Non-local Renormalization
and
Higher Twist

Gil Paz

Institute *for* Advanced Study, Princeton

In collaboration with
Mike Glatzmaier
Sonny Mantry
Michael Ramsey-Musolf

Outline

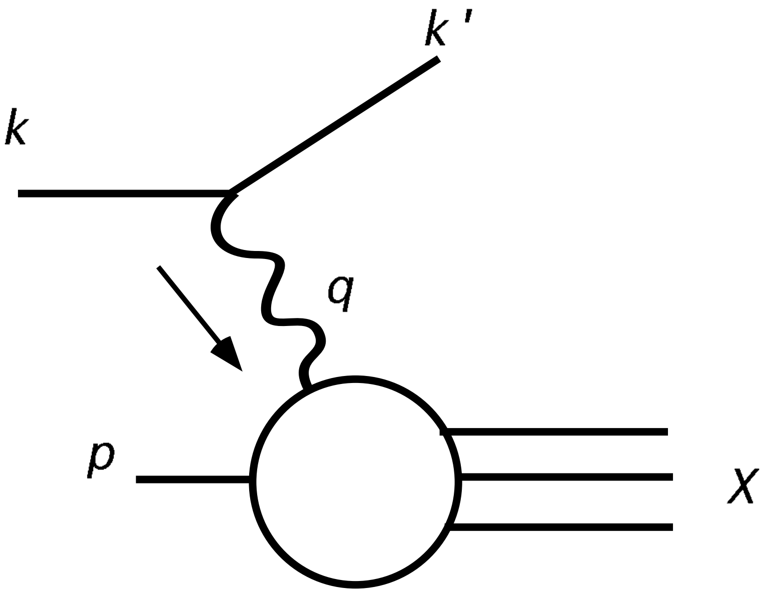
- Introduction: EFT look at DIS
- Example of non-local renormalization:

  non singlet splitting function
- Higher twist in QCD and SCET
- Lessons from inclusive charmless B decays
- Conclusions and outlook

Introduction:

EFT look at DIS

DIS



- Define

$$x = \frac{-q^2}{2p \cdot q} = \frac{Q^2}{2p \cdot q}$$

- Cross section

$$\sigma \to L^{\mu\nu} W_{\mu\nu} \to F_1,\ F_2$$

- Factorization

$$F_2(x, Q^2) = \sum_i \int d\xi\, C_2^i\left(\frac{x}{\xi}, \frac{Q}{\mu}, \frac{\mu_F}{\mu}, \alpha_s(\mu)\right) \phi_{i/h}\big(\xi, \mu_F, \alpha_s(\mu)\big) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{Q^2}\right)$$

PDF

- Factorization

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- $\phi_{i/h}$ often referred to as the “probability to find parton i in h”

  Maybe was good in 1969, what about a field theoretic definition?

PDF

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- $\phi_{i/h}$ often referred to as the "probability to find parton i in h"

  Maybe was good in 1969, what about a field theoretic definition?

- Answer: [Collins, Soper NPB **194**, 445 (1982)]

$$\boxed{\phi_q(\xi) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, e^{i\xi n\cdot pt} \langle N(p)|\bar{\psi}(0)\,[0,tn]\,\frac{\not{n}}{2}\,\psi(tn)|N(p)\rangle}$$

with n light-like vector, $n^2 = 0$ and

$$[0,tn] = W_n(0)W_n^\dagger(tn) \quad \text{where} \quad W_n(u) = P\,\exp\, ig\int_{-\infty}^{0} ds\, n\cdot A(u+sn)$$

- Evolution: DGLAP equation

$$\boxed{\frac{d\phi_b}{d\ln\mu} = \frac{\alpha_s}{\pi}\int_x^1 \frac{d\xi}{\xi}\, P_{ba}(\xi,\alpha_s)\,\phi_a}$$

Splitting Functions

- Evolution: DGLAP equation

$$\frac{d\phi_b}{d\ln\mu} = \frac{\alpha_s}{\pi}\int_x^1 \frac{d\xi}{\xi}\, P_{ba}(\xi,\alpha_s)\,\phi_a$$

P_{ba} are “splitting functions” calculated via

1) Altarelli-Parisi [Altarelli, Parisi NPB **126**, 298 (1977)]

probability of a quark to “split” to a quark and a gluon

$$\text{“}P_{qq}\text{”} = C_F\,\frac{1+\xi^2}{1-\xi}$$

where the full expression is

$$P_{qq} = C_F\left[\left(\frac{1+\xi^2}{1-\xi}\right)_+ + \frac{3}{2}\delta(1-\xi)\right]$$

singular $\xi \to 1$ added “by hand”

Splitting Functions

- Evolution: DGLAP equation

$$\frac{d\phi_b}{d\ln\mu} = \frac{\alpha_s}{\pi}\int_x^1 \frac{d\xi}{\xi}\, P_{ba}(\xi, \alpha_s)\, \phi_a$$

P_{ba} are "splitting functions" calculated via

2) Moments (" OPE ")

[Georgi, Politzer PRD **9**, 416 (1974)], [Gross, Wilczek PRD **9**, 980 (1974)]

Using translation invariance write the PDF as

$$\phi_q(\xi) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, e^{i\xi n\cdot pt}\langle N(p)|\bar{\psi}(0)\, e^{in\cdot Dt}\, \frac{\not{n}}{2}\, \psi(0)|N(p)\rangle$$

Moments are given as local matrix elements

$$\int_0^1 dx\, x^{n-1}\phi(x) = \phi_n, \qquad \frac{d\phi_n}{d\ln\mu} = -\gamma_n\phi_n, \qquad \gamma_n = -\int_0^1 dx\, x^{n-1} P_{qq}$$

EFT look at DIS

- Potential issues with standard derivation

1) AP method
   - How to generalize to higher orders in α_s/higher twist?
   - Singular terms added "by hand"

2) Moments
   - Can be complicated at higher twist

- Different approach: EFT look at DIS

EFT look at DIS

- Potential issues with standard derivation

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- Different approach: EFT look at DIS

$$F_2 = \underset{\text{Wilson coef.}}{\underset{\uparrow}{C_2(\mu)}} \otimes \underset{\text{Operator}}{\underset{\uparrow}{\phi(\mu)}} + \text{power corrections}$$

$$\frac{dF_2}{d\ln\mu} = 0 \quad \Rightarrow \quad \frac{d\phi}{d\ln\mu} = \underset{\substack{\text{anom.}\\ \text{dim.}}}{\underset{\uparrow}{P}} \otimes \phi$$

- Can't we calculate P "as usual" in EFT?

“Calculating As Usual”

- Consider one loop and Minimal subtraction scheme(s)
- standard procedure, e.g. [Buras hep-ph/9806471]
  - Calculate one loop matrix element
  - Renormalize

$$O_i = Z_{ij}\, O_j$$

where

$$Z_{ij} = I + \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\Big(\quad \Big)$$

Define $Z^{(1)}$ coefficient of $\frac{1}{\epsilon}$

  - Anomalous dimension

$$\gamma_{ij} = 2g^2 \frac{dZ_{ij}^{(1)}}{dg^2}$$

  - RGE equation

$$\frac{dO_i}{d\ln\mu} = \gamma_{ij}\, O_j$$

- Only difference, operators are non local

$\Rightarrow$ anomalous dimension matrix is “infinite”

History

- In fact, this calculation was done before!

  T. Braunschweig, J. Horejsi and D. Robaschik,

  "Nonlocal Light Cone Expansion And Its Applications

  To Deep Inelastic Scattering Processes,"

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  - [Georgi, Politzer PRD **9**, 416 (1974)] 581 citations
  - [Gross, Wilczek PRD **9**, 980 (1974)] 905 citations
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  - [Braunschweig, Horejsi, Robaschik ZPC **23**, 19 (1984)] **5** citations!

    1 self citation , 1 by Nyeo, 3 by Andrei V. Belitsky et. al.

SCET

- But there is a renewed interest...
- The appropriate EFT for DIS is: Soft Collinear Effective Theory
- Calculating the anomalous dimension of non local operators is almost standard in SCET
- Some results for anomalous dimensions from inclusive charmless B decays (analogous to DIS for $x \to 1$)

SCET

- But there is a renewed interest...
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  is almost standard in SCET
- Some results for anomalous dimensions from inclusive charmless B decays

  (analogous to DIS for $x \to 1$)
  - Two loop leading power jet function

    [Becher, Neubert PLB **637**, 251 (2006), arXiv:hep-ph/0603140]
  - one loop subleading power jet functions

    [GP, arXiv:0903.3377, to appear in JHEP]
  - Two loop Leading power shape function (B meson PDF)

    [Becher, Neubert PLB **633**, 739 (2006), arXiv:hep-ph/0512208]
  - Partial one loop subleading power shape functions (subleading twist)

    [Trott, Williamson PRD **74**, 034011 (2006), arXiv:hep-ph/0510203]

    ...

SCET Vs. QCD

- Although SCET is the appropriate EFT for generic x DIS

  the soft gluons completely cancel in the PDF

  [Bauer, Fleming, Pirjol, Rothstein, Stewart, PRD **66**, 014017 (2002), arXiv:hep-ph/0202088]

  [Becher, Neubert, Pecjak JHEP **0701**, 076 (2007), arXiv:hep-ph/0607228]

- In this case SCET is just “boosted QCD”

  Can calculate using QCD Feynman rules

- Let us demonstrate by calculating P_{qq}

  Method technically different from

  [Braunschweig, Horejsi, Robaschik ZPC **23**, 19 (1984)]

  but conceptually very similar

Example of a Non-Local Renormalization:

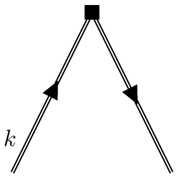
Non-singlet Splitting Function

Feynman Rules

- Definition of PDF

$$\phi_q(\xi) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, e^{i\xi n\cdot pt}\langle N(p)|\bar{\psi}(0)\,[0,tn]\,\frac{\not{n}}{2}\,\psi(tn)|N(p)\rangle$$

- Replace $N(p)$ by a free quark with momentum k



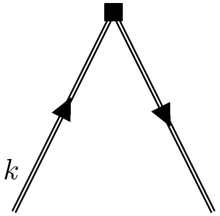
zero-gluon Feynman rule

$$\begin{aligned}
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, e^{i\xi n\cdot pt}\langle k|\bar{\psi}(0)\,\frac{\not{n}}{2}\,\psi(tn)|k\rangle \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, e^{i\xi n\cdot pt}\, e^{i0\cdot k}\, e^{-itn\cdot k}\,\bar{u}(k)\,\frac{\not{n}}{2}\,u(k) \\
&= \delta(\xi\, n\cdot p - n\cdot k)\,\bar{u}(k)\,\frac{\not{n}}{2}\,u(k)
\end{aligned}$$

- If we sum and average over spins and set $k = p$ we will find $\delta(\xi - 1)$, which is the parton model picture

Feynman Rules

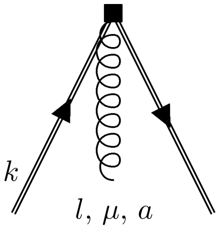
- Zero-gluon Feynman rule



$$\delta(\xi\, n\cdot p - n\cdot k)\,\frac{\not{n}}{2}$$

- In order to find one-gluon Feynman rule

  need to expand Wilson lines



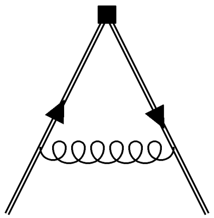
$$t^a\, g\,\frac{n^\mu}{n\cdot l}\left[\delta(\xi\, n\cdot p - n\cdot k) - \delta(\xi\, n\cdot p - n\cdot k + n\cdot l)\right]\,\frac{\not{n}}{2}$$

- Analogous to

  [Grozin, Neubert PRD **55**, 272 (1997), arXiv:hep-ph/9607366]

One Loop Calculation

- We need to calculate, for example,



- Set external momentum to p and internal quark momentum to k
- Get integrals of the form

$$\int \frac{d^d k}{(2\pi)^d} \left[\frac{1}{k^2 + i\epsilon}\right]^2 \frac{1}{(k-p)^2 + i\epsilon}\, \delta(\xi\, n\cdot p - n\cdot k)$$

Only difference from standard Feynman integral, presence of delta function

- Define light cone vectors $n^2 = \bar{n}^2 = 0$, $n\cdot\bar{n} = 2$ e.g.

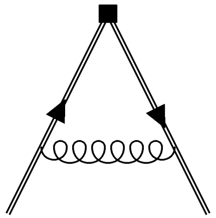
$$n = (1,0,0,-1),\; \bar{n} = (1,0,0,1) \Rightarrow n\cdot p = p_0 + p_3 \geq 0$$

- We have

$$k^\mu = \bar{n}\cdot k\frac{n^\mu}{2} + n\cdot k\frac{\bar{n}^\mu}{2} + k_\perp^\mu$$

$$d^d k = \frac{1}{2}\, dn\cdot k\; d\bar{n}\cdot k\; d^{d-2}k_\perp$$

One Loop Calculation

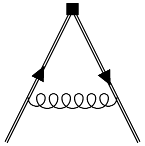


- Integral

$$\int \frac{1}{2(2\pi)^d}\, dn\cdot k\; d\bar{n}\cdot k\; d^{d-2}k_\perp \left[\frac{1}{k^2+i\epsilon}\right]^2 \frac{1}{(k-p)^2+i\epsilon}\,\delta(\xi\, n\cdot p - n\cdot k)$$

- Strategy: Integrate over
  - $\bar{n}\cdot k$: residue
  - $k_\perp$: usual dim. reg.
  - $n\cdot k$: delta function

PDF's support properties



- Integral

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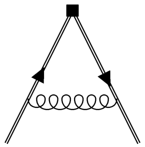
- Integrate $\bar{n}\cdot k$ using residues. Poles:

$$k^2+i\epsilon = \bar{n}\cdot k\, n\cdot k + k_\perp^2 + i\epsilon = 0 \quad \Rightarrow \quad \bar{n}\cdot k = \frac{-k_\perp^2 - i\epsilon}{n\cdot k} = 0$$

$$(k-p)^2 + i\epsilon = 0 \quad \Rightarrow \quad \bar{n}\cdot k = \bar{n}\cdot p + \frac{-(k-p)_\perp^2 - i\epsilon}{n\cdot k - n\cdot p} = 0$$

- For non zero result:

PDF's support properties



- Integral

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$$(k-p)^2+i\epsilon = 0 \quad\Rightarrow\quad \bar{n}\cdot k = \bar{n}\cdot p + \frac{-(k-p)_\perp^2 - i\epsilon}{n\cdot k - n\cdot p} = 0$$

- For non zero result:

I) $n\cdot k > 0 \quad\cup\quad n\cdot k - n\cdot p < 0 \quad\Rightarrow\quad 0 < n\cdot k < n\cdot p \quad\Rightarrow\quad 0 < \dfrac{n\cdot k}{n\cdot p} < 1$

II) $n\cdot k < 0 \quad\cup\quad n\cdot k - n\cdot p > 0 \quad\Rightarrow\quad \cancel{n\cdot p < n\cdot k < 0}$

- Conclusion

$$\delta(\xi\, n\cdot p - n\cdot k) \quad\cup\quad 0 < \frac{n\cdot k}{n\cdot p} < 1 \quad\Rightarrow\quad 0 < \xi < 1$$

Use of Method of Regions

- So far ignored numerator. Full integral

$$I=\int \frac{1}{2(2\pi)^d}\, dn\cdot k\; d\bar{n}\cdot k\; d^{d-2}k_\perp \left[\frac{1}{k^2+i\epsilon}\right]^2 \frac{\text{Num.}}{(k-p)^2+i\epsilon}\,\delta(\xi\, n\cdot p-n\cdot k)$$

  Where

$$\text{Num.}=(n\cdot k)^2\,\frac{\bar{\not{n}}}{2}+n\cdot k\,\not{k}_\perp-k_\perp^2\,\frac{\not{n}}{2}$$

- Numerator structure can be complicated. Simplify using "Method of Regions"

  [Beneke,Smirnov, NPB **522**, 321 (1998), arXiv:hep-ph/9711391]

  [Smirnov, Springer Tracts Mod. Phys. **177**, 1 (2002)]

- For PDF, only collinear region

$$n\cdot k\sim\mathcal{O}(1),\quad \bar{n}\cdot k\sim\mathcal{O}(\Lambda_{\text{QCD}}^2),\quad k_\perp\sim\mathcal{O}(\Lambda_{\text{QCD}})$$

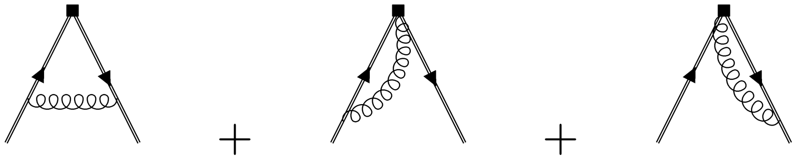
$$d^4k\sim\mathcal{O}(\Lambda_{\text{QCD}}^4),\quad k^2\sim\mathcal{O}(\Lambda_{\text{QCD}}^2)\quad\Rightarrow I\sim\Lambda_{\text{QCD}}^4\left(\frac{1}{\Lambda_{\text{QCD}}^2}\right)^3\times\text{Num.}$$

- Anomalous dimension $\mathcal{O}(1)$ quantity

  Need only Num. $\sim\Lambda_{\text{QCD}}^2\quad\Rightarrow\quad$ only $k_\perp^2$

Full Result

- Full calculation:



+ WFR

- Unlike AP, the + distribution and δ-function arise from

$$(1-\xi)^{-1-\epsilon} = -\frac{1}{\epsilon}\delta(1-\xi) + \frac{1}{(1-\xi)_+} + \mathcal{O}(\epsilon)$$

- Full result

$$D^{\text{div.}}_{\text{bare}} = \bar{u}\frac{\not{n}}{2}u\frac{1}{n\cdot p}\left\{\delta(1-\xi) + \frac{C_F\alpha_s}{4\pi}\frac{1}{\epsilon}\theta(\xi)\theta(1-\xi)2\left[\left(\frac{1+\xi^2}{1-\xi}\right)_+ + \frac{3}{2}\delta(1-\xi)\right]\right\}$$

- We now find the anomalous dimension in the "usual way"

$$\phi_{\text{bare}} = \int du\, Z\left(\frac{x}{u}\right)\phi_{\text{ren.}}(u)$$

$$\gamma = 2g^2\frac{\partial Z^{(1)}}{\partial g^2} \quad \Rightarrow \quad \frac{d\phi}{d\ln\mu} = \int du\,\gamma\left(\frac{x}{u}\right)\phi(u)$$

- We obtain finally the well known result

$$\frac{d\phi}{d\ln\mu} = \int_x^1 \frac{du}{u}\frac{C_F\alpha_s}{\pi}\left[\left(\frac{1+\xi^2}{1-\xi}\right)_+ + \frac{3}{2}\delta(1-\xi)\right]\phi\left(\frac{x}{u}\right)$$

Beyond Non Singlet

- Using similar methods can calculate

  the full 2 × 2 DGLAP
- Method can also applied to higher twist
- What is the current status of higher twist?

Higher Twist in QCD and SCET

Higher Twist in QCD

- Higher twist was considered in early 80's by various authors
- [Jaffe, Soldate PRD **26**, 49 (1982)]

  Approach similar to OPE for twist 2
- To describe $W^{\mu\nu}$ at tree level need
  - 3 four-quark operators
  - 4 operators of the form $\bar{\psi}...\psi$

    where ... combination of $in \cdot D$, $n_\alpha F^{\alpha\beta}$, $\not{n}$ etc.
- The "complete" basis
  - 6 possible four-quark operators

$$\left(\bar{\psi}\left\{\not{n}P_L, \not{n}P_R\right\}\left\{1, t^a\right\}\psi\right)\left(\bar{\psi}\left\{\not{n}P_L, \not{n}P_R\right\}\left\{1, t^a\right\}\psi\right)$$

  - 8 operators of the form $\bar{\psi}...\psi$

Higher Twist in QCD

- Higher twist was considered in early 80's by various authors
- [Ellis, Furmanski, Petronzio NPB **212**, 29 (1983)]

  Analysis using non local operator bases: "Collinear", "Transverse"
- "Collinear": non local version of Jaffe & Soldate

  Taking moments reproduces Jaffe & Soldate

  Expression for $W^{\mu\nu}$ at tree level very **complicated**
- "Transverse": To describe $W^{\mu\nu}$ at tree level need
  - 2 four-quark operators

$$(\bar\psi\, \not{n}\, t^a\, \psi)\ (\bar\psi\, \not{n}\, t^a\, \psi)\,, \quad (\bar\psi\, \not{n}\gamma_5\, t^a\, \psi)\ (\bar\psi\, \not{n}\gamma_5\, t^a\, \psi)$$

  - 2 operators of the form

$$\bar\psi\gamma_\mu\not{n}\gamma_\nu\, D_\perp^\mu\, D_\perp^\nu\,\psi, \quad \bar\psi\gamma_\mu\not{n}\gamma_\nu\, D_\perp^\nu\, D_\perp^\mu\,\psi$$

  Expression for $W^{\mu\nu}$ at tree level very **simple**
- Although operators are multi-non-local

  at tree level appear as function of one variable

Higher Twist in SCET

- 25 years later...

Higher Twist in SCET

- 25 years later...
- SCET basis of twist-4 PE DIS [Marcantonini, Stewart, arXiv:0809.1093]

$$O_1 = \bar{\chi}_{n,\omega_1} \frac{\not{\bar{n}}}{2} \chi_{n,\omega_2} , \qquad O_2 = \bar{\chi}_{n,\omega_1} \frac{\not{\bar{n}}}{2} \mathcal{P}_\perp^2 \chi_{n,\omega_2} ,$$

$$O_{3a} = \bar{\chi}_{n,\omega_1} \frac{\not{\bar{n}}}{2} (g\mathcal{B}^\mu_{n\perp})_{\omega_3} \mathcal{P}^\perp_\mu \chi_{n,\omega_2} , \qquad O_{3b} = \bar{\chi}_{n,\omega_1} \frac{\not{\bar{n}}}{2} \mathcal{P}^\perp_\mu (g\mathcal{B}^\mu_{n\perp})_{\omega_3} \chi_{n,\omega_2} ,$$

$$O_{4a} = \bar{\chi}_{n,\omega_1} \frac{\not{\bar{n}}}{2} (g\not{\mathcal{B}}_{n\perp})_{\omega_3} \not{\mathcal{P}}_\perp \chi_{n,\omega_2} , \qquad O_{4b} = \bar{\chi}_{n,\omega_1} \frac{\not{\bar{n}}}{2} \not{\mathcal{P}}_\perp (g\not{\mathcal{B}}_{n\perp})_{\omega_3} \chi_{n,\omega_2} ,$$

$$O_5 = \bar{\chi}_{n,\omega_1} \frac{\not{\bar{n}}}{2} (g\not{\mathcal{B}}_{n\perp})_{\omega_3} (g\not{\mathcal{B}}_{n\perp})_{\omega_4} \chi_{n,\omega_2} , \qquad O_6 = \bar{\chi}_{n,\omega_1} \frac{\not{\bar{n}}}{2} \mathrm{Tr}\big[(g\not{\mathcal{B}}_{n\perp})_{\omega_3} (g\not{\mathcal{B}}_{n\perp})_{\omega_4}\big] \chi_{n,\omega_2} ,$$

$$O_7 = \bar{\chi}_{n,\omega_1} \frac{\not{\bar{n}}}{2} (g\mathcal{B}^\mu_{n\perp})_{\omega_3} (g\mathcal{B}^{\perp n}_\mu)_{\omega_4} \chi_{n,\omega_2} , \qquad O_8 = \bar{\chi}_{n,\omega_1} \frac{\not{\bar{n}}}{2} \mathrm{Tr}\big[(g\mathcal{B}^\mu_{n\perp})_{\omega_3} (g\mathcal{B}^{\perp n}_\mu)_{\omega_4}\big] \chi_{n,\omega_2} ,$$

$$O_9 = \Big[\bar{\chi}_{n,\omega_1} \frac{\not{\bar{n}}}{2} \chi_{n,\omega_2}\Big] \Big[\bar{\chi}_{n,\omega_3} \frac{\not{\bar{n}}}{2} \chi_{n,\omega_4}\Big] ,$$

$$O_{10} = \Big[\bar{\chi}_{n,\omega_1} \frac{\not{\bar{n}}}{2} \gamma_5 \chi_{n,\omega_2}\Big] \Big[\bar{\chi}_{n,\omega_3} \frac{\not{\bar{n}}}{2} \gamma_5 \chi_{n,\omega_4}\Big] ,$$

$$O_{11} = \Big[\bar{\chi}_{n,\omega_1} \frac{\not{\bar{n}}}{2} \gamma_5 T^A \chi_{n,\omega_2}\Big] \Big[\bar{\chi}_{n,\omega_3} \frac{\not{\bar{n}}}{2} \gamma_5 T^A \chi_{n,\omega_4}\Big] ,$$

$$O_{12} = \Big[\bar{\chi}_{n,\omega_1} \frac{\not{\bar{n}}}{2} T^A \chi_{n,\omega_2}\Big] \Big[\bar{\chi}_{n,\omega_3} \frac{\not{\bar{n}}}{2} T^A \chi_{n,\omega_4}\Big] .$$

Anomalous Dimension of twist-4

- Emphasize again, For generic x DIS :

$$\text{QCD} = \text{SCET}$$

- SCET supplies a complete "transverse" basis

  ideal setting for calculating anomalous dimension

- What is known about anomalous dimension of twist-4?
- Using "OPE"
  - Anomalous dimension of four-quark operators

    [Gottlieb, NPB **139**, 125 (1978)]

    [Okawa, NPB **172**, 481 (1980)]
  - Anomalous dimension of "3-body" operators

    [Okawa, NPB **187**, 71 (1981)]
  - Anomalous dimension of "4-body" operators **unknown**
- We can use non local renormalization to calculate anomalous dimension of twist-4 operators [Glatzmaier, Mantry, Ramsey-Musolf, GP, *In Progress*]
- There is partial calculation of "subleading twist" for inclusive B decays

  by Trott & Williamson, what can we learn from that?

Lessons from

Inclusive Charmless B Decays

Inclusive B decays

- $\bar{B} \to X_s \gamma$ and $\bar{B} \to X_u \, l \, \bar{\nu}$ in the end point region

$$E_X \sim m_b \gg \Lambda_{\rm QCD} \quad P_X^2 \sim m_b \Lambda_{\rm QCD}$$

  analogous to DIS for $x \to 1$

- In particular similar factorization formula

$$\begin{aligned} d\Gamma &\sim H \cdot J \otimes S + \ldots \\ d\sigma &\sim H \cdot J \otimes \phi_{x\to 1} + \ldots \end{aligned}$$

  where S is B meson PDF

- Inclusive B decays are more
  - Complicated: Presence of heavy ("time-like") quarks apart from collinear ("light-like") fields
  - Simple: No mixing with gluonic operators ("non-singlet")

- At subleading power new structures arise

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \ldots$$

  (K.S.M. Lee, Stewart '04; Bosch, Neubert, GP '04; Beneke, Campanario, Mannel, Pecjak '04)

Subleading Shape Functions I

$$
\begin{aligned}
W_{ij}^{\mathrm{SSF}} &= \int d\omega\, \delta(n\cdot p+\omega)\left[\frac{\omega\, S(\omega)+t(\omega)}{m_b}\,T_2+\frac{s(\omega)}{m_b}\,T_1+\frac{t(\omega)}{\bar n\cdot p}\,T_3+\frac{u(\omega)}{\bar n\cdot p}\,T_1-\frac{v(\omega)}{\bar n\cdot p}\,T_4\right]\\
&- \pi\alpha_s \int d\omega\, \delta(n\cdot p+\omega)\left[\frac{f_u(\omega)}{\bar n\cdot p}\,T_1+\frac{f_v(\omega)}{\bar n\cdot p}\,T_4\right]
\end{aligned}
$$

where

$$
\begin{aligned}
\langle \bar h(0)\,[0,x_-]\,h(x_-)\rangle &= \int d\omega\, e^{-\frac{i}{2}\omega\bar n\cdot x}\, S(\omega)\,,\\
m_b\,\langle i\int d^4z\, T\{\bar h(0)\,[0,x_-]\,h(x_-)\,\mathcal{L}_h^{(2)}(z)\}\rangle &= \int d\omega\, e^{-\frac{i}{2}\omega\bar n\cdot x}\, s(\omega)\,,\\
-\int_0^{\bar n\cdot x/2} dt\,\langle \bar h(0)\,\frac{\not{n}}{2}\,[0,tn]\,\gamma_\mu^\perp n_\nu\, gG^{\mu\nu}(tn)\,[tn,x_-]\,h(x_-)\rangle &= \int d\omega\, e^{-\frac{i}{2}\omega\bar n\cdot x}\, t(\omega)\,,\\
-i\int_0^{\bar n\cdot x/2} dt\,\langle \bar h(0)\,[0,tn]\,(iD_\perp)^2(tn)\,[tn,x_-]\,h(x_-)\rangle &= \int d\omega\, e^{-\frac{i}{2}\omega\bar n\cdot x}\, u(\omega)\,,\\
-i\int_0^{\bar n\cdot x/2} dt\,\langle \bar h(0)\,\frac{\not{n}}{2}\,[0,tn]\,\sigma_{\mu\nu}^\perp\, gG_\perp^{\mu\nu}(tn)\,[tn,x_-]\,h(x_-)\rangle &= \int d\omega\, e^{-\frac{i}{2}\omega\bar n\cdot x}\, v(\omega)\,,
\end{aligned}
$$

- s, t unique to inclusive B decay (arise from heavy quark)
- u (“kinetic”), v (“chromomagnetic”) analogous to DIS twist-4

Subleading Shape Functions II

$$\begin{aligned} W_{ij}^{\mathrm{SSF}} &= \int d\omega\, \delta(n\cdot p+\omega) \left[\frac{\omega\, S(\omega)+t(\omega)}{m_b}\, T_2 + \frac{s(\omega)}{m_b}\, T_1 + \frac{t(\omega)}{\bar{n}\cdot p}\, T_3 + \frac{u(\omega)}{\bar{n}\cdot p}\, T_1 - \frac{v(\omega)}{\bar{n}\cdot p}\, T_4 \right] \\ &- \pi\alpha_s \int d\omega\, \delta(n\cdot p+\omega) \left[\frac{f_u(\omega)}{\bar{n}\cdot p}\, T_1 + \frac{f_v(\omega)}{\bar{n}\cdot p}\, T_4 \right] \end{aligned}$$

and

$$\begin{aligned} 2(-i)^2 & \int_0^{\bar{n}\cdot x/2} dt_1 \int_{t_1}^{\bar{n}\cdot x/2} dt_2\, \langle \left[\, \left(\bar{h}S\right)_0 t_a \right]_k \left[t_a \left(S^\dagger h\right)_{x_-} \right]_l \left[\, (\bar{q}S)_{t_2 n} \right]_l \not{n} \left[\, \left(S^\dagger q\right)_{t_1 n} \right]_k \rangle \\ &= \int d\omega\, e^{-\frac{i}{2}\omega \bar{n}\cdot x}\, f_u(\omega)\,, \\ 2(-i)^2 & \int_0^{\bar{n}\cdot x/2} dt_1 \int_{t_1}^{\bar{n}\cdot x/2} dt_2\, \langle \left[\, \left(\bar{h}S\right)_0 t_a \right]_k \not{n}\gamma_5 \left[t_a \left(S^\dagger h\right)_{x_-} \right]_l \left[\, (\bar{q}S)_{t_2 n} \right]_l \not{n}\gamma_5 \left[\, \left(S^\dagger q\right)_{t_1 n} \right]_k \rangle \\ &= \int d\omega\, e^{-\frac{i}{2}\omega \bar{n}\cdot x}\, f_v(\omega)\,, \end{aligned}$$

- f_u, f_v analogous to DIS twist-4 four quark operators
- As for “transverse” basis: although operators are multi-non-local at tree level appear as function of one variable

Towards Anomalous Dimension of SSF

- First study of anomalous dimension of subleading shape functions (SSF)

  [Trott, Williamson PRD **74**, 034011 (2006), arXiv:hep-ph/0510203]
  - Consider mostly inclusive B decays specific operators
  - No four quark operators
- Conclusions of the study
  - "Chromomagnetic"operator mixes only into itself

    with γ of leading twist
  - "Kinetic" operator seems to mix into new operators

    Closure of basis not established!

What is (probably) the problem

- Need to consider genuine multi non local operator
- similar conclusion for subleading jet functions

  [GP, arXiv:0903.3377, to appear in JHEP]
- Multi non local

  $$\langle\Omega|\, T\left\{\psi(0),\, n\cdot\mathcal{A}(y),\, \bar\psi(x)\right\}|\Omega\rangle$$

  finite in general but divergent for $y \to x$ or $y \to 0$
- Ren. of bi-local operators is well known

$\phi(\xi)$	F.T.	of	$\langle P\vert\bar\psi(0)\dots\psi(tn)\vert P\rangle$
$S(\omega)$	F.T.	of	$\langle \bar B\vert h(0)\dots h(tn)\vert\bar B\rangle$
$J(p^2)$	F.T.	of	$\langle\Omega\vert\, T\left\{\psi(0),\, \bar\psi(x)\right\}\vert\Omega\rangle$

- Ren. of multi-local operators:

  subleading jet, shape, or twist

  still an open problem!

Outlook and Conclusions

Gluoinc operators

- Another possible complication: gluonic operators

  [Marcantoninі, Stewart, arXiv:0809.1093]

$$
\begin{aligned}
O_1 &= \mathrm{Tr}\big[(g\mathcal{B}_{n\perp})_{\omega_1}\cdot(g\mathcal{B}_{n\perp})_{\omega_2}\big]\,,\\
O_2 &= \mathrm{Tr}\big[(g\mathcal{B}^{\mu}_{n\perp})_{\omega_1}\mathcal{P}_\perp^2(g\mathcal{B}^{\perp}_{n\,\mu})_{\omega_2}\big]\,,\\
O_{3,4} &= \mathrm{Tr}\big[(g\mathcal{B}^{\mu}_{n\perp})_{\omega_1}(g\mathcal{B}^{\nu}_{n\perp})_{\omega_2}\mathcal{P}^{\alpha}_\perp(g\mathcal{B}^{\beta}_{n\perp})_{\omega_3}\big]\Gamma^{1,2}_{\mu\nu\alpha\beta}\,,\\
O_{5,6} &= \mathrm{Tr}\big[(g\mathcal{B}^{\mu}_{n\perp})_{\omega_1}(g\mathcal{B}^{\nu}_{n\perp})_{\omega_2}(g\mathcal{B}^{\alpha}_{n\perp})_{\omega_3}(g\mathcal{B}^{\beta}_{n\perp})_{\omega_4}\big]\Gamma^{1,2}_{\mu\nu\alpha\beta}\,,\\
O_{7,8} &= \mathrm{Tr}\big[(g\mathcal{B}^{\mu}_{n\perp})_{\omega_1}(g\mathcal{B}^{\nu}_{n\perp})_{\omega_2}\big]\mathrm{Tr}\big[(g\mathcal{B}^{\alpha}_{n\perp})_{\omega_3}(g\mathcal{B}^{\beta}_{n\perp})_{\omega_4}\big]\Gamma^{1,2}_{\mu\nu\alpha\beta}\,,\\
O_9 &= \mathrm{Tr}\big[(g\mathcal{B}^{\mu}_{n\perp})_{\omega_1}\mathcal{P}^{\perp}_{\mu}\mathcal{P}^{\nu}_{\perp}(g\mathcal{B}^{\perp}_{n\,\nu})_{\omega_2}\big]\,,
\end{aligned}
$$

where $\Gamma^{1,2}_{\mu\nu\alpha\beta}=\{g_{\mu\nu}g_{\alpha\beta},g_{\mu\alpha}g_{\nu\beta}\}$ and the traces are over color

Outlook

- Using non local renormalization

  to calculate anomalous dimension of higher twist
  - Not an easy task ...
  - Most likely easier and more feasible than “OPE”
  - Must consider genuine multi-non-local operators

Conclusions

- Although DIS among the oldest examples of (non-local) OPE

  EFT point of view relatively new
- DGLAP equations can be calculated directly from non local operators themselves
- Approach almost unknown ($5 \ll 3800$!) but very common in SCET
- Higher twist was considered since early 80's

  but *complete* anomalous dimension is still lacking
- Using non-local QCD/SCET approach for renormalization

  will allow us to calculate this anomalous dimension
- Interest beyond DIS:

  closure of *any* non local basis beyond leading power *not established*

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  will allow us to calculate this anomalous dimension
- Interest beyond DIS:

  closure of *any* non local basis beyond leading power *not established*
- Stay tuned for results!