) is not sensitive to trial function value or tolerance value.

Solving SD Equation - Extrapolation

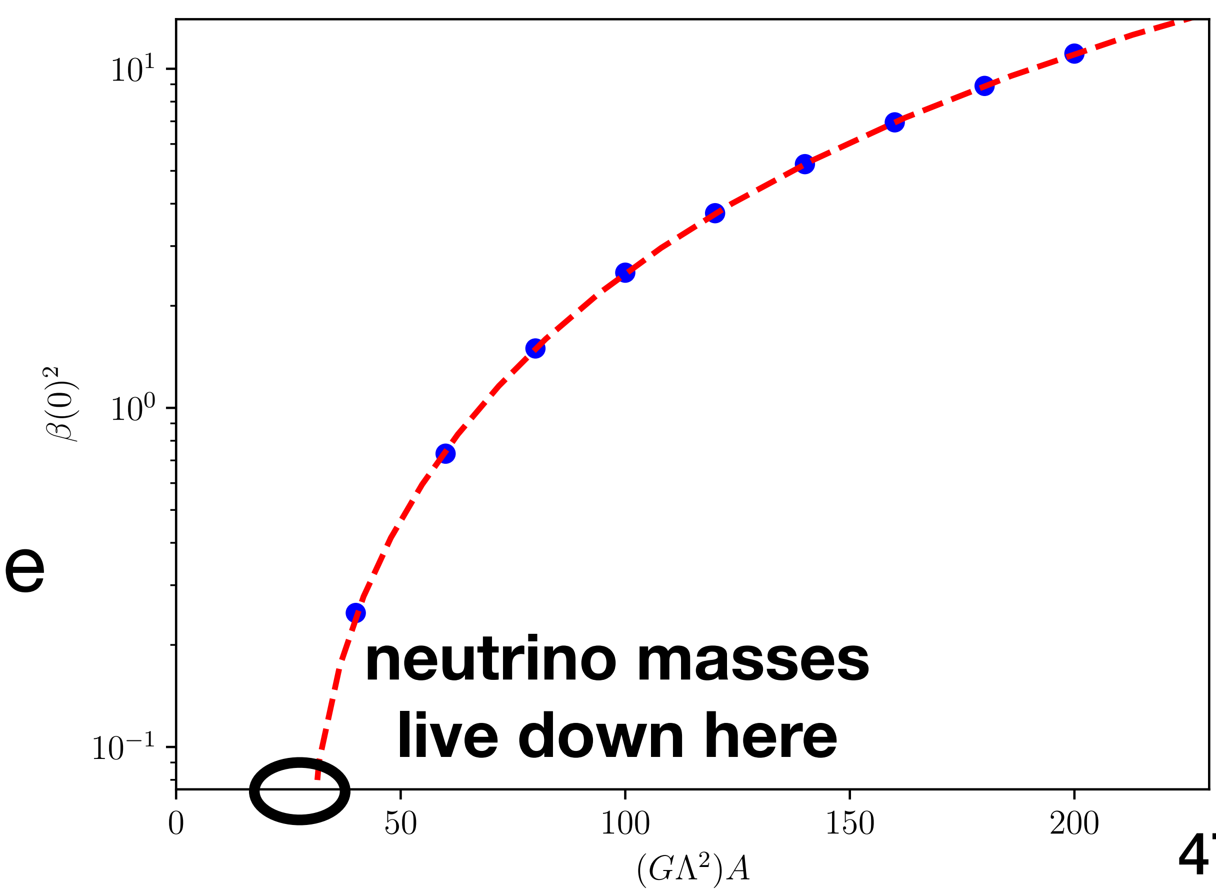
1. Choose a value of $G\Lambda^2$, A, B, tolerance and trial function values.
2. Subdivide the x-interval $[x_{\mathrm{IR}}, 1]$ into n bins, where x_{IR} is infrared boundary of the theory.
3. Iteratively solve SDE for each bin.
4. For each bin calculate the tolerance and summate this measure over all bins.
5. Require the tolerance to be close to 0.0. For example, for $G\Lambda^2 = 1.0$, $A = 50.0$ and $B = 43.5$ we choose a tolerance of 10^{-6}.

$$m_\nu = \frac{\beta(0)}{\alpha(0)}\Lambda$$

$$\beta(0) \approx 10^{-29} \text{ for } \Lambda \approx M_{\mathrm{pl}}$$

$$G\Lambda^2 \implies A \gtrsim 23$$

Requires beyond SM particle content to support the condensate even if scale is high, also we are tuning around chiral preserving point.



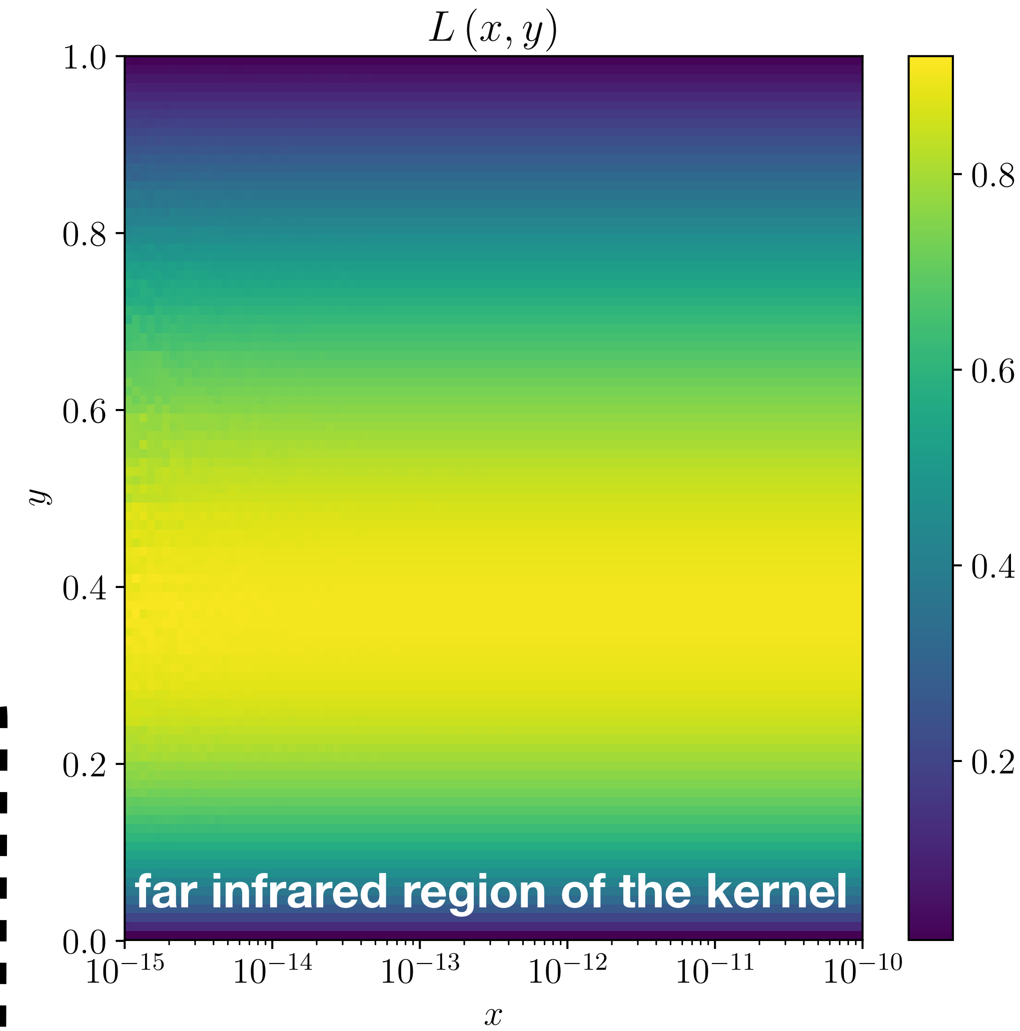
Solving SD Equation - Ansatz

Take quenched limit for simplicity i.e $\alpha \approx 1$

β depends on L(x,y). This kernel is flat in the x-direction even for tiny momentum. Make Ansatz that β is a step function of magnitude 'a'.

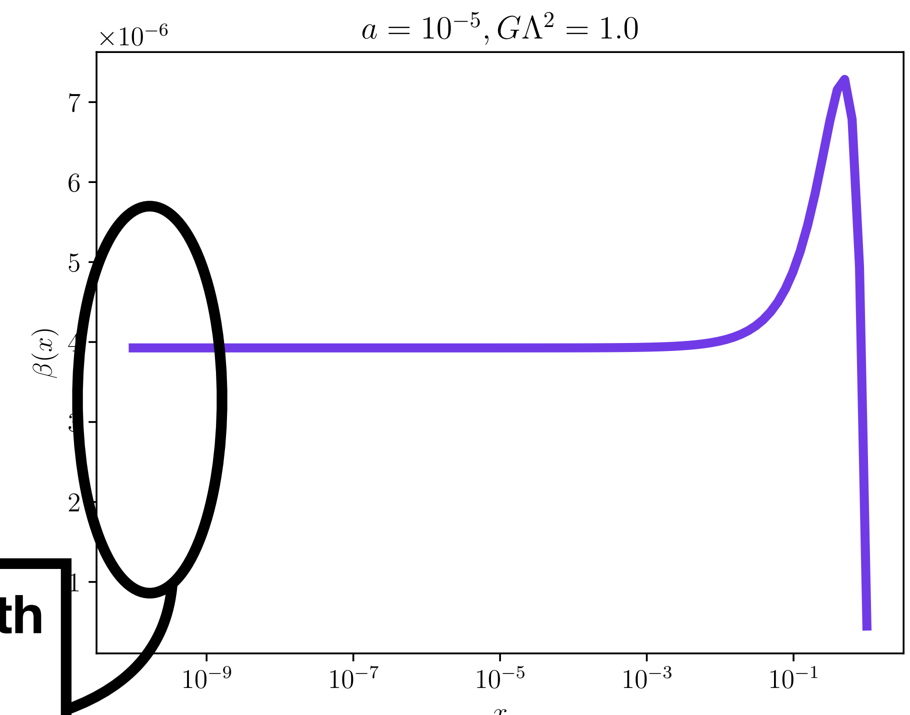
$$\beta(x) = \frac{8(G\Lambda^2)^2}{(2\pi)^3}\int_0^1 \frac{aydy}{a^2+y} L_A(x,y)$$

$$AL_A(x,y) \gg BL_B(x,y)$$



Solve SDE for this Ansatz

Checks for self-consistency of postulated form of β with kernel structure.
As before the mass can be tuned but this this approach it is tuned via 'a'.



concerned with the deep IR neutrino mass

Summary

- Neutrinos are unique amongst the Standard Model (SM) fermions in the tininess of their mass, the weakness of their interactions and their capacity to be their own anti-particles. Such features suggest neutrinos acquire their mass in a different way from the quarks and charged leptons.

- Neutrino masses from gravity is an intriguing idea and we have made a first calculational attempt at exploring this possibility.

- An interesting feature is new d.o.fs are necessary to provide finite support to the condensate even if it occurs at a very high scale. SM + gravity is not sufficient unless there are large ED which lowers Planck scale.

- As gravity does not discriminate between the neutrinos, they are mass degenerate, one needs some additional mechanism to induce a mass splitting.

- However a high level of fine-tuning is required if the Planck scale is at $\sim 10^{19}$ GeV.

Thank you for your attention

Back up slides

Formulae

$$D_1(z) = \frac{\Gamma_1(T)}{Hz} = K_1 z \langle \frac{\mathcal{K}_1(z)}{\mathcal{K}_2(z)} \rangle$$

$$N_1^{eq} = \frac{1}{2} z^2 K_2(z)$$

$$W_1(z) = \frac{1}{2} \frac{\Gamma_1^{ID}}{Hz} = \frac{1}{4} K_1 \mathcal{K}_\infty z^3$$

$$\eta_B(T = T_{rec}) = \frac{28}{29} \frac{N_{B-L}^f}{N_\gamma} \frac{g_*(T = T_{rec})}{g_*(T = T_{lep})} \simeq 0.96 \times 10^{-2} N_{B-L}^f$$

Formulae for resonant leptogenesis

$$\epsilon_{\alpha\alpha}^{(i)} = \sum_{j \neq i} \frac{\mathrm{Im}\left[Y_{i\alpha}^{\dagger} Y_{\alpha j} \left(Y^{\dagger} Y\right)_{ij}\right] + \frac{M_i}{M_j} \mathrm{Im}\left[Y_{i\alpha}^{\dagger} Y_{\alpha j} \left(Y^{\dagger} Y\right)_{ji}\right]}{\left(Y^{\dagger} Y\right)_{ii} \left(Y^{\dagger} Y\right)_{jj}} \left(f_{ij}^{\mathrm{mix}} + f_{ij}^{\mathrm{osc}}\right)$$

$$f_{ij}^{\mathrm{mix}} = \frac{\left(M_i^2 - M_j^2\right) M_i \Gamma_j}{\left(M_i^2 - M_j^2\right)^2 + M_i^2 \Gamma_j^2}$$

$$f_{ij}^{\mathrm{osc}} = \frac{\left(M_i^2 - M_j^2\right) M_i \Gamma_j}{\left(M_i^2 - M_j^2\right)^2 + \left(M_i \Gamma_i + M_j \Gamma_j\right)^2 \frac{\det[\mathrm{Re}(Y^{\dagger} Y)]}{(Y^{\dagger} Y)_{ii} (Y^{\dagger} Y)_{jj}}}$$

More complete formulae for Neutrino Option

MSbar renormalisation scheme:

$$\Delta M_H^2 = \frac{M_1^2}{8\pi^2}\left(|Y_1|^2 + x_M^2|Y_2|^2\right) ,$$

$$\Delta\lambda = -\frac{1}{32\pi^2}\left[5|Y_1|^4 + 5|Y_2|^4 + 2\,\mathrm{Re}(Y_1\cdot Y_2^*)^2\left(1-\frac{2\log x_M^2}{1-x_M}\right)\right.$$
$$\left. +2\,\mathrm{Im}(Y_1\cdot Y_2^*)^2\left(1-\frac{2\log x_M^2}{1+x_M}\right)\right] ,$$

inputs	value (GeV)
v	174.10
M_H	125.09
M_t	173.2

RGE boundary conditions at $\mu = M_t$			
λ	0.1258	M_H(GeV)	131.431
g_1	0.461	Y_t	0.933
g_2	0.644	Y_b	0.024
g_3	1.22029	Y_τ	0.0102

Analytic Argument for mass scale difference in IO and NO scenario

$$\Delta M_H^2 = \frac{1}{8\pi^2}\mathrm{Tr}\left[Y M^2 Y^\dagger\right] \implies \Delta M_H^2 = \frac{1}{8\pi^2 v^2}\cosh(2y) M^3 (m_1 + m_2 + m_3)$$

Casas Ibarra

If I make the mass scale smaller, I need to increase 'y' to compensate

Simple analytic estimate

$$n_{B-L} \approx \frac{\pi^2}{6 z_d} n^{\mathrm{eq}}(0)$$

$$\frac{\epsilon^{(1)}_{\alpha\alpha}}{K_1 p_{1\alpha}} \approx 16 m_* \left(f_{\mathrm{osc}} + f_{\mathrm{mix}}\right) \frac{m_2 - m_3}{(m_2 + m_3)^2} e^{-4y} \sin 2x$$

Normal Ordering

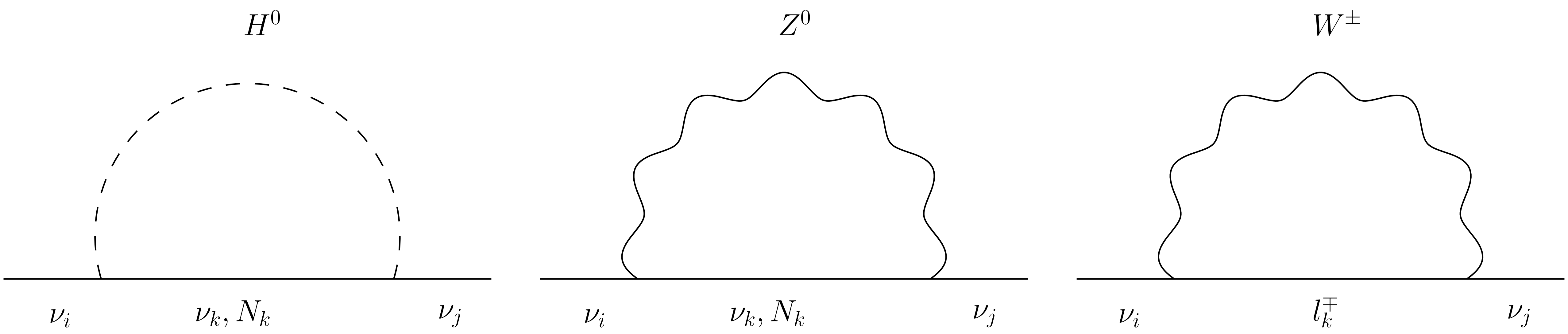
$$\frac{\epsilon^{(1)}_{\alpha\alpha}}{K_1 p_{1\alpha}} \approx 16 m_* \left(f_{\mathrm{osc}} + f_{\mathrm{mix}}\right) \frac{m_1 - m_2}{(m_1 + m_2)^2} e^{-4y} \sin 2x$$

Inverted Ordering

$$\frac{m_2 - m_3}{(m_2 + m_3)^2} > \frac{m_1 - m_2}{(m_1 + m_2)^2}$$

If I increase y I get an exponential suppression which reduces baryon asymmetry

Parametrisation: Radiative Corrections



$$m_\nu = m^{\text{tree}} + m^{\text{1-loop}}.$$

$$Y = \frac{1}{v} m_D = \frac{1}{v} U \sqrt{\hat{m}_\nu} R^T \sqrt{f(M)^{-1}},$$

contains loop contributions

Light Neutrino Mass

$$m^{\text{tree}} \approx m_D M^{-1} m_D^T \qquad m_\nu = m^{\text{tree}} + m^{\text{1-loop}}$$

$$m^{\text{1-loop}} =$$

$$- m_D \left(\frac{M}{32\pi^2 v^2} \left(\frac{\log\left(\frac{M^2}{m_H^2}\right)}{\frac{M^2}{m_H^2} - 1} + 3\frac{\log\left(\frac{M^2}{m_Z^2}\right)}{\frac{M^2}{m_Z^2} - 1} \right) \right) m_D^T,$$

$$= -\frac{1}{32\pi^2 v^2} m_D \text{diag}\left(g\left(M_1\right), g\left(M_2\right), g\left(M_3\right)\right) m_D^T,$$

$$g\left(M_i\right) \equiv M_i \left(\frac{\log\left(\frac{M_i^2}{m_H^2}\right)}{\frac{M_i^2}{m_H^2} - 1} + 3\frac{\log\left(\frac{M_i^2}{m_Z^2}\right)}{\frac{M_i^2}{m_Z^2} - 1} \right),$$

$$f(M) \equiv M^{-1} - \frac{M}{32\pi^2 v^2} \left(\frac{\log\left(\frac{M^2}{m_H^2}\right)}{\frac{M^2}{m_H^2} - 1} + 3\frac{\log\left(\frac{M^2}{m_Z^2}\right)}{\frac{M^2}{m_Z^2} - 1} \right) = \text{diag}\left(\frac{1}{M_1}, \frac{1}{M_2}, \frac{1}{M_3}\right) - \frac{1}{32\pi^2 v^2} \text{diag}\left(g\left(M_1\right), g\left(M_2\right), g\right.$$

Back up slides - the action

$$S_g = \int d^4x \sqrt{-g}(\frac{1}{4\pi G} R + \mathcal{L}_m)$$

$$\mathcal{L}_m = D_\mu \phi^* g^{\mu\nu} D_\nu \phi + \frac{i}{2}\left[\bar{\psi}\gamma^a e^\mu_a D_\mu \psi + (D_\mu \bar{\psi})\gamma^a e^\mu_a \psi\right] - \frac{1}{4} g^{\mu\nu} g^{\rho\sigma} F_{\mu\rho} F_{\nu\sigma}$$

where $F_{\mu\nu} = D_\mu A_\nu - D_\nu A_\mu$ and D_μ denotes the covariant derivative with respect to the gravitational field and gauge fields, and e^μ_a is the vierbein to shift frame to the local Minkowski flat frame.

$$g_{\mu\nu} \rightarrow \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

Perturb the metric, the classical gravitational field is fixed at zero.

$$\text{Graviton propagator} : G_{\mu\nu\rho\sigma}(p) = \frac{i\mathcal{P}_{\mu\nu\rho\sigma}}{p^2}$$

$$\mathcal{P}^{\mu\nu\rho\sigma} = \frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma})$$

Back up slides - kernel structure

$$K(x,y) = \frac{1}{x}\int_0^{\pi} \sin^2\theta d\theta \frac{2xy\cos^2\theta + 4xy + 3\sqrt{xy}(x+y)\cos\theta}{x+y-2\sqrt{xy}\cos\theta}$$

$$L(x,y) = \int_0^{\pi} \sin^2\theta d\theta \left[A\,(x+y+2\sqrt{xy}\cos\theta) - B\,\frac{(x-y)^2}{8(x+y-2\sqrt{xy}\cos\theta)}\right] \times \log\left[x+y-2\sqrt{xy}\cos\theta\right]$$

$$K(x,y) = \frac{\pi}{x}\frac{(x+y)^3 - [(x+y)^2+2xy]|x-y|}{2xy}\,,$$

$$L_A(x,y) = \frac{\pi}{12}\left\{\frac{5(x^2+y^2) - 5(x+y)|x-y| - 6xy}{(x+y)+|x-y|} - 6(x+y)\log\left[\frac{(x+y)+|x-y|}{2}\right]\right\}\,,$$

$$L_B(x,y) = \frac{\pi}{8}\frac{(x-y)^2}{xy}\left\{\frac{(x+y)-|x-y|}{2} - \frac{(x+y)+|x-y|}{2}\log\left[\frac{(x+y)+|x-y|}{2}\right]\right.$$

$$\left. + |x-y|\log(|x-y|)\right\}\,,$$