Finite-Q^2 corrections to parity-violating deep-inelastic scattering

T. Hobbs, Indiana U. & JLab
with
Wally Melnitchouk, JLab

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program

- PVDIS sensitive to nonperturbative features of nucleon structure
- e.g., flavor dependence (proton target); isospin dependence (deuteron target)
- at leading twist, finite-Q^2 corrections (e.g., R^γ, target mass) scale as $1/Q^2$
- how large are these corrections at low to intermediate Q^2 (5 – 10 GeV^2)?

general PVDIS formalism

PVDIS lagrangian with Born-level EW couplings

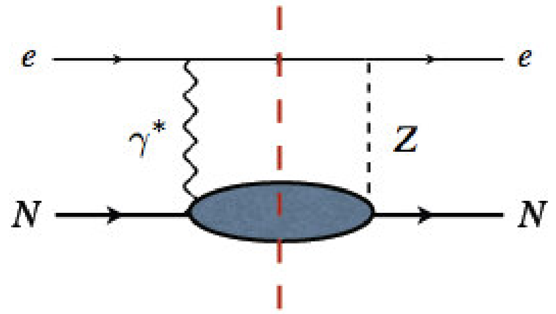
$$\mathcal{L}^{\rm PV} = \frac{G_F}{\sqrt{2}}[\bar{e}\gamma^\mu\gamma_5 e(C_{1u}\bar{u}\gamma_\mu u + C_{1d}\bar{d}\gamma_\mu d) + \bar{e}\gamma^\mu e(C_{2u}\bar{u}\gamma_\mu\gamma_5 u + C_{2d}\bar{d}\gamma_\mu\gamma_5 d)]$$

$$\begin{aligned}
C_{1u} &= g_A^e \cdot g_V^u = -\tfrac{1}{2} + \tfrac{4}{3}\sin^2\theta_W,\\
C_{1d} &= g_A^e \cdot g_V^d = \tfrac{1}{2} - \tfrac{2}{3}\sin^2\theta_W,\\
C_{2u} &= g_V^e \cdot g_A^u = -\tfrac{1}{2} + 2\sin^2\theta_W,\\
C_{2d} &= g_V^e \cdot g_A^d = \tfrac{1}{2} - 2\sin^2\theta_W.
\end{aligned}$$

hadronic tensor:

$$W^i_{\mu\nu} = -\frac{g_{\mu\nu}}{M} F^i_1 + \frac{p_\mu p_\nu}{M\ p\cdot q} F^i_2 + \frac{i\varepsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta}{2Mp\cdot q} F^i_3$$
$$+ \frac{i\varepsilon_{\mu\nu\alpha\beta}}{p\cdot q}\left(q^\alpha S^\beta\ g^i_1 + 2xp^\alpha S^\beta\ g^i_2\right) - \frac{p_\mu S_\nu + S_\mu p_\nu}{2p\cdot q} g^i_3$$
$$+ \frac{S\cdot q\ p_\mu p_\nu}{(p\cdot q)^2} g^i_4 + \frac{S\cdot q\ g_{\mu\nu}}{p\cdot q} g^i_5\ ,$$

- i selects virtual photon current, interference current, etc.



$$A^{\mathrm{PV}} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

- numerator sensitive to the interference current; denominator is purely electromagnetic

$$A^{\mathrm{PV}} = -\left(\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\right) \frac{g_A^e (2xyF_1^{\gamma Z} - 2[1 - 1/y + xM/E]F_2^{\gamma Z}) + g_V^e x(2-y)F_3^{\gamma Z}}{2xyF_1^{\gamma} - 2[1 - 1/y + xM/E]F_2^{\gamma}}$$

- sums and differences of inclusive cross sections select the relevant SFs from the hadron tensor unknown phenomenology

$$R^{\gamma(\gamma Z)} \equiv \frac{\sigma_L^{\gamma(\gamma Z)}}{\sigma_T^{\gamma(\gamma Z)}} = r^2 \frac{F_2^{\gamma(\gamma Z)}}{2xF_1^{\gamma(\gamma Z)}} - 1$$

$$r^2 = 1 + \frac{Q^2}{\nu^2} = 1 + \frac{4M^2x^2}{Q^2}$$

- acts as measure of violation of Callan-Gross; in principle, $R^{\gamma Z}$ should deviate from R^{γ}

$$A^{\rm PV} = -\left(\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\right)\left[g_A^e\, Y_1\, \frac{F_1^{\gamma Z}}{F_1^{\gamma}} + \frac{g_V^e}{2}\, Y_3\, \frac{F_3^{\gamma Z}}{F_1^{\gamma}}\right]$$

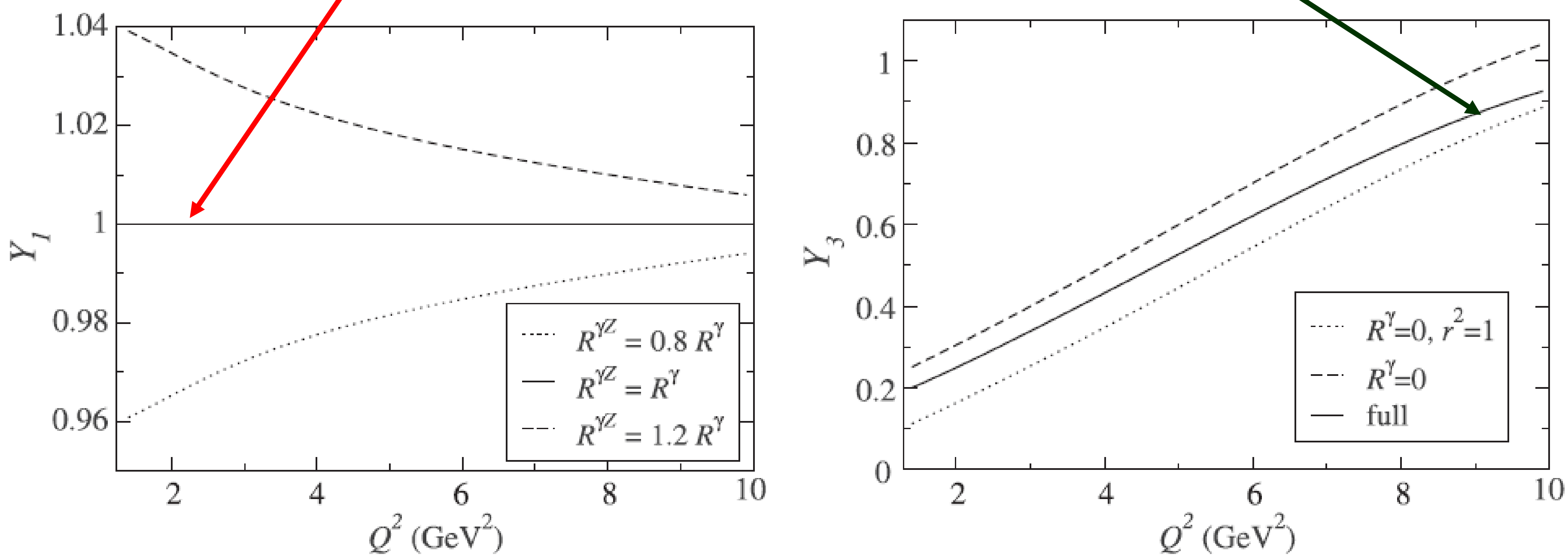
$$Y_1 = \frac{1+(1-y)^2 - y^2(1-r^2/(1+R^{\gamma Z})) - 2xyM/E}{1+(1-y)^2 - y^2(1-r^2/(1+R^{\gamma})) - 2xyM/E}\left(\frac{1+R^{\gamma Z}}{1+R^{\gamma}}\right)$$

$$Y_3 = \frac{1-(1-y)^2}{1+(1-y)^2 - y^2(1-r^2/(1+R^{\gamma})) - 2xyM/E}\left(\frac{r^2}{1+R^{\gamma}}\right).$$

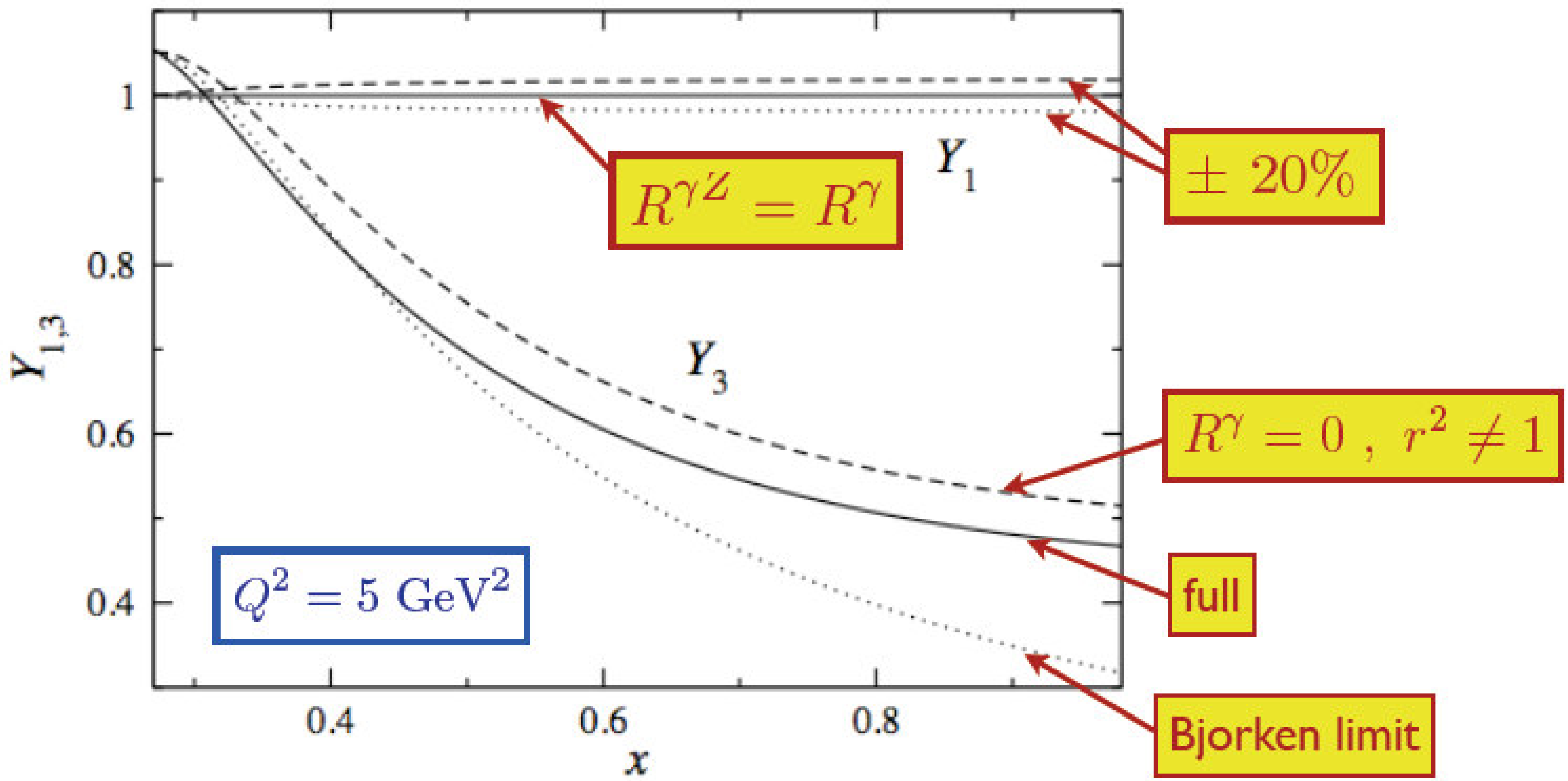
▪in the Bjorken limit, Y_1 & Y_3 return the familiar DIS result

$$Y_1 \longrightarrow 1,$$

$$Y_3 \longrightarrow \frac{1-(1-y)^2}{1+(1-y)^2} \equiv f(y).$$



▪in general, deviation from Bj.-limit kinematics more significant in "F_3" term:



anticipating finite-Q^2 behavior of $R^{\gamma Z}$?

- in $Q^2 \to \infty$ limit, pQCD suggests $R^{\gamma Z} \to R^{\gamma}$

- in $Q^2 \to 0$ limit, $R^{\gamma Z}$,$R^{\gamma} \to 0$ by vector current conservation

- not the case in $Q^2 \to 0$ limit for R^Z, due to axial current non-conservation

Kulagin, Petti, PRD ***76*** *(2007) 094023*

GOAL: interpolate $R^{\gamma Z}$ at finite Q^2 from VMD in vector current and pQCD behavior

▪using QPM expressions for SFs in this analysis; that is,

$$F_1^{\gamma}(x) = \frac{1}{2}\sum_q e_q^2(q(x) + \bar{q}(x))$$

$$F_2^{\gamma}(x) = 2xF_1^{\gamma}(x),$$

photon exchange

$$F_1^{\gamma Z}(x) = \sum_q e_q g_V^q(q(x) + \bar{q}(x)),$$

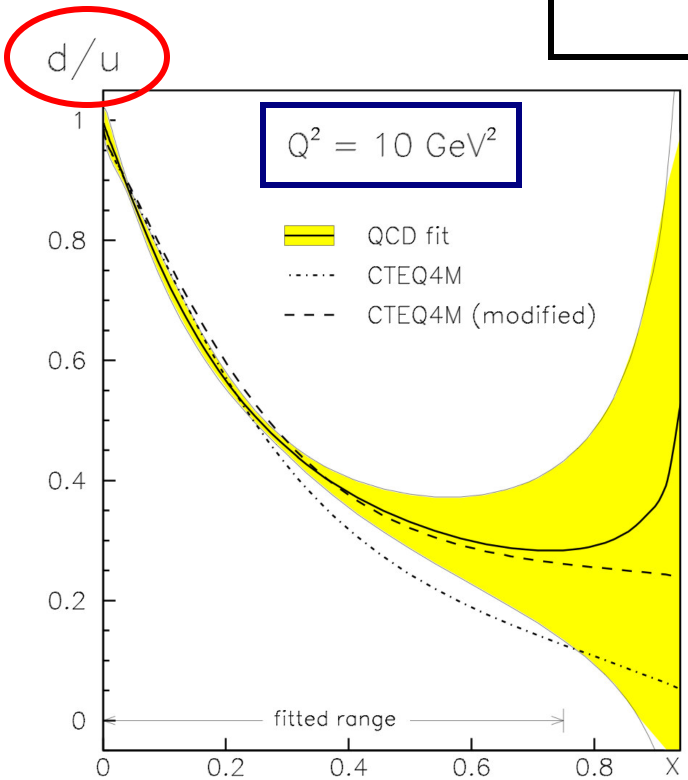
$$F_2^{\gamma Z}(x) = 2xF_1^{\gamma Z}(x),$$

$$F_3^{\gamma Z}(x) = 2\sum_q e_q g_A^q(q(x) - \bar{q}(x))$$

photon / Z boson exchange

flavor structure of the proton

motivation



- LARGE uncertainty at high Bjorken x
- consequence mostly of nuclear corrections

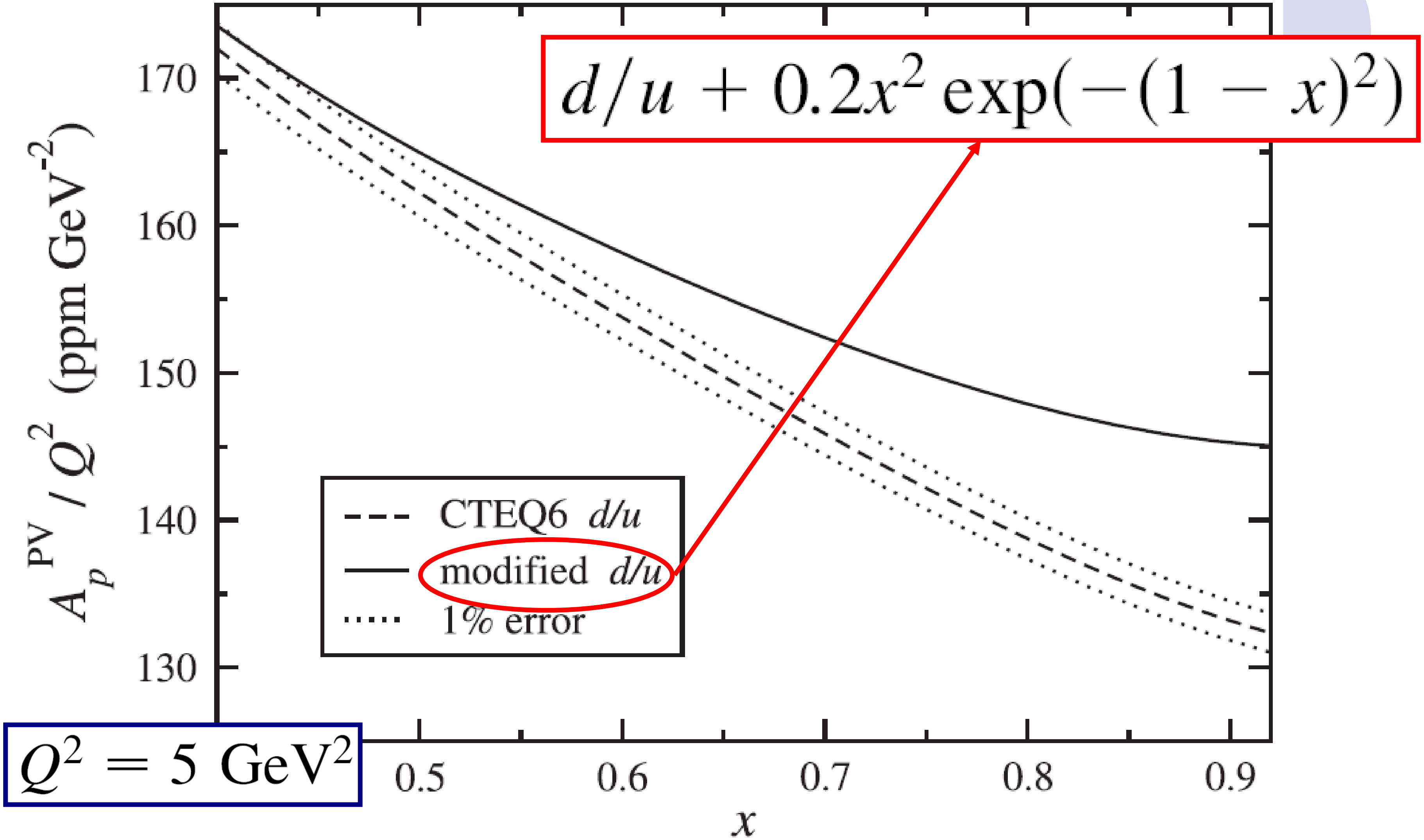
▪ various unconstrained scenarios for high-x behavior; e.g., scalar diquark dominance (S = 0): d/u → 0, S_Z = 0 dominance: d/u → 1/5

$$A^{\mathrm{PV}} = -\left(\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\right)(Y_1 a_1 + Y_3 a_3)$$

specializing to valence-level quarks of the proton,

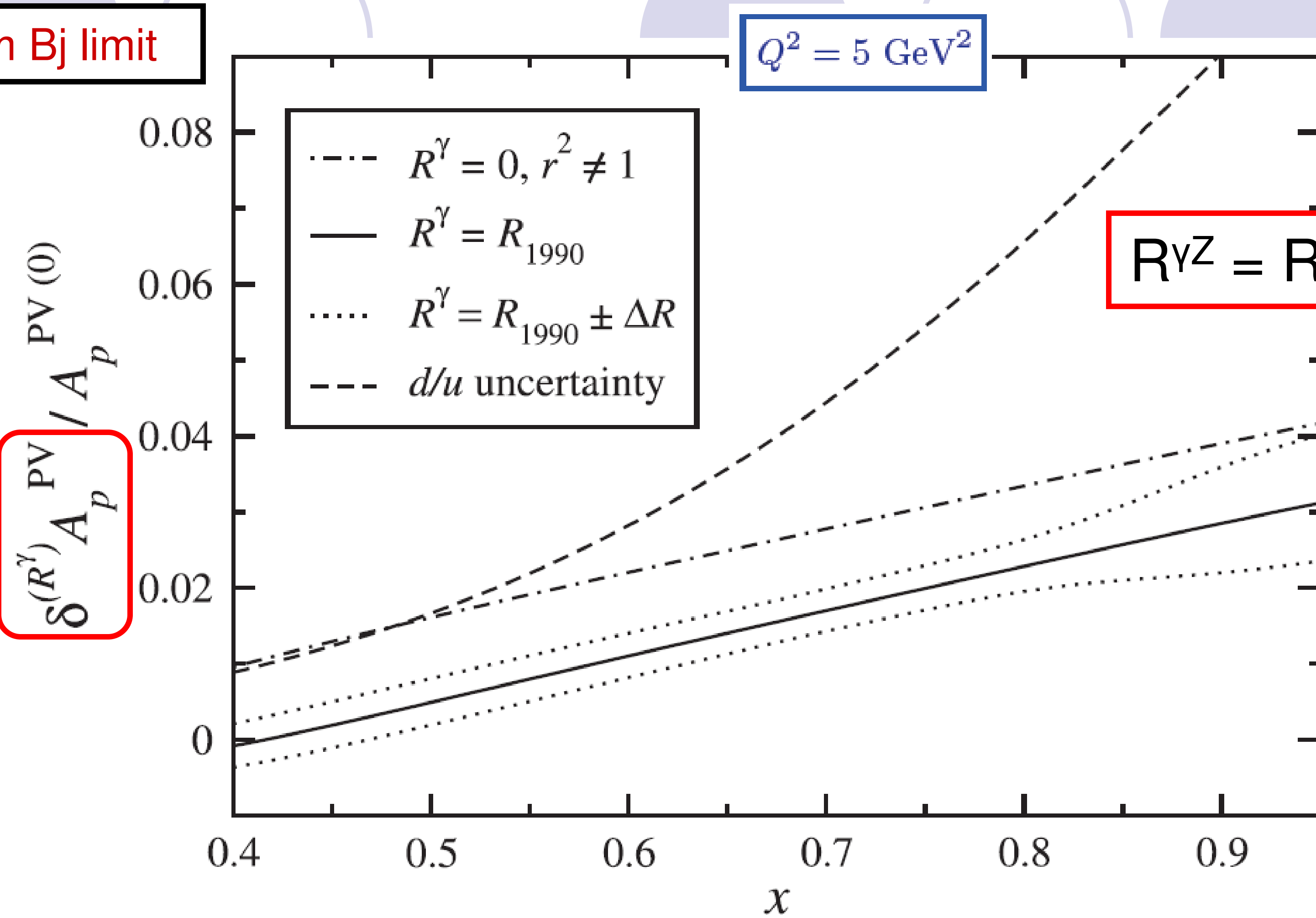
$$a_1 = \frac{2\sum_q e_q C_{1q}(q+\bar{q})}{\sum_q e_q^2 (q+\bar{q})} \qquad a_3 = \frac{2\sum_q e_q C_{2q}(q-\bar{q})}{\sum_q e_q^2 (q+\bar{q})}$$

$$a_1^p = \frac{12C_{1u} - 6C_{1d}d/u}{4 + d/u} \qquad a_3^p = \frac{12C_{2u} - 6C_{2d}d/u}{4 + d/u}$$

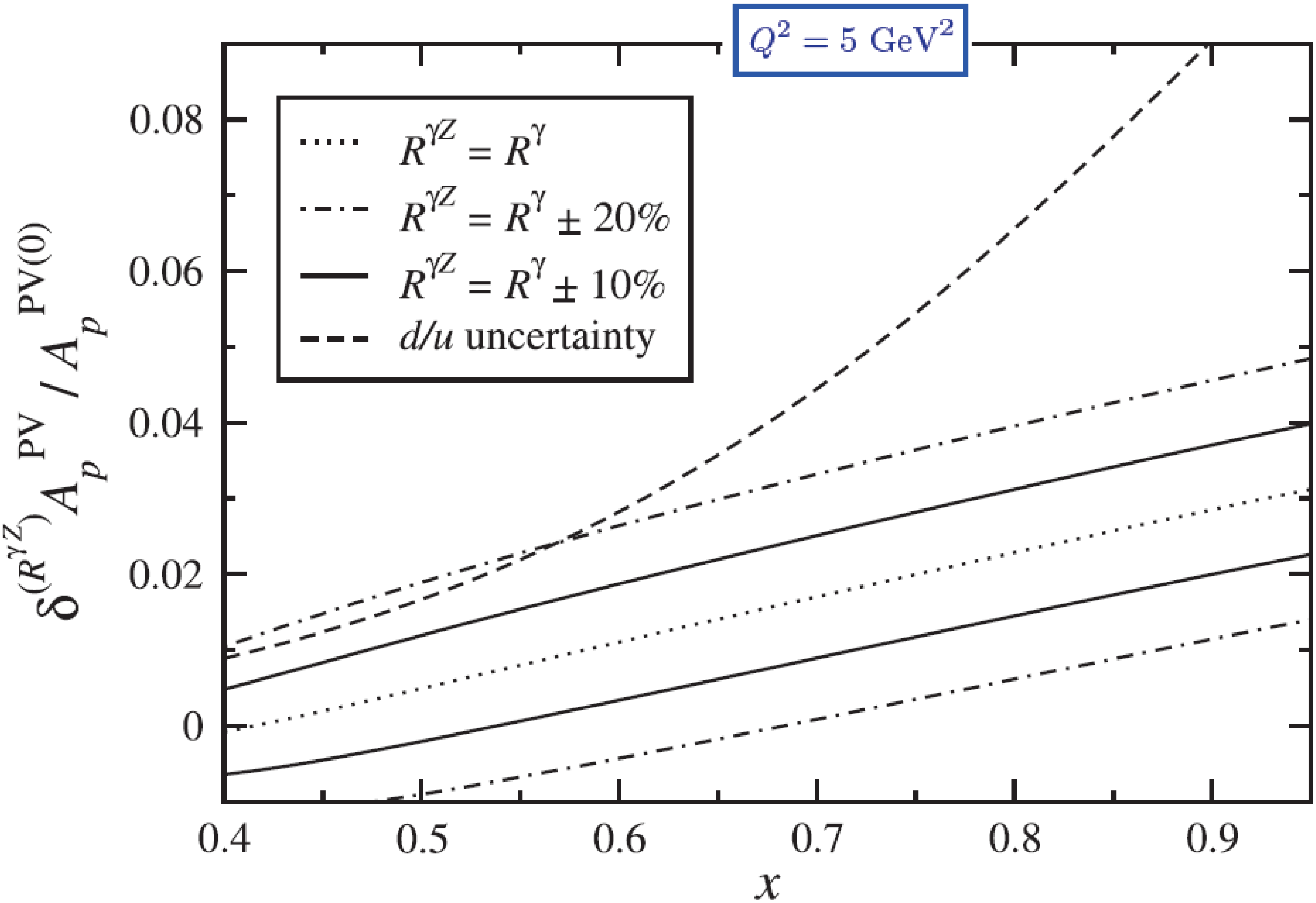

$d/u + 0.2x^2 \exp(-(1-x)^2)$
170
160
150
140
130
A_p^{PV}/Q^2 (ppm GeV^{-2})
CTEQ6 d/u
modified d/u
1% error
0.5
0.6
0.7
0.8
0.9
x
$Q^2 = 5$ GeV^2

deviation from Bj limit

$R^{\gamma Z} = R^{\gamma}$



→ at large x, d/u signal “safe” from uncertainty introduced by R^{γ}



→ $R^{\gamma Z}$ must be controlled

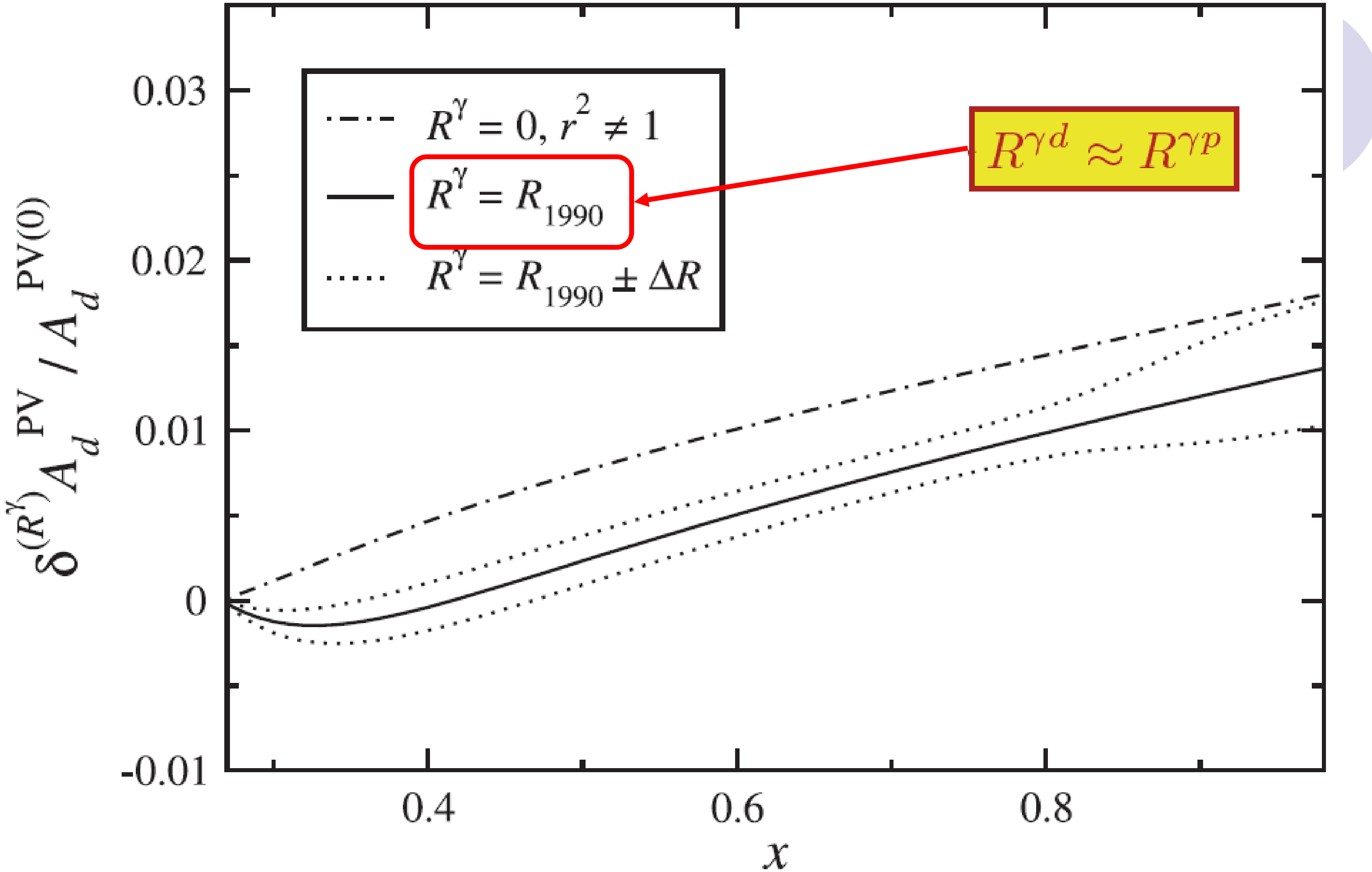
isospin dependence from the deuteron

- deuteron is isoscalar → no flavor dependence in vector & axial-vector terms:

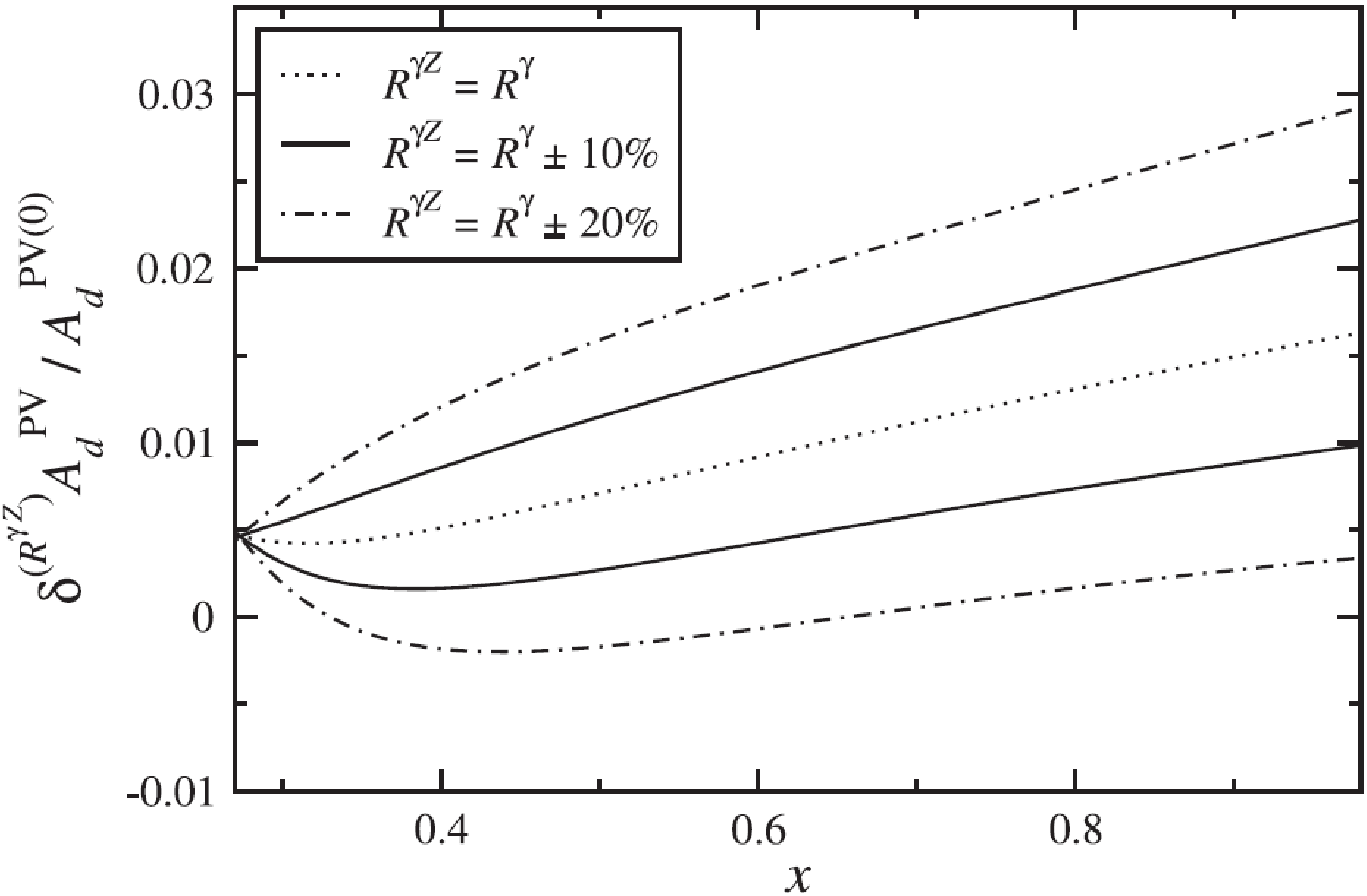
$$a_1^d = \tfrac{6}{5}(2C_{1u} - C_{1d})$$
$$a_3^d = \tfrac{6}{5}(2C_{2u} - C_{2d})$$

- x-dependence in correction originates solely through Y_1 & Y_3:

$$A^{PV} = -\left(\frac{3G_F Q^2}{10\sqrt{2}\pi\alpha}\right)[Y_1(2C_{1u} - C_{1d}) + Y_3(2C_{2u} - C_{2d})]$$



▪contributes uncertainty ~ 0.5%



- $R^{\gamma Z}$ contrib. error ~ 2% even in conservative scenario → may be a problem

Thanks to KK:

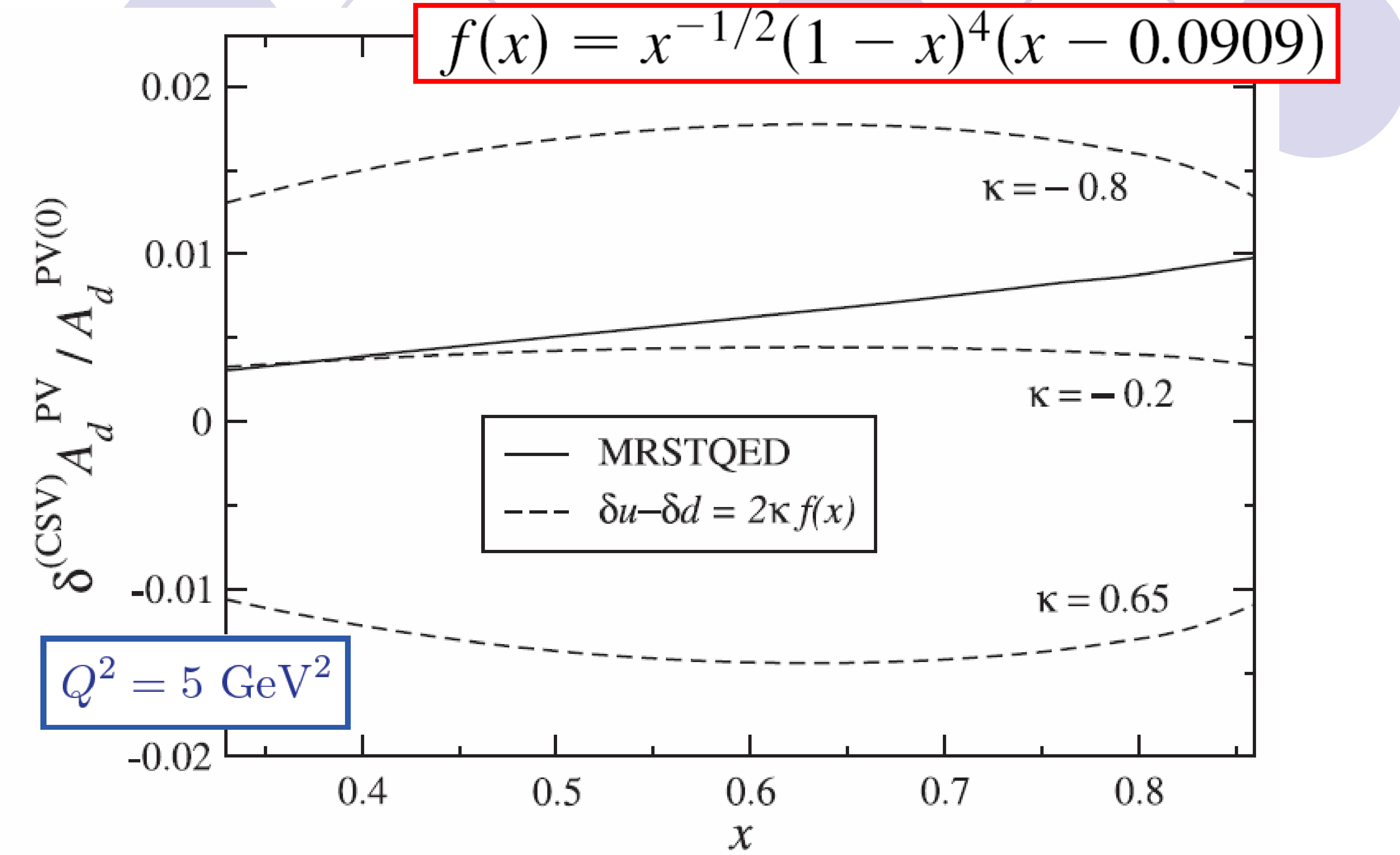
$$\delta u = u^p - d^n$$
$$\delta d = d^p - u^n$$

$$u \equiv u^p - \frac{\delta u}{2} = d^n + \frac{\delta u}{2}$$
$$d \equiv d^p - \frac{\delta d}{2} = u^n + \frac{\delta d}{2}$$

- deuteron asym. terms defined in the presence of CSV:

$$a_1^d = a_1^{d(0)} + \delta^{(\mathrm{CSV})} a_1^d$$
$$a_3^d = a_3^{d(0)} + \delta^{(\mathrm{CSV})} a_3^d$$

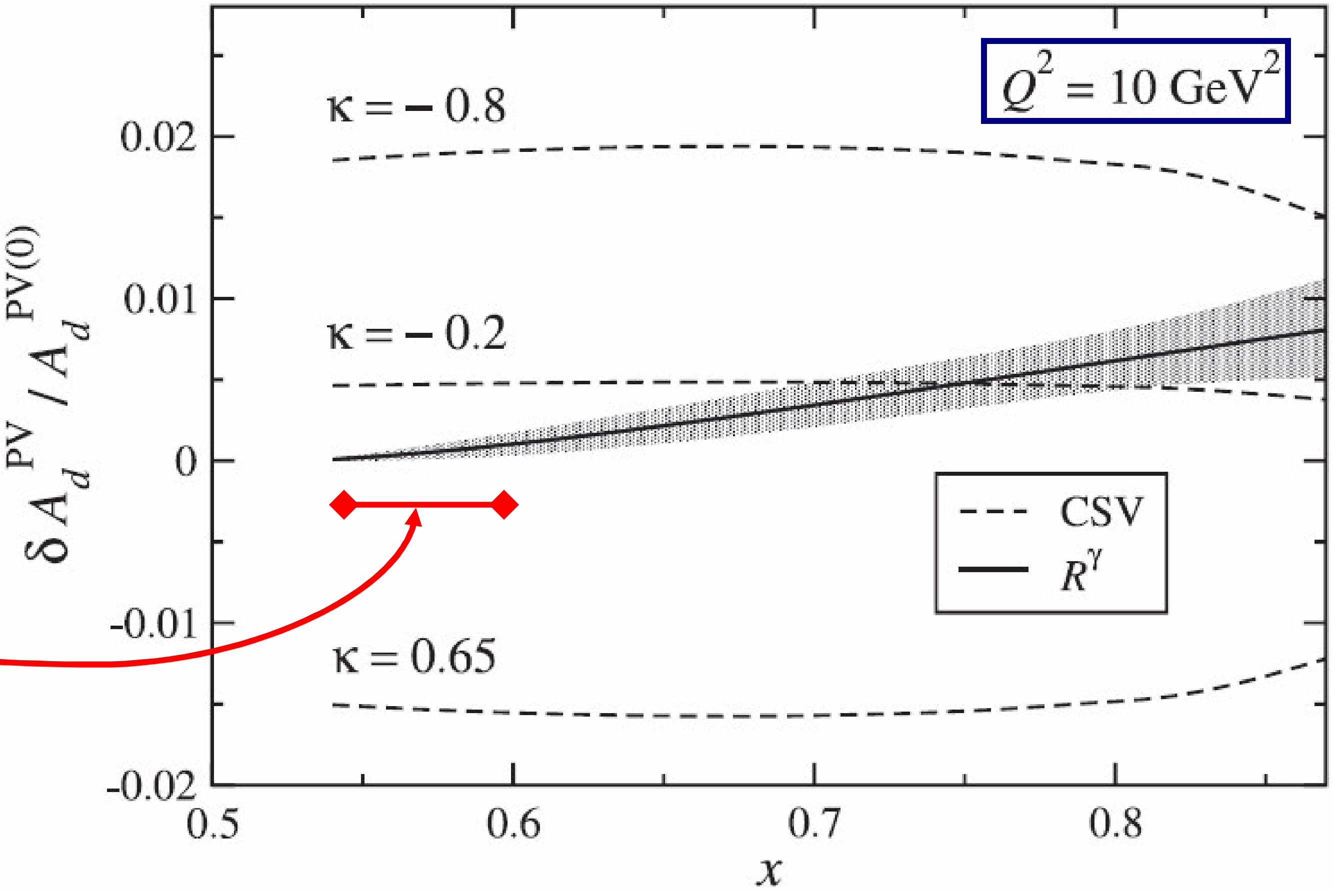
$$\frac{\delta^{(\mathrm{CSV})} a_1^d}{a_1^{d(0)}} = \left(-\frac{3}{10} + \frac{2C_{1u} + C_{1d}}{2(2C_{1u} - C_{1d})}\right)\left(\frac{\delta u - \delta d}{u + d}\right)$$
$$\frac{\delta^{(\mathrm{CSV})} a_3^d}{a_3^{d(0)}} = \left(-\frac{3}{10} + \frac{2C_{2u} + C_{2d}}{2(2C_{2u} - C_{2d})}\right)\left(\frac{\delta u - \delta d}{u + d}\right)$$



separate LT & HT contributions to CSV at higher Q^2

- R^{γ} does not threaten observation of CSV at large Q^2 ($\approx$ 10 GeV^2), provided CSV effects $\geq$ 0.5 – 1.0%

intermediate x most promising if CSV small



- still, $R^{\gamma Z}$ » R^{γ} or $R^{\gamma Z}$ « R^{γ} problematic

target mass effects

OPE formalism

O. Nachtmann, Nucl. Phys. B **63**, 237 (1973)

$$\int d^4x \; e^{iq\cdot x} \langle N | T(J^\mu(x) J^\nu(0)) | N \rangle$$

$$= \sum_k \left(-g^{\mu\nu} q^{\mu_1} q^{\mu_2} + g^{\mu\mu_1} q^\nu q^{\mu_2} + q^\mu q^{\mu_1} g^{\nu\mu_2} + g^{\mu\mu_1} g^{\nu\mu_2} Q^2 \right)$$

$$\times q^{\mu_3} \cdots q^{\mu_{2k}} \frac{2^{2k}}{Q^{4k}} A_{2k} \Pi_{\mu_1 \cdots \mu_{2k}}$$

$$\langle N | \mathcal{O}_{\mu_1 \cdots \mu_{2k}} | N \rangle$$

local operators

$$\Pi_{\mu_1 \cdots \mu_{2k}} = p_{\mu_1} \cdots p_{\mu_{2k}} - (g_{\mu_i \mu_j} \text{ terms})$$

$$= \sum_{j=0}^{k} (-1)^j \frac{(2k-j)!}{2^j (2k)^j} g \cdots g \; p \cdots p$$

traceless, symmetric rank-$2k$ tensor

e.g.: F_2 in the OPE setting

$$M_2^n(Q^2) = \int dx\ x^{n-2}\ F_2(x, Q^2)$$

$$= \sum_{j=0}^{\infty} \left(\frac{M^2}{Q^2}\right)^j \frac{(n+j)!}{j!(n-2)!} \frac{A_{n+2j}}{(n+2j)(n+2j-1)}$$

- TM corrected SF extracted from Cornwall-Norton moment with inverse Mellin transform

LT OPE-corrected SFs:

$$F_1^{\mathrm{OPE}}(x,Q^2) = \frac{x}{\xi\rho}F_1^{(0)}(\xi,Q^2) + \frac{M^2x^2}{Q^2\rho^2}\int_\xi^1 du\frac{F_2^{(0)}(u,Q^2)}{u^2} + \frac{2M^4x^3}{Q^4\rho^3}\int_\xi^1 dv(v-\xi)\frac{F_2^{(0)}(v,Q^2)}{v^2},$$

$$F_2^{\mathrm{OPE}}(x,Q^2) = \frac{x^2}{\xi^2\rho^3}F_2^{(0)}(\xi,Q^2) + \frac{6M^2x^3}{Q^2\rho^4}\int_\xi^1 du\frac{F_2^{(0)}(u,Q^2)}{u^2} + \frac{12M^4x^4}{Q^4\rho^5}\int_\xi^1 dv(v-\xi)\frac{F_2^{(0)}(v,Q^2)}{v^2},$$

$$F_3^{\mathrm{OPE}}(x,Q^2) = \frac{x}{\xi\rho^2}F_3^{(0)}(\xi,Q^2) + \frac{2M^2x^2}{Q^2\rho^3}\int_\xi^1 du\frac{F_3^{(0)}(u,Q^2)}{u},$$

- modified scaling via nucleon mass-dependent variable:

$$\xi(x,Q^2) = \frac{2x}{1+\rho}, \qquad \rho = \sqrt{1+4M^2x^2/Q^2}$$

▪alternatively, LT collinear factorization (CF) separates hard and soft processes

S. Kretzer and M. H. Reno, Phys. Rev. D **69**, 034002 (2004)

▪amplitude then scales with external (i.e., not partonic) momenta

$$F_1^{\mathrm{CF}}(x,Q^2) = F_1^{(0)}(\xi,Q^2) \ ,$$
$$F_2^{\mathrm{CF}}(x,Q^2) = \frac{x}{\xi\rho^2} F_2^{(0)}(\xi,Q^2),$$
$$F_3^{\mathrm{CF}}(x,Q^2) = \frac{1}{\rho} F_3^{(0)}(\xi,Q^2) \ .$$

▪again, rescaled SFs with Nachtmann variable; kinematical prefactor

- TMC introduces some non-physical behavior at high x:

$$\xi_0 \equiv \xi(x=1) < 1$$

$$F(\xi_0) > 0 \quad \Rightarrow \quad F_i^{\mathrm{TMC}}(x=1, Q^2) > 0$$

- by definition, SFs should approach zero at large x

- Why not? Higher twists a plausible culprit

somewhat ad hoc "solution" due to Kulagin-Petti:

S. A. Kulagin and R. Petti, Nucl. Phys. A **765** (2006) 126

$$F_1^{\rm OPE}(x,Q^2) \approx F_1^{(0)}(x,Q^2) + \frac{x^3M^2}{Q^2}\left(2\int_x^1 \frac{dz}{z^2}F_1^{(0)}(z,Q^2) - \frac{\partial}{\partial x}F_1^{(0)}(x,Q^2)\right)$$

$$F_2^{\rm OPE}(x,Q^2) \approx \left(1-\frac{4x^2M^2}{Q^2}\right)F_2^{(0)}(x,Q^2) + \frac{x^3M^2}{Q^2}\left(6\int_x^1 \frac{dz}{z^2}F_2^{(0)}(z,Q^2) - \frac{\partial}{\partial x}F_2^{(0)}(x,Q^2)\right),$$

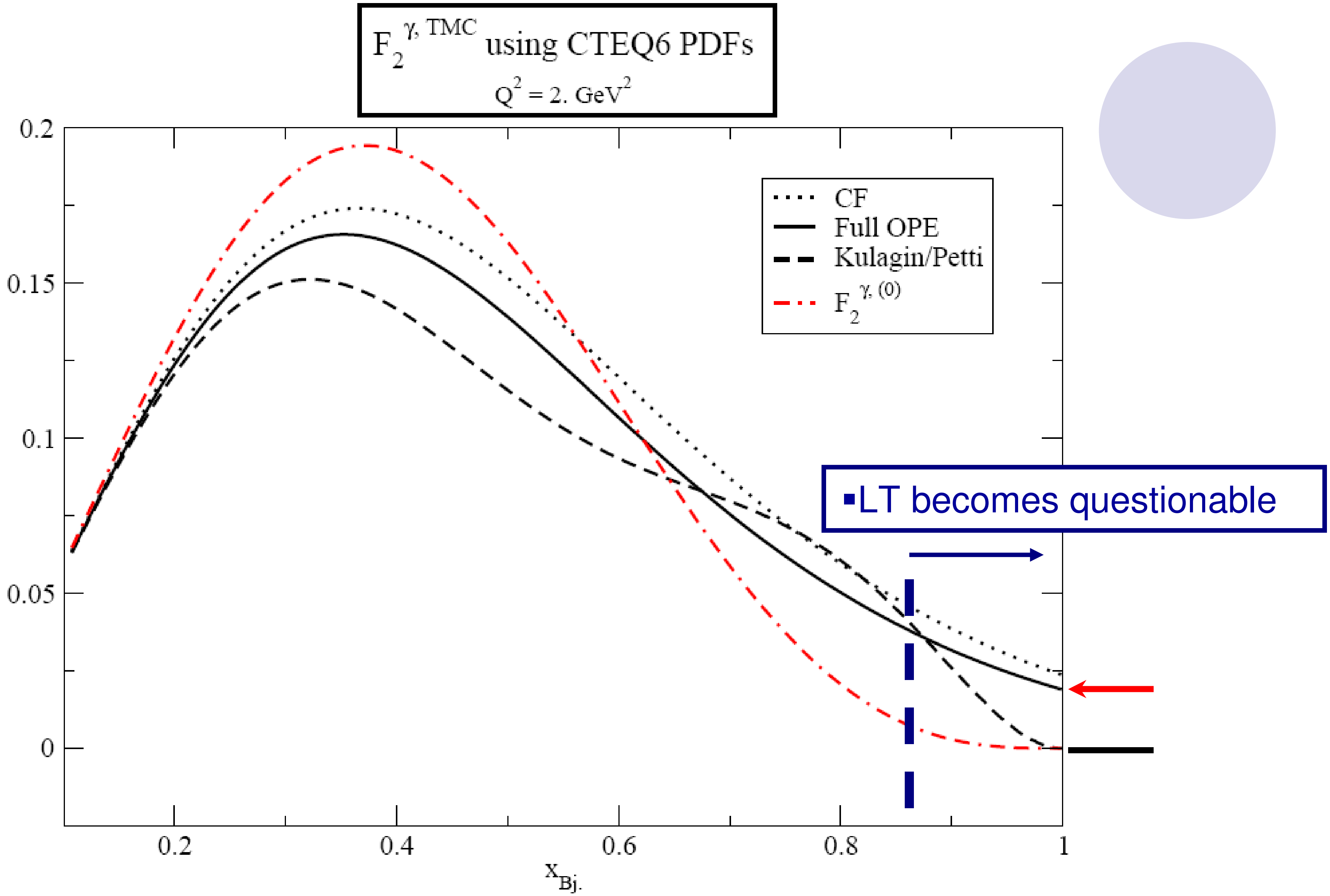
$$F_3^{\rm OPE}(x,Q^2) \approx \left(1-\frac{2x^2M^2}{Q^2}\right)F_3^{(0)}(x,Q^2) + \frac{x^3M^2}{Q^2}\left(2\int_x^1 \frac{dz}{z^2}F_3^{(0)}(z,Q^2) - \frac{\partial}{\partial x}F_3^{(0)}(x,Q^2)\right).$$

- expanded LT OPE in $1/Q^2$ to next-to-leading term
- circumvents threshold problem

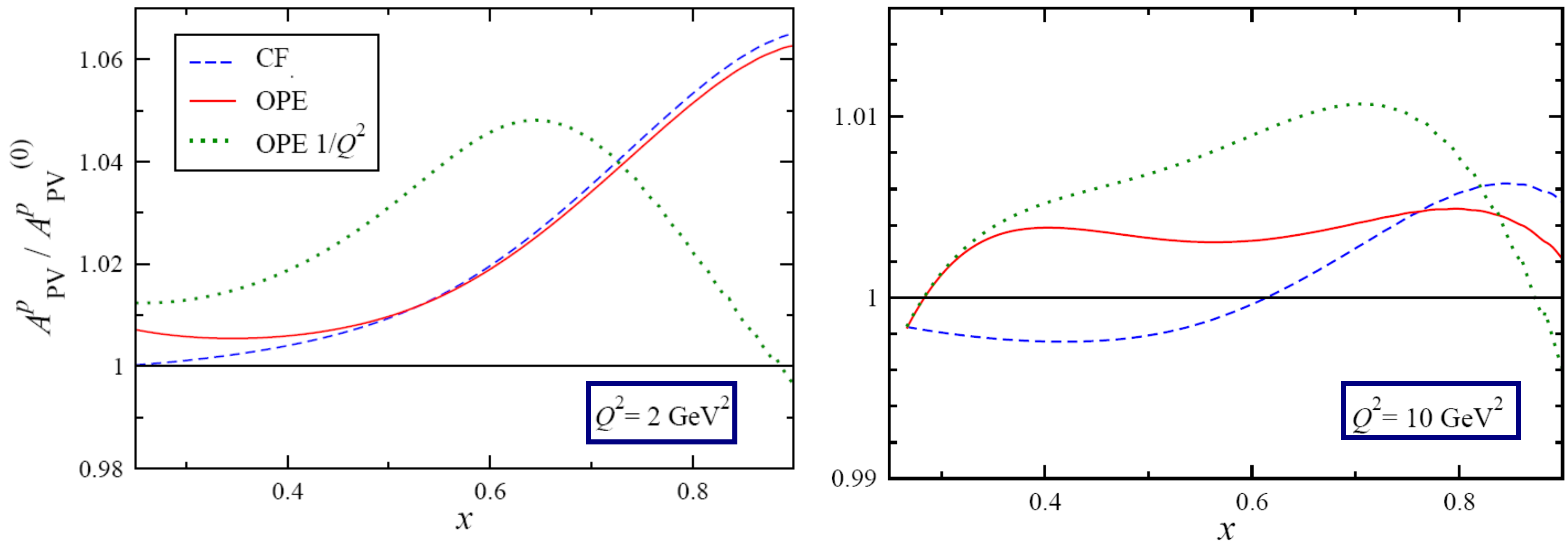
▪K/P prescription naively simulates inclusion of HT at large-x

▪i.e., insofar as K/P “brute-forces” TM corrected SFs to observe proper threshold behavior

▪simple comparison of LT OPE and Kulagin-Petti may suggest where in x HTs become important



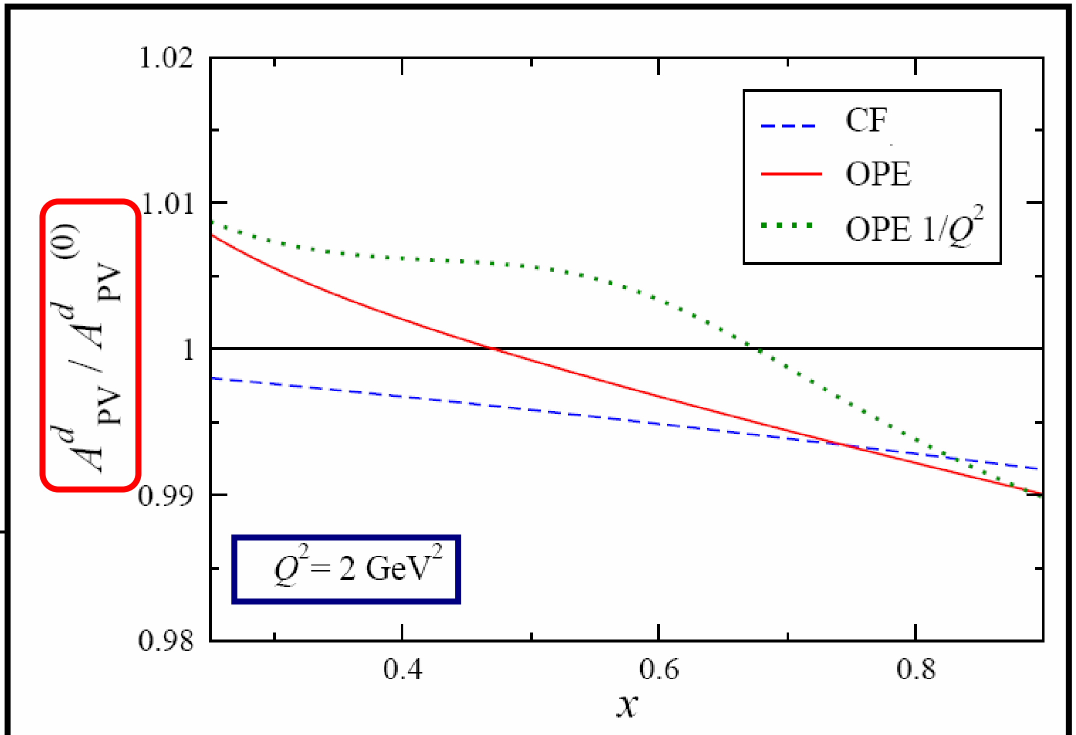
K/P prescription consistent with other LT treatments for x < 0.8

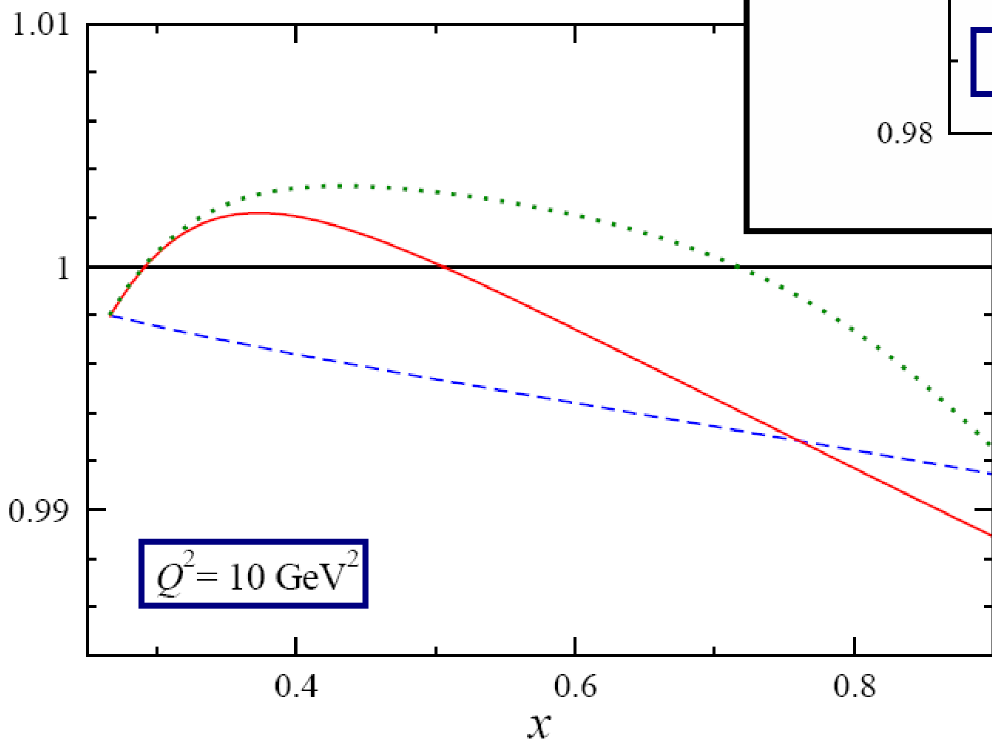


size of TMC highly Q^2 dependent in proton A^{PV}

yet another source of uncertainty (contrib. ~ 2 – 3 %) at low Q^2

deuteron asymmetry calculation





similar story, but weaker Q^2 dependence

spin-polarized DIS

unpolarized electron on longitudinally polarized hadron

- helicity-averaged hadron tensor → purely spin-dependent cross section:

$$\frac{d^2\sigma^{\rm PV}}{dxdy}(\bar{\lambda}, S_L) = 2x\left(2 - y - \frac{xyM}{E}\right)g_1^{\gamma Z} - \frac{4x^2M}{E}g_2^{\gamma Z} + \frac{2}{y}\left(1 - y - \frac{xyM}{2E}\right)g_3^{\gamma Z} - \frac{2}{y}\left(1 + \frac{xM}{E}\right)\left(1 - y - \frac{xyM}{2E}\right)g_4^{\gamma Z} + 2xy\left(1 + \frac{xM}{E}\right)g_5^{\gamma Z},$$

$$\Delta A^{\rm PV} = \frac{\sigma^{\rm PV}(\bar{\lambda}, S_L) - \sigma^{\rm PV}(\bar{\lambda}, -S_L)}{\sigma^{\rm PV}(\bar{\lambda}, S_L) + \sigma^{\rm PV}(\bar{\lambda}, -S_L)}$$

$$\Delta A^{\rm PV} = \frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\left(g_A^e f(y)\frac{g_1^{\gamma Z}}{F_1^{\gamma}} + g_V^e \frac{g_5^{\gamma Z}}{F_1^{\gamma}}\right)$$

- spin SFs encode target polarization in parton level expressions -

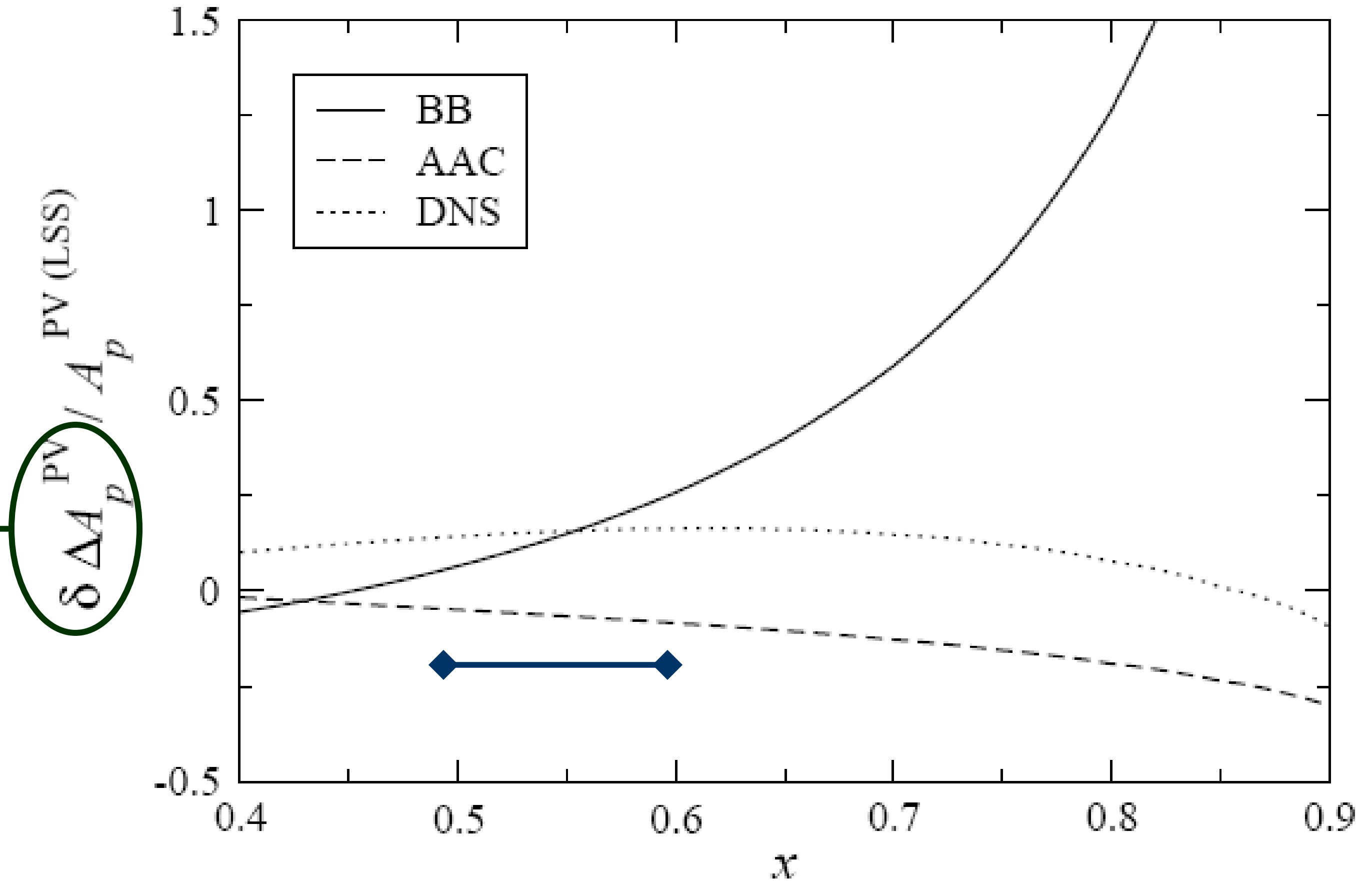
$$g_1^{\gamma Z} = \sum_q e_q g_V^q (\Delta q + \Delta \bar{q}),$$
$$g_5^{\gamma Z} = \sum_q e_q g_A^q (\Delta q - \Delta \bar{q}),$$

- asymmetry dependence upon valence quark spin PDFs analogous to the unpolarized case:

$$\Delta A_p^{\mathrm{PV}} = \frac{6 G_F Q^2}{4\sqrt{2}\pi\alpha} [(2C_{1u}\Delta u - C_{1d}\Delta d) f(y) + (2C_{2u}\Delta u - C_{2d}\Delta d)] \left(\frac{1}{4u + d} \right),$$

sensitivity to Δu, Δd:

deviation of phenom. fit from LSS



•over (0.5≤x≤0.6), sens. ~ 20%; large x dependence

summary

- finite-Q^2 corrections may threaten flavor/isospin info. in few-GeV^2 region
- such corrections should receive contrib. from HT
- → more work on higher twist effects needed