

# Physics 623: Transmission Lines and Characteristic Impedance: Lab Worksheet

*Please complete before coming to the lab on September 13*

In this lab, we are going to build the circuit of Figure 1 to investigate propagation characteristics of transmission lines. From reflection measurements, we will determine the propagation speed of waves inside a transmission line, and measure its impedance.

Noting Figure 1, consider a transmission line of length  $L$ , impedance  $Z_0$ . Let's denote the propagation speed of electromagnetic waves inside the line as  $u_0$ . We supply a voltage pulse with time duration of  $T$  at point **C** of the circuit. When the pulse hits point **B**, due to impedance mismatch, part of the wave is reflected. This reflected wave encounters a similar phenomenon when it reaches point **A**. As a result, multiple reflections can be observed at points **A** and **B**

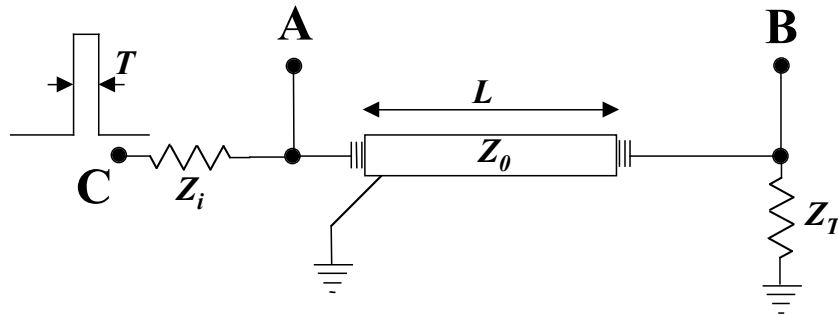


Figure 1: The circuit diagram for the transmission line experiment

1) Assume that  $T \ll L/u_0$ , and  $Z_T, Z_i \gg Z_0$ . Plot what you would expect to see on points **A** and **B** (if we look on a scope for example).

2) Repeat (1), but now assume that  $Z_T = 0$  (short circuit). What would you expect to see on points **A** and **B**.

3) Repeat (1), but now let's introduce some loss to our system (in reality every circuit is lossy). Let's assume that, while propagating from **A** to **B**, 10% of the wave is attenuated. What would you expect to see on points **A** and **B**.

4) In the lecture, we have derived the telegraph equations for a transmission line. The plane wave solution for these equations is of the form

$$V(z, t) = V_0 \cos(\omega_0 t - k_0 z), \quad (1)$$

where  $\omega_0$  is the angular frequency of the wave. As we have also derived in the lecture, the phase velocity for a monochromatic wave is  $u_0 = \omega_0/k_0 = 1/\sqrt{L_L C_L}$ , where  $L_L$  is the inductance per unit length, and  $C_L$  is the capacitance per unit length. When the wave contains multiple frequencies, its velocity can be different than the phase velocity. For the simple case of a two frequency wave,

$$V(z, t) = V_0 \cos(\omega_0 t - k_0 z) + V_0 \cos(\omega_1 t - k_1 z), \quad (2)$$

show that the wave will propagate with a velocity of

$$u_g = \frac{\omega_1 - \omega_0}{k_1 - k_0} \equiv \frac{\Delta\omega}{\Delta k} \quad (3)$$

The quantity,  $u_g$  is called the group velocity of the wave.

5) In the laboratory, we are going to measure the velocity of a square pulse, by measuring the time it takes for the pulse to travel from point **A** to point **B**. Are we measuring the phase velocity or the group velocity? (In our laboratory experiment these two velocities are almost identical  $u_g \approx u_0$ , and we will not worry about the difference between the two velocities).

6) (Optional) Rigorously prove your answer to question (5), by taking the Fourier transform of a time domain square pulse, propagating each Fourier component and then reconstructing the pulse. What is the condition that the pulse doesn't change its shape while propagating through the transmission line?