Higher twists from polarized structure functions

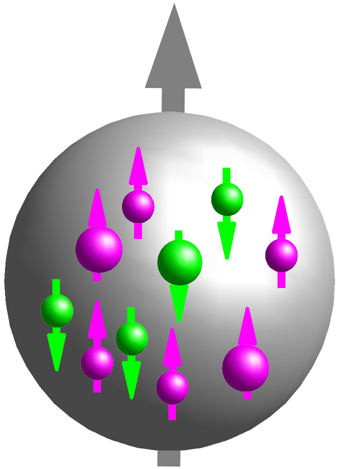
Alexandre Deur

Jefferson Lab

PVDIS workshop

Madison, June 09

Simplest view of nucleon spin



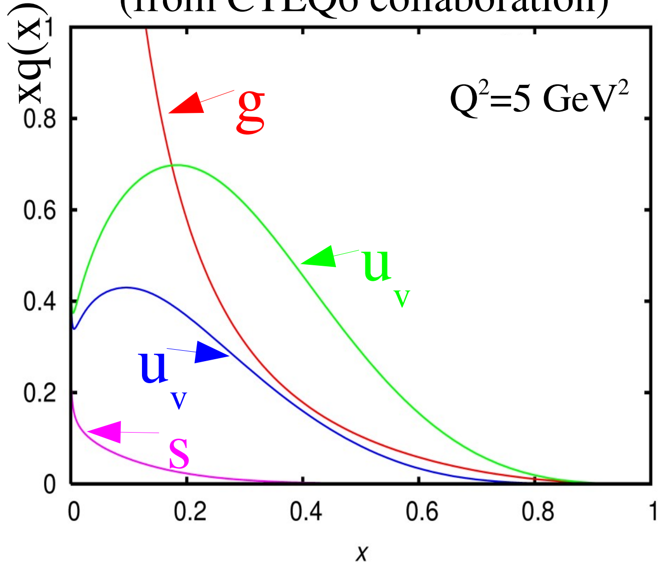
Nucleon spin $=\Sigma$quark spins

(with relativistic correction: $0.7\times$nucleon spin $=\Sigma$quark spins.
In a less naive model, gluons contribute as well.)

Nucleon: bag of free quarks with probabiliy densities and polarizations:

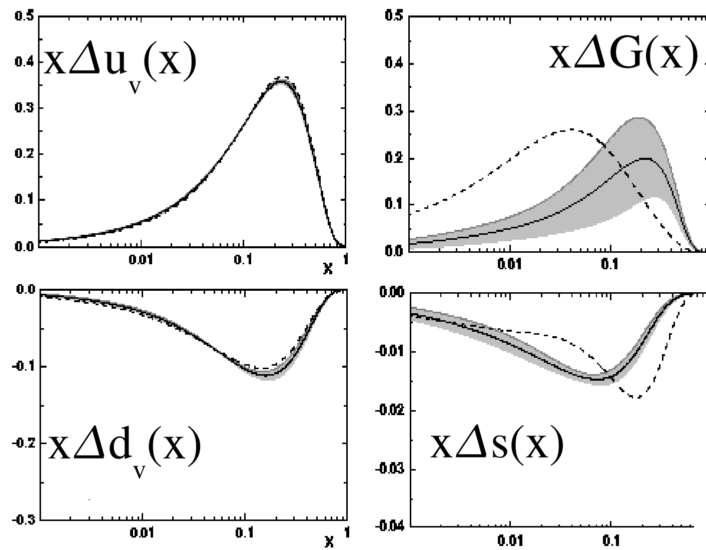
Quark (and gluon) densities

(from CTEQ6 collaboration)



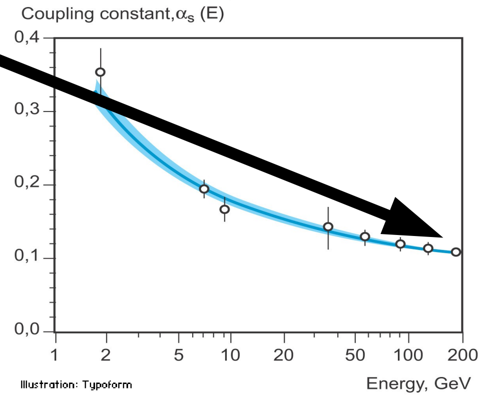
Quark (and gluon) polarizations

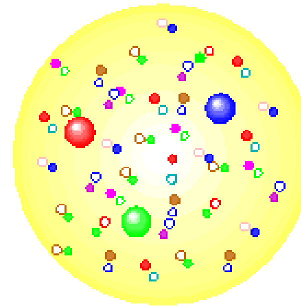
(from Leader, Sidorov & Stamenov)



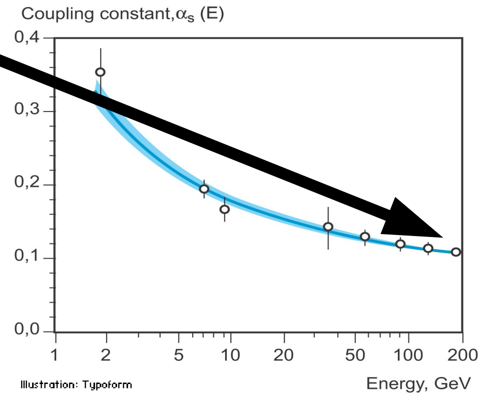
Jefferson Lab

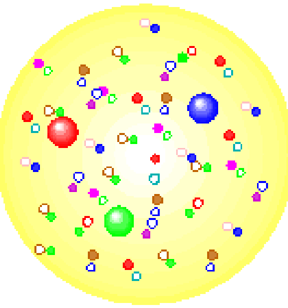
Valid at $Q^2 \to \infty$ only where $\alpha_s = 0$ (quarks are free).





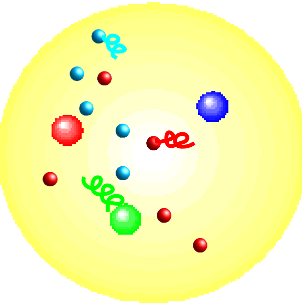
Valid at $Q^2 \rightarrow \infty$ only where $\alpha_s = 0$ (quarks are free).



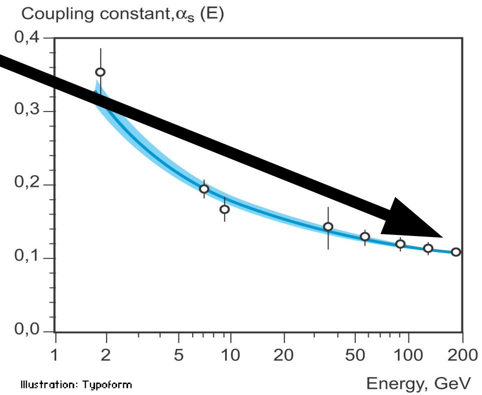


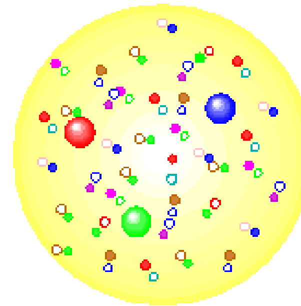
At finite (but still large) Q^2:

- quarks start to interact: gluon corrections
- $M^2/Q^2 \neq 0$: Mass corrections
- ...

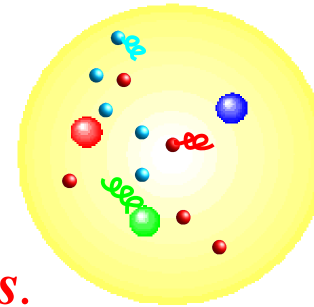


Valid at $Q^2 \rightarrow \infty$ only where $\alpha_s = 0$ (quarks are free).





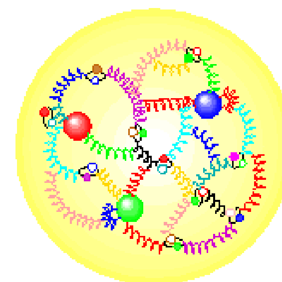
At finite (but still large) Q^2:



- quarks start to interact: gluon corrections
- $M^2/Q^2 \neq 0$: Mass corrections

At smaller Q^2 (~GeV^2): correlations between quarks ***higher twists***.

Like quark densities and quark polarizations, higher twists are **non perturbative quantities.**



⇒Cannot be computed analytically within QCD.

Hard to compute on the lattice (expressions are not local).

HT are interesting: they are the next step after the naive free quark model and should be related to quark confinement. Important ingredient to understand the parton↔hadron transition. Also, HT need to be under control to extract quark and gluon distributions (especially for scarcer pol. data).

Jefferson Lab

Q^2 evolutions

- Gluons emissions induce a $\log(Q^2)$ behavior.

- Higher twists and mass corrections induce $(Q^2)^{2-t}$ power corrections

$$A(x,Q^2)=A(x,\log(Q^2))_{pQCD} + \text{Power Correction}(x,\ Q^2,\ \text{quark \& gluon distributions})$$

⇒In principle we can separate log and power corrections (dynamical HT and mass corrections) by fitting data.

In practice, the 3 series involved:
- pQCD series in α_s for all non perturbative quantities.
- α_s series in β in the pQCD series.
- The power correction series in $(Q^2)^{2-t}$

are all truncated to finite (and not high) orders ⇒ Ambiguities.

To fit and check the ambiguities, we need accurate data with large Q^2 lever arm.

Higher twists from moments of spin structure functions

I first start with moments of spin structure function: simpler (x-dep. integrated)

Twists series: $\int_0^1 g_1 dx = \sum_{\text{twist}=2,4...} \frac{\mu_{\text{twist}}}{Q^{\text{twist}-2}}$

$$\int g_1 dx = \left(\pm 12 g_a + \frac{a_8}{36}\right)\left(1 - \frac{\alpha_s(\ln(Q^2))}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - ...\right) + \frac{a_0}{9}\left(1 - \frac{\alpha_s}{\pi} - 1.10\left(\frac{\alpha_s}{\pi}\right)^2 - ...\right) + \frac{\mu_4}{Q^2} + \frac{\mu_6}{Q^4} + ...$$

Triplet axial charge — Octet axial charge — singlet axial charge

Here: $\overline{\text{MS}}$ (no gluon contribution to Γ_1) and a_0 is Q^2-independent. In other schemes, the gluon distribution comes in.

$$\frac{\mu_4}{Q^2} = \frac{M^2}{9}\left(a_2(\ln(Q^2)) + 4d_2(\ln(Q^2)) + 2f_2(\ln(Q^2))\right)$$

Leading twist (known from high energy data) — Twist 3 (known from SLAC + JLab@large x) — Twist 4

(So you may say that we have a fourth series as well.)

Higher twists from moments of spin structure functions

I first start with moments of spin structure function: simpler (x-dep. integrated)

Twists series: $\int_0^1 g_1 dx = \sum_{\mathrm{twist}=2,4...} \frac{\mu_{\mathrm{twist}}}{Q^{\mathrm{twist}-2}}$

$$\int g_1 dx = \left(\pm 12 g_a + \frac{a_8}{36}\right)\left(1 - \frac{\alpha_s(\ln(Q^2))}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - ...\right) + \frac{a_0}{9}\left(1 - \frac{\alpha_s}{\pi} - 1.10\left(\frac{\alpha_s}{\pi}\right)^2 - ...\right) + \frac{\mu_4}{Q^2} + \frac{\mu_6}{Q^4} + ...$$

Triple axial charge — Octet axial charge — singlet axial charge

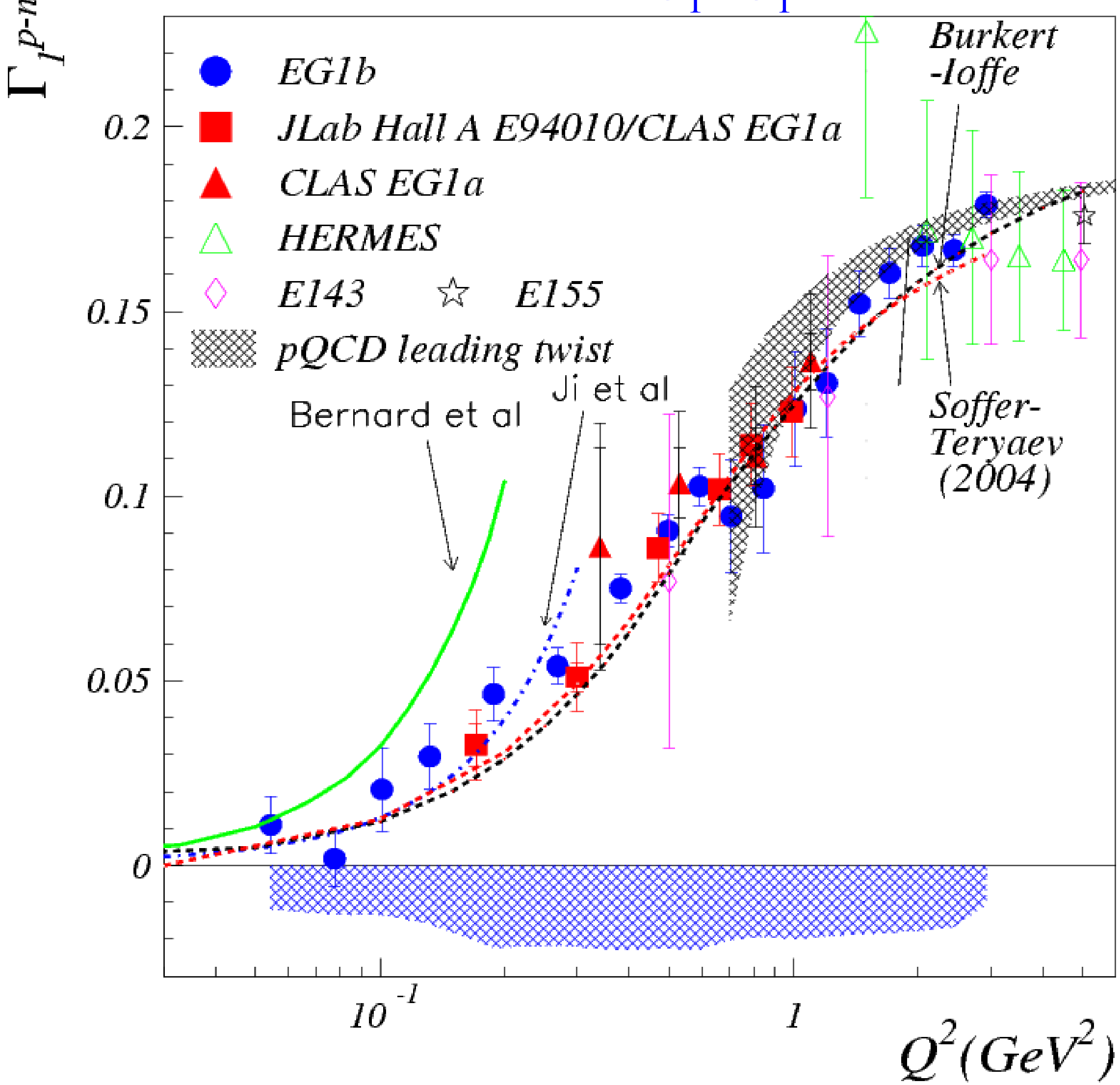
Here: $\overline{\mathrm{MS}}$ (no gluon contribution to Γ_1) and a_0 is Q^2-independent. In other schemes, the gluon distribution comes in.

$$\frac{\mu_4}{Q^2} = \frac{M^2}{9}\left(a_2 + 4d_2 + 2f_2\right)$$

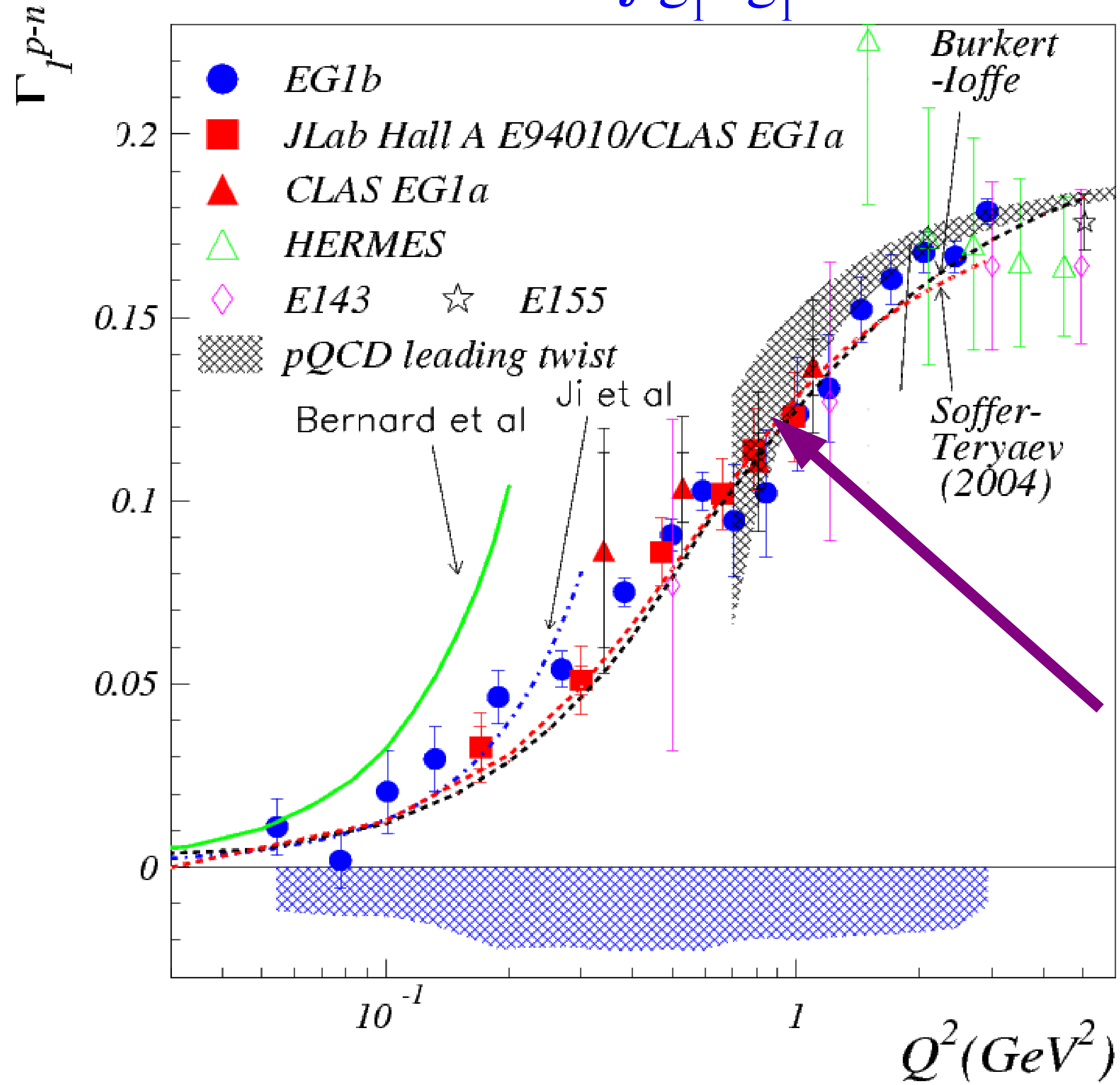
The isovector part of it (Bjorken sum) is even simpler: no gluon contribution

$$\int g_1^{p-n} dx = \frac{1}{6} g_a \left(1 - \frac{\alpha_s(\ln(Q^2))}{\pi} + ...\right) + \frac{\mu_4}{Q^2} + \frac{\mu_6}{Q^4} + ...$$

Results on $\int g_1^p - g_1^n \, dx$



Results on $\int g_1^p - g_1^n \, dx$



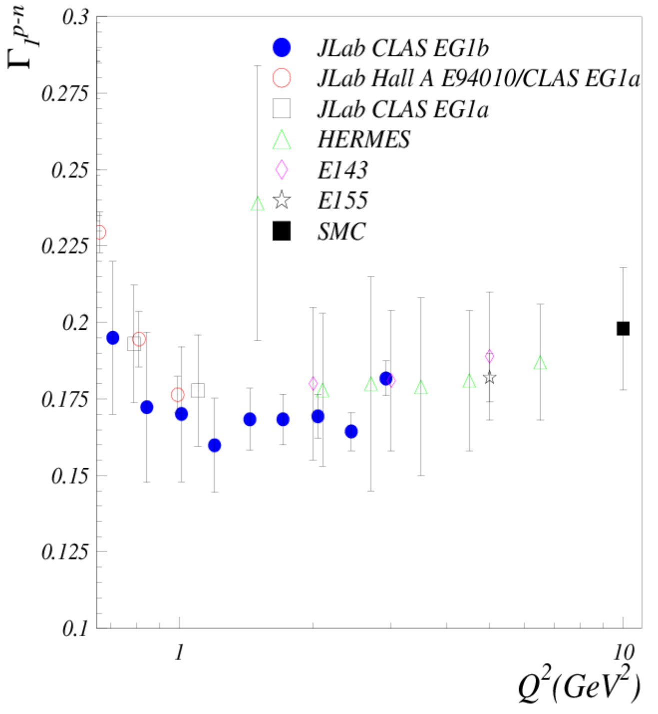
pQCD without higher twists follow the data surprisingly well

Confinement is a strong effect. It is surprising that higher twists are so elusive.

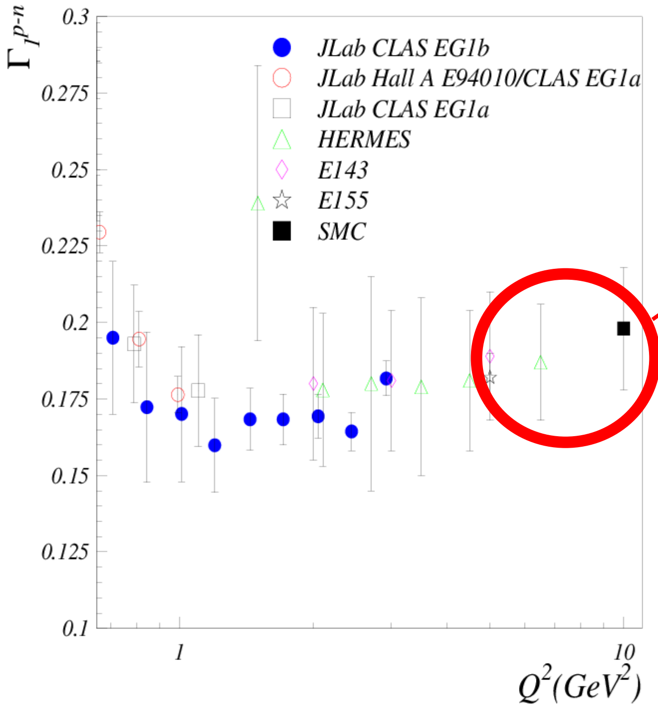
Jefferson Lab

Results on $\int g_1^p - g_1^n \, dx$

To extract power corrections, the elastic has to be added:

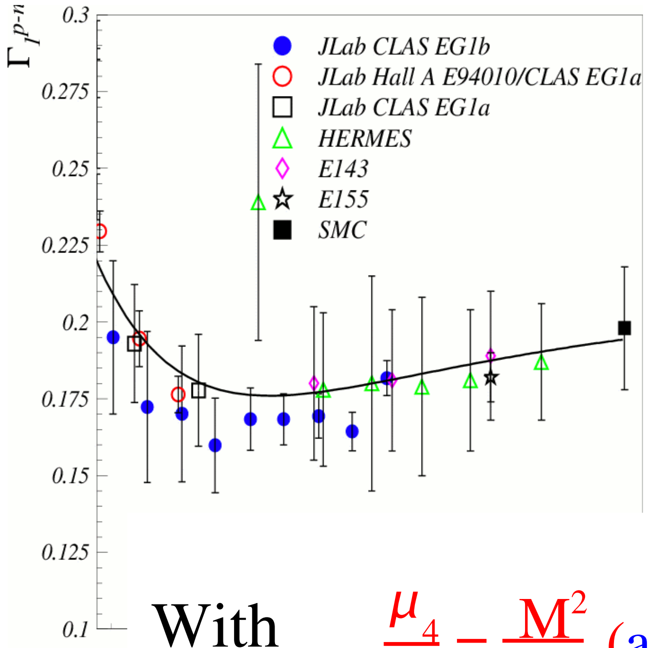


Results on $\int g_1^{p}-g_1^{n}\, dx$



μ_2 (here g_a) is obtained from large Q^2 fit, with an uncertainty Δg_a. (no assumption on sum rule validity).

Results on $\int g_1^p - g_1^n \, dx$



Fits done with forms:

$$\frac{1}{6} g_a \left(1 - \frac{\alpha_s(Q^2)}{\pi} + \ldots\right) + \frac{\mu_4}{Q^2}$$

$$\frac{1}{6} g_a \left(1 - \frac{\alpha_s(Q^2)}{\pi} + \ldots\right) + \frac{\mu_4}{Q^2} + \frac{\mu_6}{Q^4}$$

$$\frac{1}{6} g_a \left(1 - \frac{\alpha_s(Q^2)}{\pi} + \ldots\right) + \frac{\mu_4}{Q^2} + \frac{\mu_6}{Q^4} + \frac{\mu_8}{Q^6}$$

With $\frac{\mu_4}{Q^2} = \frac{M^2}{9} (a_2 + 4d_2 + 2f_2)$

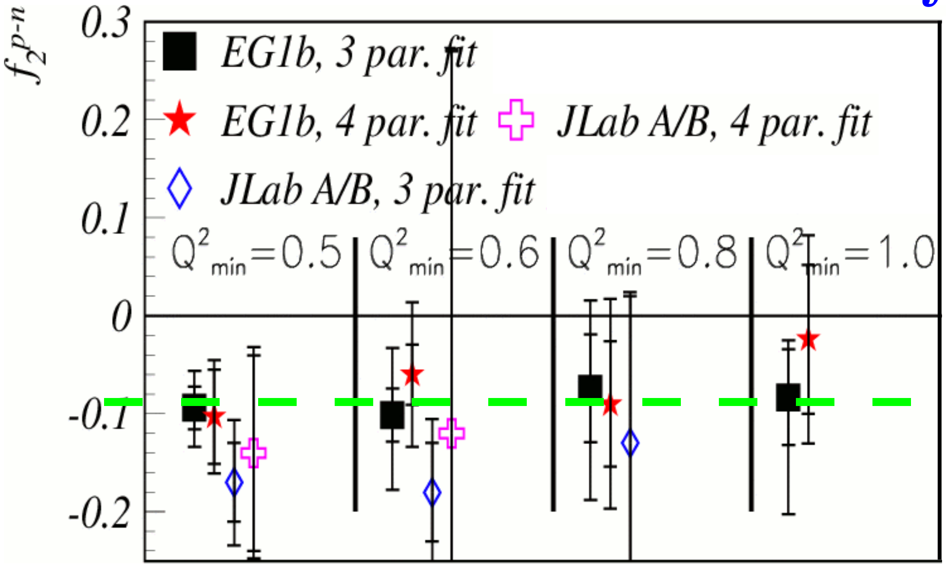
$$a_2 = \int x^2 [g_{1,LT}{}^p - g_{1,LT}{}^n] dx$$

$$d_2 = \int x^2 [(2g_1 + 3g_2)^p - (\ldots)^n] dx$$

and Δg_a, f_2, μ_6 and μ_8 are fit parameters.

The lower limit of the Q^2 range on which the fit is performed is also varied.

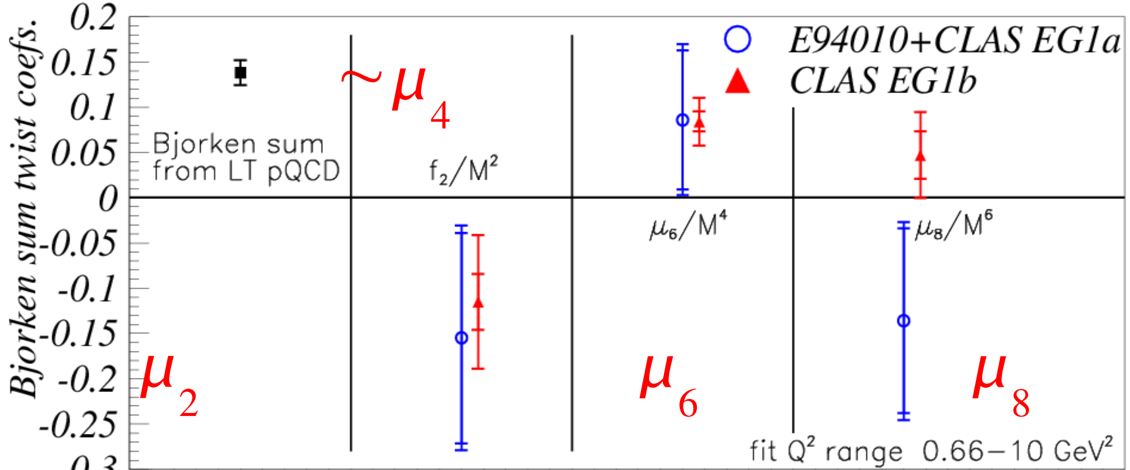
Results on $\int g_1^p - g_1^n \, dx$



Results for f_2 for different fit forms and data sets.

Consistency

Values of the coefficients of the twist series.



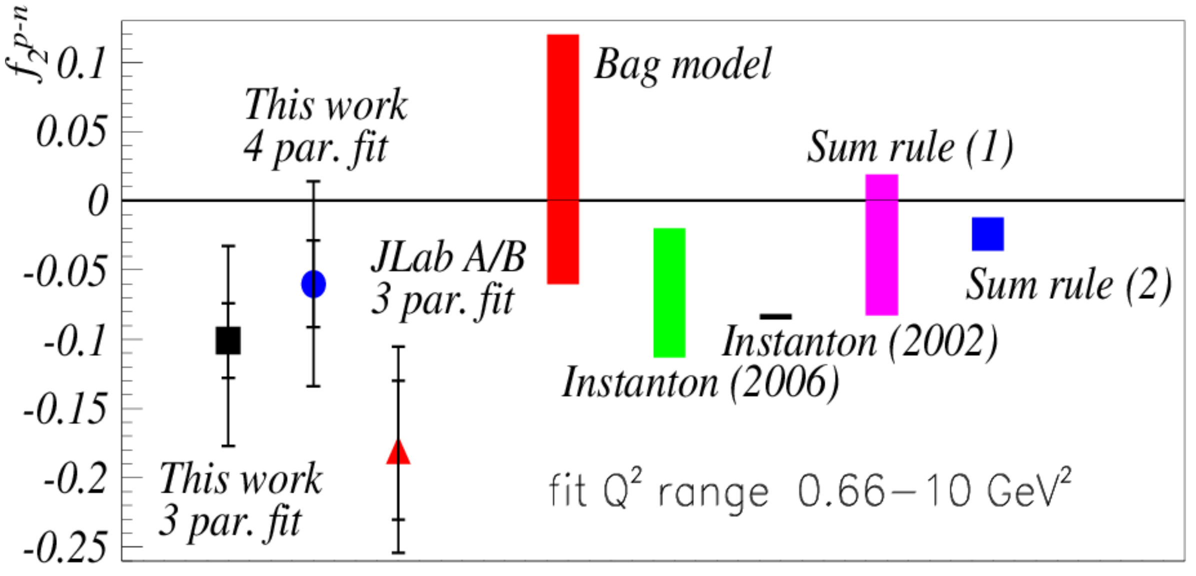
Higher twists are large!

Sign is alternating !

Systematic studies done with Γ_1 models support these conclusions and legitimacy our analysis with Jlab's data. Not a given because the convergence of the HT series is not obvious.

Results on $\int g_1^p - g_1^n \, dx$

Comparison with models:

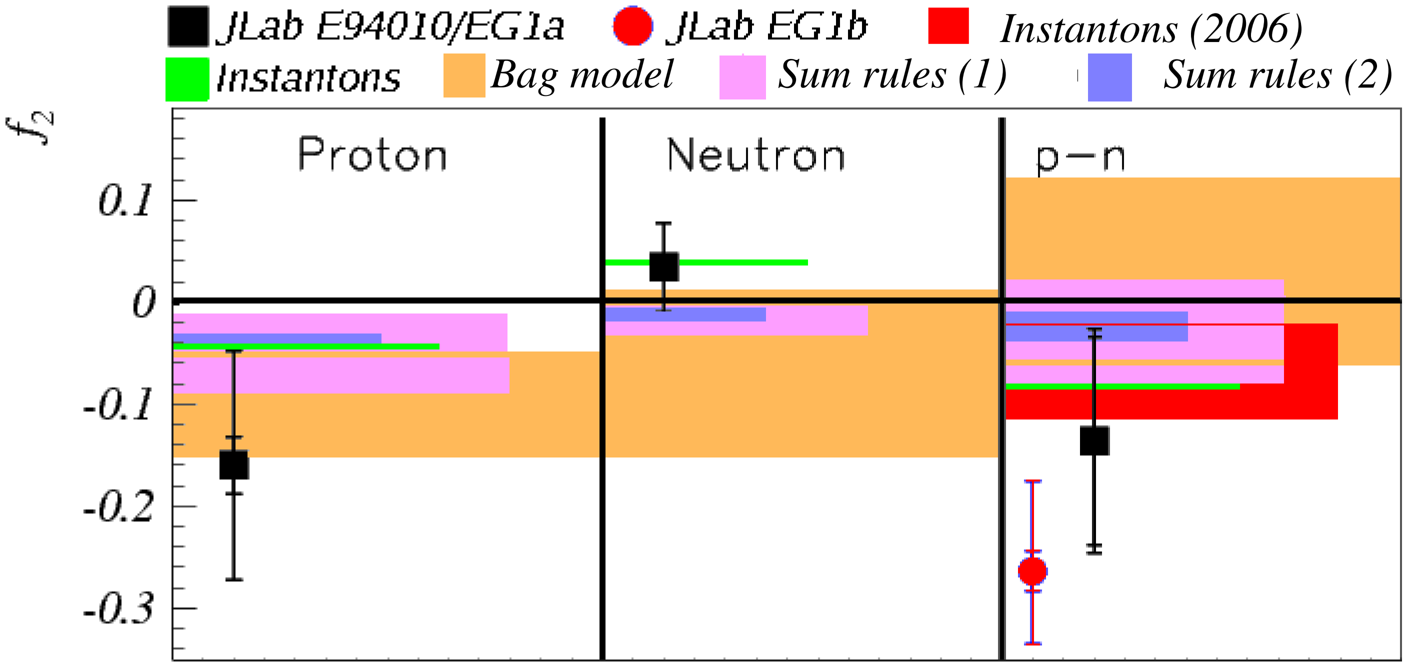


Models:
- Bag model: X. Ji and W. Melnitchouk, PRD 56, 1 (1997)
- Instanton (2006): A. V. Sidorov, C. Weiss. PRD 73 074016 (2006)
- Instanton (2002): N-Y. Lee, K. Goeke and C. Weiss, PRD 65 054008 (2002)
- Sum rule (1): I. Balitsky et al. PLB 242 245 (1990)
- Sum rule (2): E. Stein et al. PLB 353 107 (1995)

Analysis: A.D. PRL 93 212001 (2004), PRD 78 032001 (2008)

Results on $\int g_1 dx$

The analyses for $\int g_1 dx$ on proton and neutron proceed similarly. The large Q^2 fit gives $\Delta\Sigma = a_0$, the contribution from quarks to the nucleon spin.



Proton: A. D. nucl-ex/0508022

Neutron: Z-E. Meziani et al. PLB 613 148 (2005)

Signs seems to be different for neutron and proton.

Expected from isospin symmetry and dominance of g_a in the twist 2 term:

$$\int g_1 dx = \left(\pm 12 g_a + \frac{a_8}{36}\right)\left(1 - \frac{\alpha_s(\ln(Q^2))}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - \ldots\right) + \frac{a_0}{9}\left(1 - \frac{\alpha_s}{\pi} - 1.10\left(\frac{\alpha_s}{\pi}\right)^2 - \ldots\right)$$

and ***if*** we do have sign flip between μ_2 and μ_4.

Jefferson Lab

Conclusions from $\int g_1 dx$

- f_2 large (similar to leading twist), in accordance to intuition.

  μ_6/M^4 similar size as f_2 but opposite sign.

  ⇨ Overall, higher twist contribution small at $Q^2 = 1$ GeV^2.

- Simple explanation for HT sign flip in context of vector meson dominance (C. Weiss):

$$\frac{1}{Q^2+M^2} = \frac{1}{Q^2}\;\frac{1}{1+M^2/Q^2} = \frac{1}{Q^2}\left(1 - \frac{M^2}{Q^2} + \frac{M^4}{Q^4} - \ldots\right) = \left(\frac{1}{Q^2} - \frac{M^2}{Q^4} + \frac{M^4}{Q^6} - \ldots\right)$$

- Models and data are in general in good agreement but uncertainties are large.

- The instanton model is in good agreement and also predicted that $f_2 \gg d_2$.

Moments of g_2.

$\Gamma_2(Q^2)=\int g_2 dx$ is given by the Burkhardt-Cottingham sum rule: $\Gamma_2(Q^2)=0$

$\Rightarrow$No direct HT information from Γ_2.

Higher moment: $d_2=\int x^2(2g_1+3g_2)dx$

- Twist 3 quantities (at large enough Q^2).
- Has been computed in lattice QCD.
- Along with f_2, linked to color polarizabilities.

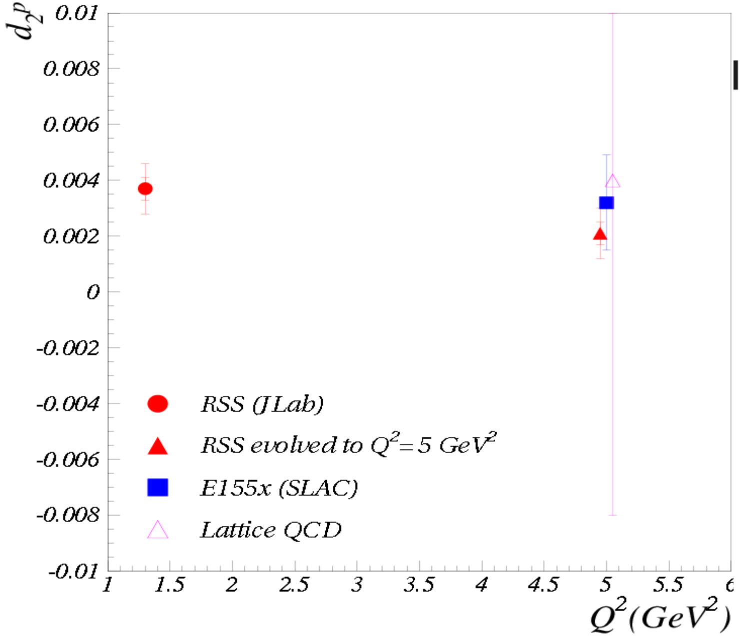
$$\chi_E=2/3\ (2d_2+f_2)$$

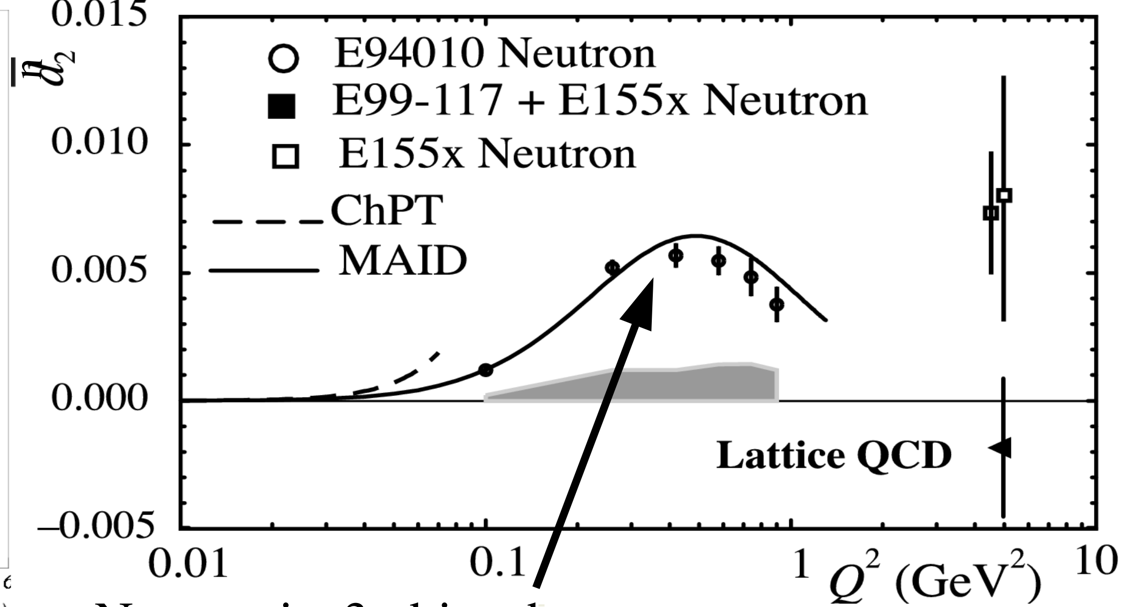
$$\chi_M=1/3\ (4d_2-f_2)$$

Response of the nucleon spin to the color electric and magnetic fields.

Can also be interpreted as transverse force on struck quark: M. Burkardt (arXiv:0810.3589)

Moments of g_2.





Not a twist 3 object here

In the DIS domain, d_2 is small (compare to $|f_2|$~0.1). Predicted by the instanton calculation.

From d_2 and f_2, color polarizabilities can be extracted: χ_E^p ~-0.08 , χ_M^p~ 0.06

χ_E^n~ 0.03 , χ_M^n~-0.003

(with almost 100% uncertainties)

Sign changes expected since $f_2 \gg d_2$ and f_2^p and f_2^n signs are opposite (isospin symmetry).

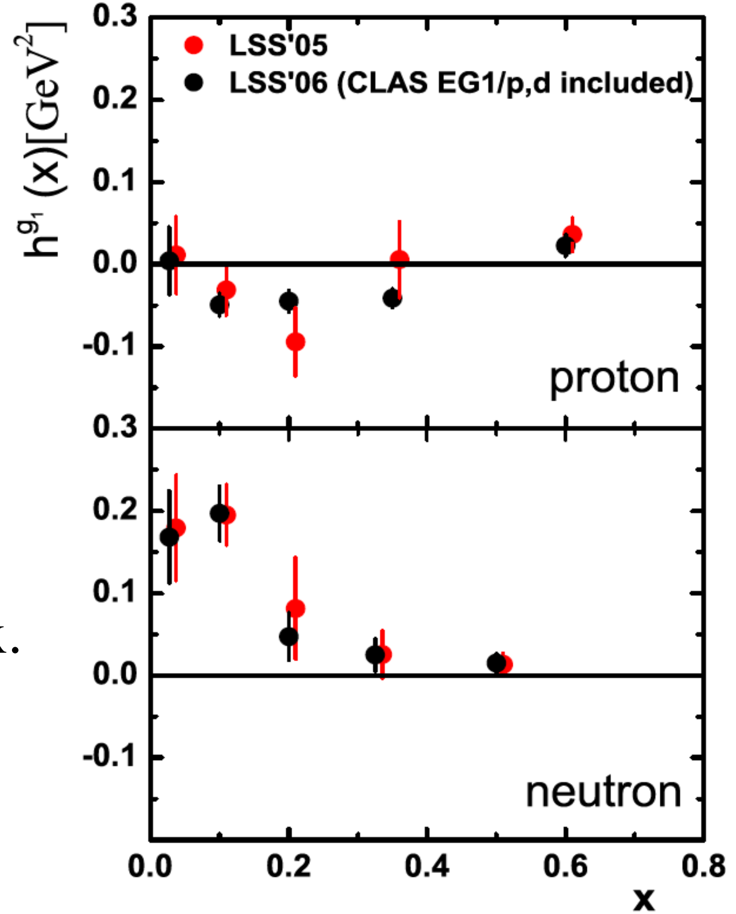
Jefferson Lab

Results on g_1

Leader, Sidorov and Stamenov global analysis of g_1 in DIS includes a HT term:

LSS PRD 75 074027 (2007)

$$g_1(x,Q^2) = g_1(x,Q^2)_{pQCD} + g_1(x,Q^2)_{TMC} + h(x,Q^2)/Q^2.$$



The analysis gives the x-dependence of the twist-4.

Again, opposite sign for proton and neutron.

h large for moderate x. Smaller at high and low x.

Since g_1 becomes small at large x, HT may still contribute significantly.

$\int h(x,Q^2)dx$ is compatible with f_2.

Results on g_2

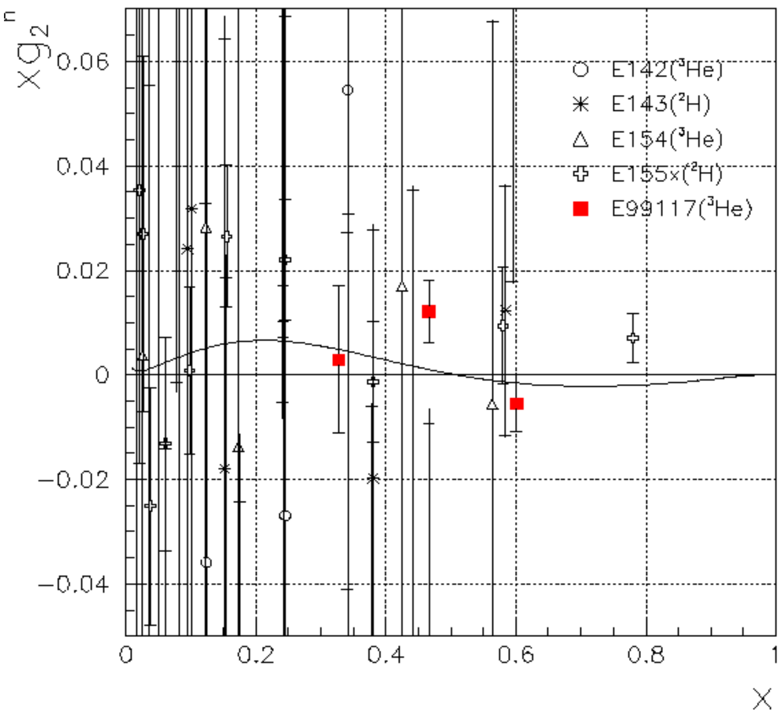
g_2 an be cleanly separated between a twist-2 term and twist-3 (and higher) terms:

$$g_2 = g_2^{WW} + g_2^{HT} \quad \text{with } g_2^{WW} = -g_1 + \int_x^1 g_1/y\, dy \qquad (\text{note: } d_2 = 3\int x^2 (g_2 - g_2^{WW}) dx)$$

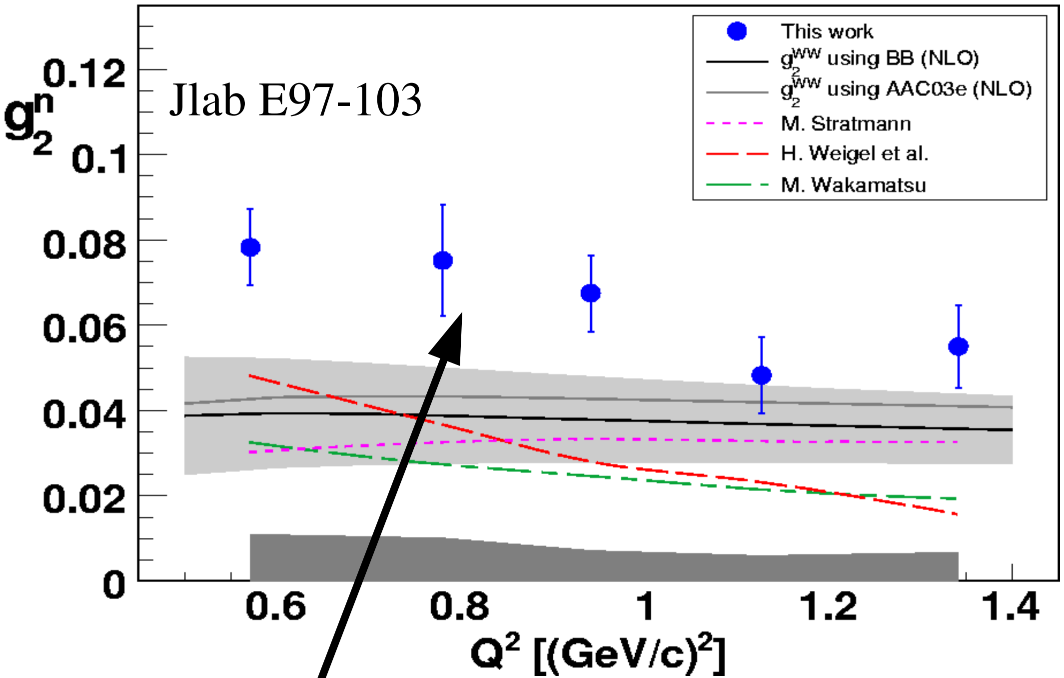
Twist-2   Higher twist. Not suppressed as a power correction !

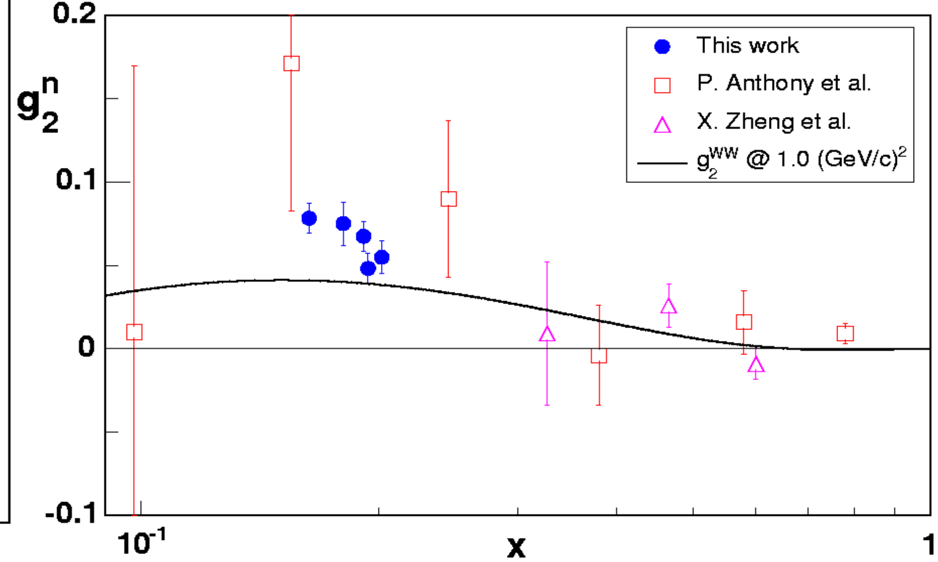
Empirically, $g_2^{WW} \gg g_2^{HT}$, so that $g_2 \sim g_2^{WW}$.

Neutron:



Results on g_2





First deviation from g_2^{WW} seen! Sign is opposite to what models predict.

Results on g_2

The Burkhardt-Cottingham sum rule teaches us indirectly something about HT.

Both g_2 and g_2^{WW} fulfill the BC sum rule: $\int g_2 dx=0$ & $\int g_2^{WW} dx=0$.

⇒At Q^2 large enough so that twist 3 dominates over HT, $\int g_2^{twist\ 3} dx=0$.

Since the sum rule is valid at any Q^2, and twist contributes as $1/Q^{t-2}$, the BC sum rule should be valid twist order by twist order.

⇒Constraint on moment of HT.

⇒The assumptions underlying the derivation of the BC sum rule (g_2 analytic enough and no singular contribution at $x=0$) are valid twist order by twist order.

Summary/Conclusions

- HT are interesting because they are coming next after the naive free quark model. They should be related to quark confinement. Also connected to transversity.
- Overall effects of HT are found to be small at the moderate Q^2 of Jlab. Surprising but understood for Γ_1 in term of large individual components (of the size of leading twist) and alternative signs of the twist series.
- Hierarchy: LT~f_2~$\mu_6 \gg d_2$~a_2
- x-dependence of g_1 HT extracted by LSS. HT important at moderate and large x.
- First deviation from g_2^{WW} seen (neutron), but twist order unclear. Small effects for Q^2>1 GeV^2 of sign opposite to predictions. The Burkhardt-Cottingham sum rule allows to make statements on 1st moment and on the good behavior of g_2 twist order by twist order.
- Smallness of overall HT effects is a bless for DIS practitioners, but this makes HT hard to extract and study.

Jefferson Lab