

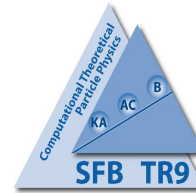


Good Morning to Madison !

# Large $x$ Higher Twist Contributions in DIS

## Polarized Twist-2 and Twist-3 Integral Relation

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- Non-Singlet QCD Analysis of  $l^\pm N$  Data
- Extraction of Non-leading Twist Contribution
- Polarized Twist-2 and Twist-3 Integral Relations

[1 ] J.B. and H. Böttcher, arXiv:0802.0408 [hep-ph]; Phys. Lett. **B662** (2008) 336

[2 ] J.B., A. Guffanti, and H. Böttcher, Nucl.Phys.**B774** (2007) 182.

[3 ] J.B., and A. Tkabladze, Nucl.Phys.**B553** (1999) 427.

## Introduction

- Higher Twist Contributions are expected to contribute to DIS Structure Functions both
  - at large  $x$ 

e.g. S. Gottlieb, Nucl. Phys. **B139** (1978) 125; H.D. Politzer, Nucl. Phys. **B172** (1980) 349;  
R.K. Ellis, W. Furmanski, R. Petronzio, Nucl. Phys. **B207** (1982) 1; **B212** (1983) 29
  - at small  $x$ 

e.g. L.V. Gribov, E.M. Ryskin, and M.G. Ryskin, Nucl. Phys. **B188** (1981) 555; A.H. Mueller and J.W. Qiu, Nucl. Phys. **B268** (1986) 427; J. C. Collins and J. Kwiecinski, Nucl. Phys. **B335** (1990) 89;  
J. Bartels, J. Blümlein, and G.A. Schuler, Z. Phys. **C50** (1991) 91
- In this talk we will investigate the Large  $x$  Region.

- How to separate **Twist-2** and **Higher Twist** contributions ?
  - Systematic investigation of  $\ln(Q^2)$ -slopes of  $F_i(x, Q^2)$  cutting from large  $Q^2$
  - A cut of  $W^2 > 12.5 \text{ GeV}^2$  allows the separation (see below).
- No **sufficient** Theoretical Description of Higher Twist Contributions is available for :
  - Anomalous Dimensions
  - Wilson Coefficients
  - HT Correlation Functions
- Fits starting with a **phenomenological Ansatz** are therefore not possible.

## Fits under Phenomenological Assumptions :

e.g. K. Varvell et al. [BEBC] Z. Phys. **C36** (1987) 1; S.I. Alekhin, A.L. Kataev, Phys. Lett. **B452** (1999) 402; M. Botje, Eur. Phys. J. **C14** (2000) 285; S. Simula, Phys. Lett. **B493** (2000) 325; A.L. Kataev, Nucl. Phys. B (Suppl. Proc.) **116** (2003) 105; A. Martin et al., Eur. Phys. J. **C35** (2004) 325; S.I. Alekhin, S.A. Kulagin, S. Liuti, Phys. Rev. **D69** (2004) 114009; M. Osipenko et al., Nucl. Phys. **A766** (2006) 142;

- How to separate **Twist-2** and **Higher Twist** contributions ?
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### Extraction of HT Terms :

- Twist-2 QCD Fit in the large  $W^2$  region
- Application of Target Mass Corrections
- Extrapolation of the Leading Twist Results into the lower  $W^2$  region:  
 $W^2, Q^2 \geq 4 \text{ GeV}^2$ .

## Twist-2 Non-Singlet Analysis [2]

- Separate the kinematic region  $x < 0.4$ ,  $x > 0.4$
- $x < 0.4$  :  $F_{2\text{NS}}$  from  $p$  and  $d$  data and  $\bar{d} - \bar{u}$  (Drell-Yan process.)
- $x > 0.4$  : Valence approximation for  $F_2^{p,d}$  from  $p$
- 3-Loop QCD Analysis
  - 3L Anom. Dim. S. Moch, J. Vermaseren, A. Vogt. Nucl. Phys. **B688** (2004) 101
- Known Heavy Flavor contributions are taken into account (NLO)
  - 1% and smaller
- Effective 4-Loop QCD Analysis possible
  - 3L WC: J. Vermaseren, A. Vogt., S. Moch Nucl. Phys. **B724** (2005) 3

Finally:

Account for the dominant large  $x$  Terms in the Wilson Coefficient to  $O(\alpha_s^4)$

S. Moch, J. Vermaseren, A. Vogt. Nucl. Phys. **B726** (2005) 317;

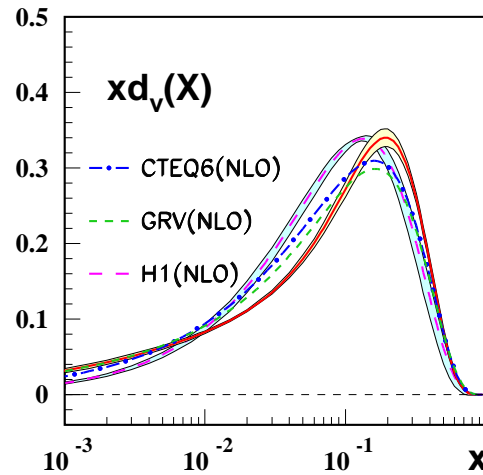
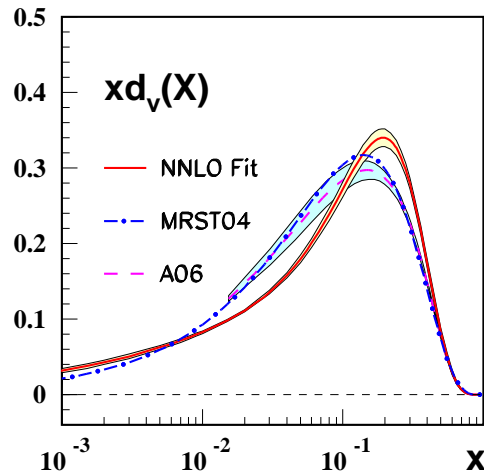
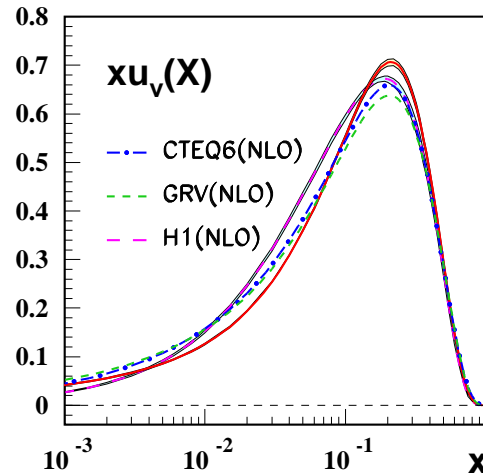
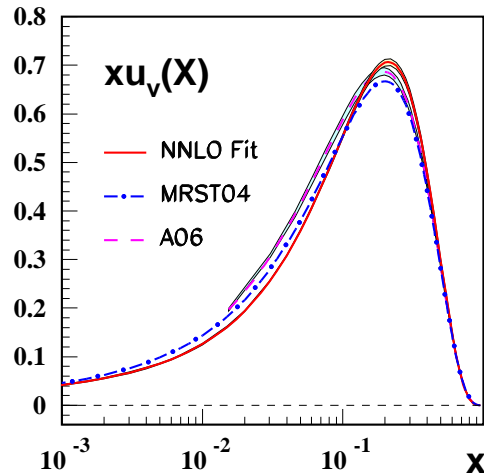
V. Ravindran, Nucl. Phys. **B752** (2006) 173

## Formalism

$$F_2^{p,d;NS}(N, Q^2) = \sum_{k=0}^{\infty} a_s^{k-1}(Q^2) C_{k-1}^{NS}(N) f_2^{p,d;NS}(N, Q^2),$$

$$\begin{aligned} f_2^{p,d;NS}(N, Q^2) &= f_2^{p,d;NS}(N, Q_0^2) \left( \frac{a}{a_0} \right)^{-\hat{P}_0(N)/\beta_0} \left\{ 1 - \frac{1}{\beta_0} (a - a_0) \left[ \hat{P}_1^+(N) - \frac{\beta_1}{\beta_0} \hat{P}_0(N) \right] \right. \\ &\quad - \frac{1}{2\beta_0} (a^2 - a_0^2) \left[ \hat{P}_2^+(N) - \frac{\beta_1}{\beta_0} \hat{P}_1^+(N) + \left( \frac{\beta_1^2 - \beta_0\beta_2}{\beta_0^2} \right) \hat{P}_0(N) \right] \\ &\quad + \frac{1}{2\beta_0^2} (a - a_0)^2 \left( \hat{P}_1^+(N) - \frac{\beta_1}{\beta_0} \hat{P}_0(N) \right)^2 \\ &\quad - \frac{1}{3\beta_0} (a^3 - a_0^3) \left[ \hat{P}_3^+(N) - \frac{\beta_1}{\beta_0} \hat{P}_2^+(N) + \left( \frac{\beta_1^2 - \beta_0\beta_2}{\beta_0^2} \right) \hat{P}_1^+(N) \right. \\ &\quad \left. + \left( \frac{\beta_1^3}{\beta_0^3} - 2\frac{\beta_1\beta_2}{\beta_0^2} + \frac{\beta_3}{\beta_0} \right) \hat{P}_0(N) \right] \frac{(a - a_0)(a_0^2 - a^2)}{2\beta_0^2} \left( \hat{P}_1^+(N) - \frac{\beta_1}{\beta_0} \hat{P}_0(N) \right) \\ &\quad \times \left[ \hat{P}_2(N) - \frac{\beta_1}{\beta_0} \hat{P}_1(N) - \left( \frac{\beta_1^2 - \beta_0\beta_2}{\beta_0^2} \right) \hat{P}_0(N) \right] \\ &\quad \left. - \frac{(a - a_0)^3}{6\beta_0^3} \left( \hat{P}_1^+(N) - \frac{\beta_1}{\beta_0} \hat{P}_0(N) \right)^3 \right\}. \end{aligned}$$

# World Data Analysis: Valence Distributions



World data:

NS-analysis

$$W^2 > 12.5 \text{ GeV}^2, Q^2 > 4 \text{ GeV}^2$$

$N^3\text{LO}$

$$\alpha_s(M_Z^2) = 0.1141^{+0.0020}_{-0.0022}$$

J.B., H. Böttcher, A. Guffanti,

Nucl. Phys. **B774** (2007) 182

## Why an $O(\alpha_s^4)$ analysis can be performed?

Assume an  $\pm 100\%$  error on the Padé approximant  $\longrightarrow \pm 2$  MeV in  $\Lambda_{QCD}$

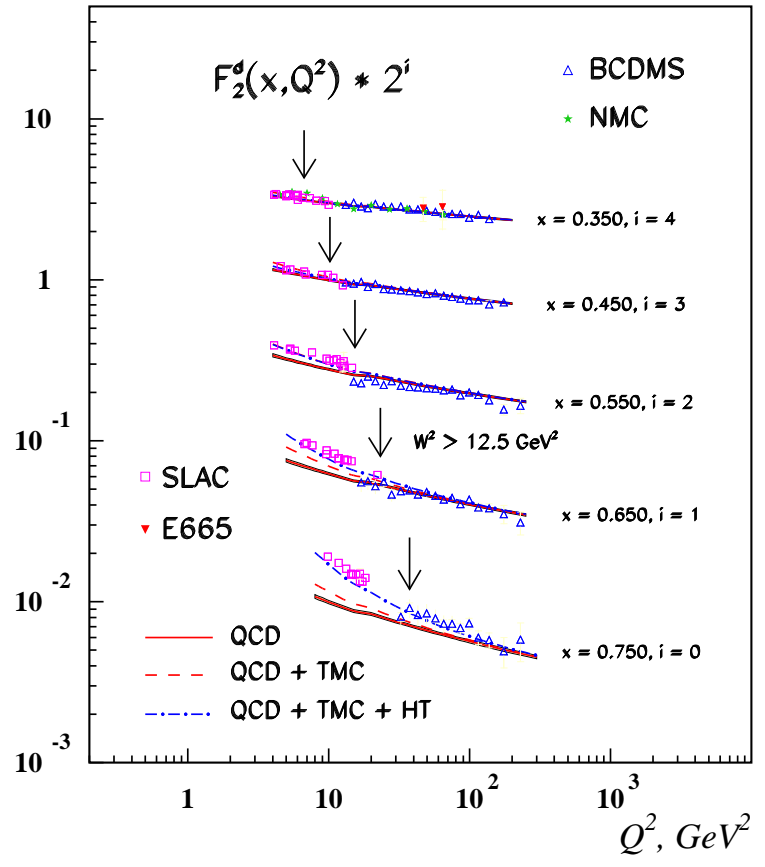
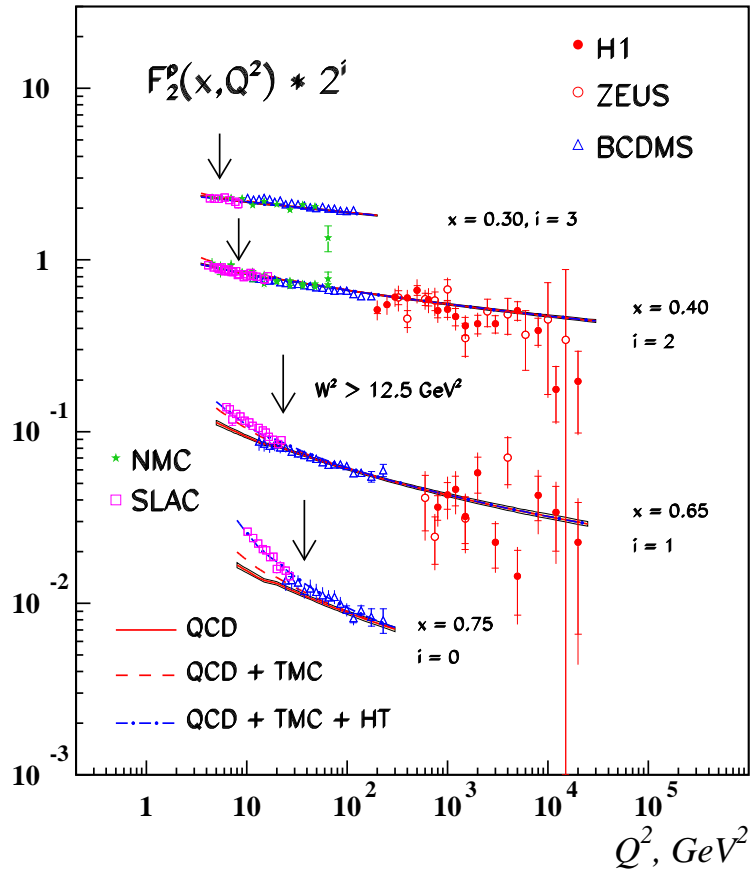
$$\gamma_n^{approx:3} = \frac{\gamma_n^{(2)^2}}{\gamma_n^{(1)}}$$

P. Baikov, K. Chetyrkin Nucl.Phys.Proc.Suppl. **160** (2006) 76.

$$\begin{aligned} \gamma_2^{3;NS} = & \frac{32}{9}a_s + \frac{9440}{243}a_s^2 + \left[ \frac{3936832}{6561} - \frac{10240}{81}\zeta_3 \right] a_s^3 \\ & + \left[ \frac{1680283336}{1777147} - \frac{24873952}{6561}\zeta_3 + \frac{5120}{3}\zeta_4 - \frac{56969}{243}\zeta_5 \right] a_s^4 \end{aligned}$$

The results agree better than 20%.

# $F_2(x, Q^2)$ in the Valence Region



No validity of the twist-2 approximation left from the arrow.

$$\alpha_s(M_Z^2)$$

$$\frac{da(\mu^2)}{d \ln(\mu^2)} = - \sum_{k=0}^{\infty} \beta_k a^{k+2}(\mu^2) .$$

$\alpha_s(M_Z^2)$  is to be determined together with the valence distributions in the same analysis.

## Overview of the analyzes :

- Various NLO analyses;  $\Rightarrow$  Precision requires NNLO analysis and higher!
- Mixed S- and NS-NNLO analyses  $e(\mu)N$  world data
- S- and NS-NNLO moment analyses  $\nu N$  world data
- NS-N<sup>3</sup>LO analysis  $e(\mu)N$  world data
- NLO analyses polarized  $e(\mu)N$  world data
- Lattice measurements

$$\alpha_s(M_Z^2)$$

<b>NNLO</b>	$\alpha_s(M_Z^2)$	expt	theory	Ref.
MRST03	0.1153	$\pm 0.0020$	$\pm 0.0030$	[2]
A02	0.1143	$\pm 0.0014$	$\pm 0.0009$	[3]
SY01(ep)	0.1166	$\pm 0.0013$		[8]
SY01( $\nu$ N)	0.1153	$\pm 0.0063$		[8]
GRS	0.111			[10]
A06	0.1128	$\pm 0.0015$		[11]
BBG	0.1134	$+0.0019 / - 0.0021$		[9]
<b>N<sup>3</sup>LO</b>	$\alpha_s(M_Z^2)$	expt	theory	Ref.
BBG	0.1141	$+0.0020 / - 0.0022$		[9]

### NNLO and N<sup>3</sup>LO

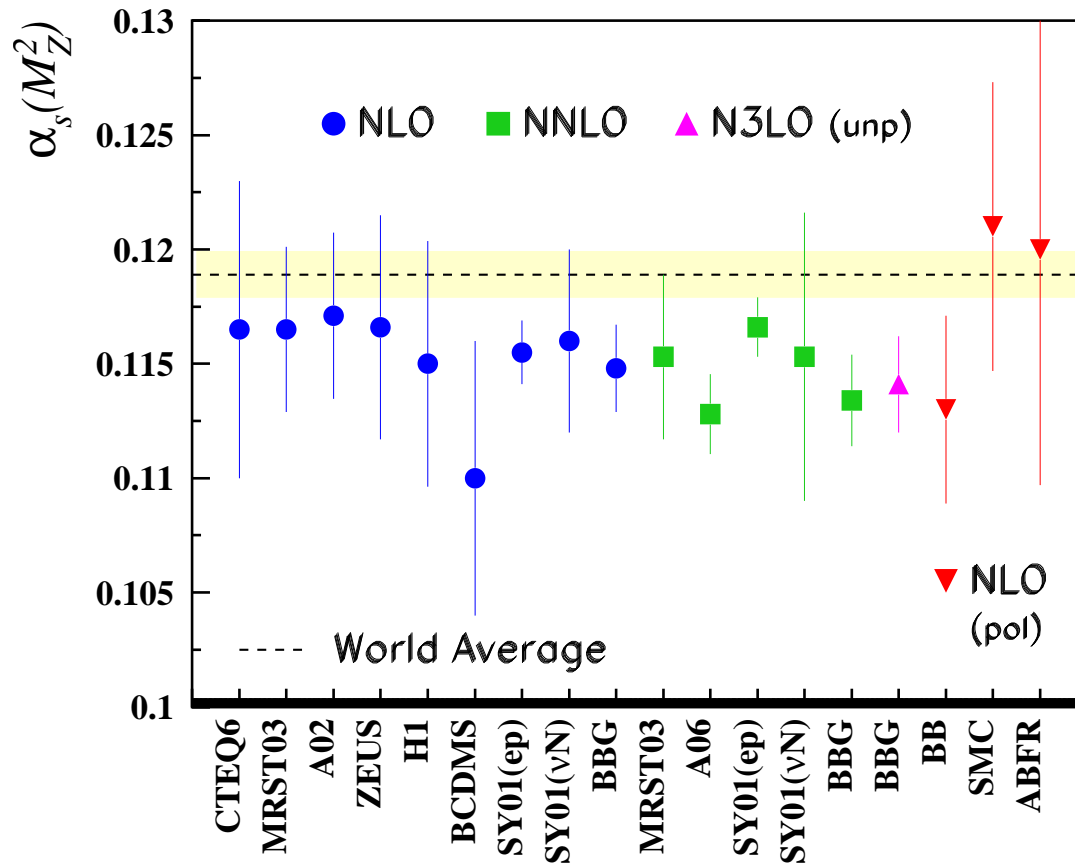
BBG:  $N_f = 4$ : non-singlet data-analysis at  $O(\alpha_s^4)$ :  $\Lambda = 234 \pm 26 \text{MeV}$

#### Lattice results :

Alpha Collab:  $N_f = 2$  Lattice; non-pert. renormalization  $\Lambda = 245 \pm 16 \pm 16 \text{MeV}$

QCDSF Collab:  $N_f = 2$  Lattice, pert. reno.  $\Lambda = 261 \pm 17 \pm 26 \text{MeV}$

# $\alpha_s(M_Z^2)$



J.B., H. Böttcher, A. Guffanti, 2006 [2]

# Higher Twist at Large $x$

- Non-leading twist terms :

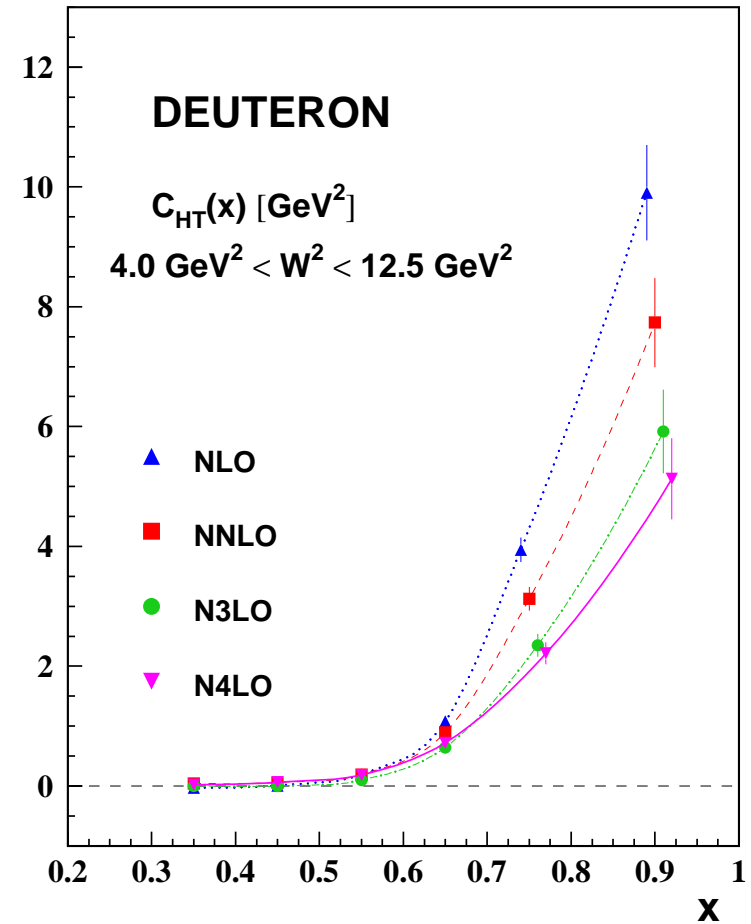
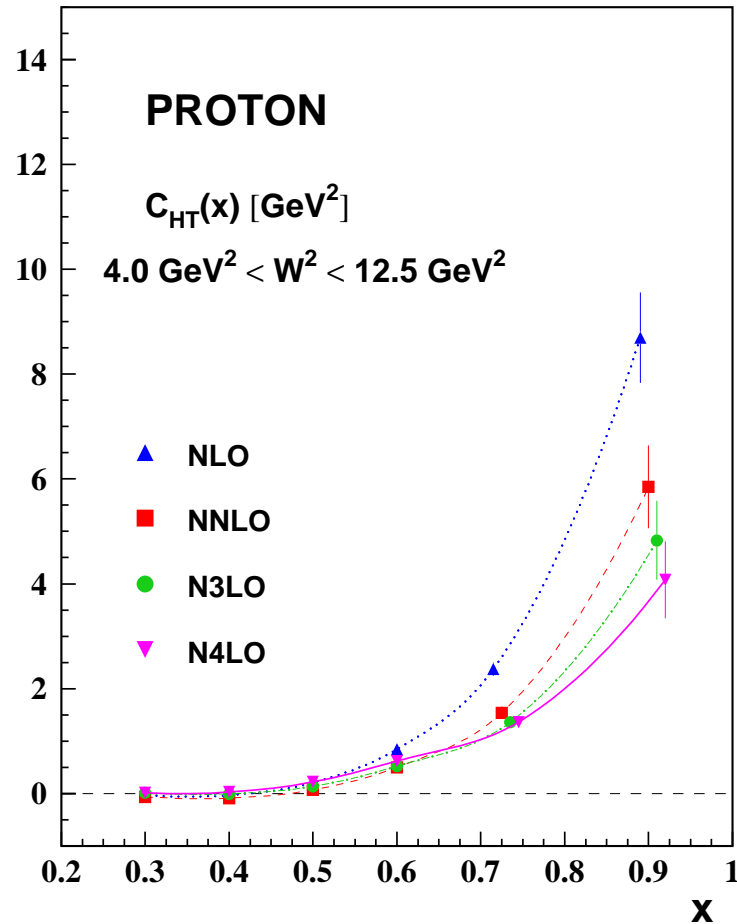
$$F_2^{\text{exp}}(x, Q^2) = F_2^{\text{tw}2}(x, Q^2) \cdot \left[ \frac{O_{\text{TMC}} [F_2^{\text{tw}2}(x, Q^2)]}{F_2^{\text{tw}2}(x, Q^2)} + \frac{C_{\text{HT}}(x, Q^2)}{Q^2 [1 \text{ GeV}^2]} \right] .$$

- Extraction for all  $(x, Q^2)$  bins
- No HT-assumptions, evolution etc. needed
- Complete Analysis : 2-loop, 3-loop
- 4-Loop analysis Padé for the 4-loop anomalous dimension with 100% error assumed
- Beyond that level: Large  $x$  contributions to the Wilson coefficient at  $O(\alpha_s^4)$ ; neglecting anomalous dimension effects. The latter term is used for HT extraction only.
- Mathematical Structure of the large  $x$  terms in Mellin Space

$$S_{1,\dots,1}(N) = \frac{1}{n} [S_n(N) + S_1(N)S_{n-1}(N) + S_{1,1}(N)S_{n-2}(N) + \dots] .$$

$$S_1(N) \propto \ln(N) + \gamma_E; \quad S_l(N) \propto \zeta_l, \quad l \geq 2 ,$$

# Higher Twist at Large $x$



Each higher order leads to a depletion. N4LO converged for  $x \leq 0.8$

- Agreement between  $p$  and  $d$  analysis
- LGT simulations of these terms are of interest

## Twist-2 Integral Relations

$$g_2^{\tau=2}(x, Q^2) = -g_1^{\tau=2}(x, Q^2) + \int_x^1 \frac{dy}{y} g_1^{\tau=2}(y, Q^2)$$

$$g_3^{\tau=2}(x, Q^2) = 2x \int_x^1 \frac{dy}{y^2} g_4^{\tau=2}(y, Q^2)$$

$$g_5^{\tau=2}(x, Q^2) = \frac{1}{2x} [g_4^{\tau=2}(x, Q^2) + \Delta_{CG}(x, M^2/Q^2)]$$

[1 ] S. Wandzura and F. Wilczek, Phys. Lett. B **72** (1977) 195.

[2 ] J.B., and N. Kochelev, Nucl.Phys.**B498** (1997) 285.

[3 ] J.B., and A. Tkabladze, Nucl.Phys.**B553** (1999) 427.

[4 ] for the gluonic case (pure photon exchange): J.B., V. Ravindran and W. L. van Neerven, Phys. Rev. D **68** (2003) 114004

## Twist-3 Integral Relations

$$\begin{aligned}g_1^{\tau=3}(x, Q^2) &= \frac{4M^2 x^2}{Q^2} \left[ g_2^{\tau=3}(x, Q^2) - 2 \int_x^1 \frac{dy}{y} g_2^{\tau=3}(y, Q^2) \right] \\ \frac{4M^2 x^2}{Q^2} g_3^{\tau=3}(x, Q^2) &= g_4^{\tau=3}(x, Q^2) \left( 1 + \frac{4M^2 x^2}{Q^2} \right) + 3 \int_x^1 \frac{dy}{y} g_4^{\tau=3}(y, Q^2) \\ g_5^{\tau=3}(x, Q^2) &= -\frac{1}{2x} \int_x^1 \frac{dy}{y} g_4^{\tau=3}(y, Q^2)\end{aligned}$$

One may therefore unfold both the twist-2 and -3 contributions to all the five polarized structure functions using the above relations.

# Conclusions

- We performed a NS QCD Analysis to **Three Loop Order**, and under a weak assumption, to **Four Loop Order** determining the valence quark distributions and  $\Lambda_{\text{QCD}}$ .
- Within this analysis we determined, **free of assumptions**, the higher twist contributions in the large  $x$  region. We also considered the large  $x$  contributions to the NS Wilson coefficient in  $O(\alpha_s^4)$ .
- HT extractions based on a **Phenomenological Ansatz** have the problem of **yet unknown HT anomalous dimensions, Wilson Coefficients, and correlation functions**.
- We observe **gradual depletion** of the higher twist contributions with increasing order of  $\alpha_s$ . The present description converges for values  $x \leq 0.8$
- The **twist-2 and twist-3 contributions** to the polarized **electro-weak** structure functions can be unfolded using the respective **Integral Relations**.