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[^0]
## RULES OF THE LAB:

Please leave the lab in at least as nice a condition as you found it, with equipment put away so that it is ready for the next group to use. When you are done and ready to leave, please help us keep the labs neat:
_power off and disconnect all lab equipment
_return all lab equipment to its original condition
_straighten chairs
_keep tables reasonably clean
_clean up bits of string, tape, eraser crumbs...
_put away weights
_throw away garbage
_take writing utensils with you!
During winter: please do not put your feet on bottom shelves of computer carts or on chairs.

If you are in the last lab of the day, please help us maximize the computer lifetime by also turning off the computer equipment:

```
_turn off interface
_turn off power strip for interface
_turn off computer
_turn off monitor
```

Thank you for helping preserve and protect the physics lab room and equipment!

## MC-1 WELCOME TO THE PHYSICS LABORATORY

What do we do in this lab?
This lab has four parts:
A. Make a graph by hand of data given in a table;
B. Make a straight-line fit by eye, and a best-fit straight line, to a series of data;
C. Take data using the "motion sensor": an electronic sensor connected to, and controlled by, a computer;
D. Use the computer to calculate fits to, and present, motion sensor data.

## Why are we doing this lab?

Physics is an experimental science. The results of an experiment are founded on accurate measurements. One presents one's measurements for review by one's peers in the form of graphs and tables. This allows people to check each other's work. After taking Physics 103, we hope you feel you have gained confidence in estimating the accuracy of your measurements, presenting your measurements to others using graphs and tables, and checking the measurements of others.
In Physics 103 we will often take measurements with electronic sensors. Electronic sensors can make measurements more accurately than a human could, but they have limitations. When presented with a sensor you have never used before, you should take some time to make sure that its measurements are consistent with what you see and feel and hear (part C).

You also will need to be familiar with Capstone, the data acquisition software used in this lab. Capstone controls the electronic sensors, records data taken by them, and plots it on the screen (part C). It can also calculate fits to data (part D).

In this course we'll often present data for other people to review using Excelyou may use Excel after you're done taking physics, so maybe the skill of making good graphs using Excel will be useful to you in the future (part D).
Safety concerns: none

## YOU NEED TO KNOW:

error bars graphical indication of the uncertainty of a measurement.
speed the magnitude of velocity. Always a positive number. In this course, speed will be measured in units of meters/second. Some examples:

| Ordinary walking speed | 3.0 miles/hour | 1.3 meters $/$ second |
| :--- | :--- | :--- |
| Typical freeway speed | 60 miles $/$ hour | 27 meters/second |

velocity rate of change of position. To specify velocity, one must specify both the speed of motion and the direction of motion. An object moving at 10 meters/second towards the north has the same velocity as an object moving at -10 meters/second toward the south. This lab will be concerned with instantaneous velocity, but not with average velocity.

MC-1 Welcome to the Physics Lab!

## Pre-lab questions

1. How many lab-based science classes have you taken, in which you worked together with one or more lab partners to do science experiments?
2. How many laboratory-based Physics courses have you taken before now?
3. Have you ever been asked to make a graph based on data from a table?
4. Have you ever been asked to measure the slope of a line that is plotted on a graph?
5. Do you have any experience fitting data to a curve using the method of least squares?
6. Have you ever used Excel as part of a class assignment (in any grade)?
7. Are you comfortable using Excel?

## A: Representing data by graphs and tables

In this course you will use both graphs and tables to present data. It's important to be able to represent the same data in both a graph and a table. Appendix C contains instructions on how to make graphs using Excel. Here's an example:

Data source: US Department of Agriculture, National Agriculture Statistics Service http://quickstats.nass.usda.gov/results/E5DD00AF-62BF-3B1F-B2C8-A10D38F42174


Note nine of the features of this graph:

1. Descriptive title
2. Data points not connected by lines
3. Data points have error bars
4. Ordinate tickmark labels are not too many and not too few
5. Ordinate labels use scientific notation, and have a reasonable number of significant figures
6. Ordinate title clearly describes what is being plotted
7. Ordinate includes zero
8. Abscissa title clearly describes what is being plotted
9. Abscissa tickmark labels not too many and not too few

The data has an uncertainty of $+/-5 \%$. Thus the error bars on the data point for 2013 are about three times larger than the error bars on the data point for 1970.

Note that the table contains more information than the graph: if given the table, you can make the graph, but not vice versa.

## Map of Wisconsin Counties



Source: US Census Bureau (especially http://www.census.gov/population/cencounts/wi190090.txt)

| Year: | 2010 | 2000 | 1990 | 1980 | 1970 | 1960 | 1950 | 1940 | 1930 | 1920 | 1910 | 1900 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| United States | 308,747,508 | 281,421,906 | 248,709,873 | 226,545,805 | 203,211,926 | 179,323,175 | 151,325,798 | 132,164,569 | 123,202,624 | 106,021,537 | 92,228,496 | 76,212,168 |
| Wisconsin | 5,686,986 | 5,363,675 | 4,891,769 | 4,705,767 | 4,417,731 | 3,951,777 | 3,434,575 | 3,137,587 | 2,939,006 | 2,632,067 | 2,333,860 | 2,069,042 |
| Adams County | 20,875 | 18,643 | 15,682 | 13,457 | 9,234 | 7,566 | 7,906 | 8,449 | 8,003 | 9,287 | 8,604 | 9,141 |
| Ashland County | 16,157 | 16,866 | 16,307 | 16,783 | 16,743 | 17,375 | 19,461 | 21,801 | 21,054 | 24,538 | 21,965 | 20,176 |
| Barron County | 45,870 | 44,963 | 40,750 | 38,730 | 33,955 | 34,270 | 34,703 | 34,289 | 34,301 | 34,281 | 29,114 | 23,677 |
| Bayfield County | 15,014 | 15,013 | 14,008 | 13,822 | 11,683 | 11,910 | 13,760 | 15,827 | 15,006 | 17,201 | 15,987 | 14,392 |
| Brown County | 248,007 | 226,778 | 194,594 | 175,280 | 158,244 | 125,082 | 98,314 | 83,109 | 70,249 | 61,889 | 54,098 | 46,359 |
| Buffalo County | 13,587 | 13,804 | 13,584 | 14,309 | 13,743 | 14,202 | 14,719 | 16,090 | 15,330 | 15,615 | 16,006 | 16,765 |
| Burnett County | 15,457 | 15,674 | 13,084 | 12,340 | 9,276 | 9,214 | 10,236 | 11,382 | 10,233 | 10,735 | 9,026 | 7,478 |
| Calumet County | 48,971 | 40,631 | 34,291 | 30,867 | 27,604 | 22,268 | 18,840 | 17,618 | 16,848 | 17,228 | 16,701 | 17,078 |
| Chippewa County | 62,415 | 55,195 | 52,360 | 52,127 | 47,717 | 45,096 | 42,839 | 40,703 | 37,342 | 36,482 | 32,103 | 33,037 |
| Clark County | 34,690 | 33,557 | 31,647 | 32,910 | 30,361 | 31,527 | 32,459 | 33,972 | 34,165 | 35,120 | 30,074 | 25,848 |
| Columbia County | 56,833 | 52,468 | 45,088 | 43,222 | 40,150 | 36,708 | 34,023 | 32,517 | 30,503 | 30,468 | 31,129 | 31,121 |
| Crawford County | 16,644 | 17,243 | 15,940 | 16,556 | 15,252 | 16,351 | 17,652 | 18,328 | 16,781 | 16,772 | 16,288 | 17,286 |
| Dane County | 488,073 | 426,526 | 367,085 | 323,545 | 290,272 | 222,095 | 169,357 | 130,660 | 112,737 | 89,432 | 77,435 | 69,435 |
| Dodge County | 88,759 | 85,897 | 76,559 | 75,064 | 69,004 | 63,170 | 57,611 | 54,280 | 52,092 | 49,742 | 47,436 | 46,631 |
| Door County | 27,785 | 27,961 | 25,690 | 25,029 | 20,106 | 20,685 | 20,870 | 19,095 | 18,182 | 19,073 | 18,711 | 17,583 |
| Douglas County | 44,159 | 43,287 | 41,758 | 44,421 | 44,657 | 45,008 | 46,715 | 47,119 | 46,583 | 49,771 | 47,422 | 36,335 |
| Dunn County | 43,857 | 39,858 | 35,909 | 34,314 | 29,154 | 26,156 | 27,341 | 27,375 | 27,037 | 26,970 | 25,260 | 25,043 |
| Eau Claire County | 98,736 | 93,142 | 85,183 | 78,805 | 67,219 | 58,300 | 54,187 | 46,999 | 41,087 | 35,771 | 32,721 | 31,692 |
| Florence County | 4,423 | 5,088 | 4,590 | 4,172 | 3,298 | 3,437 | 3,756 | 4,177 | 3,768 | 3,602 | 3,381 | 3,197 |
| Fond du Lac County | 101,633 | 97,296 | 90,083 | 88,964 | 84,567 | 75,085 | 67,829 | 62,353 | 59,883 | 56,119 | 51,610 | 47,589 |
| Forest County | 9,304 | 10,024 | 8,776 | 9,044 | 7,691 | 7,542 | 9,437 | 11,805 | 11,118 | 9,850 | 6,782 | 1,396 |
| Grant County | 51,208 | 49,597 | 49,264 | 51,736 | 48,398 | 44,419 | 41,460 | 40,639 | 38,469 | 39,044 | 39,007 | 38,881 |
| Green County | 36,842 | 33,647 | 30,339 | 30,012 | 26,714 | 25,851 | 24,172 | 23,146 | 21,870 | 21,568 | 21,641 | 22,719 |
| Green Lake County | 19,051 | 19,105 | 18,651 | 18,370 | 16,878 | 15,418 | 14,749 | 14,092 | 13,913 | 14,875 | 15,491 | 15,797 |
| Iowa County | 23,687 | 22,780 | 20,150 | 19,802 | 19,306 | 19,631 | 19,610 | 20,595 | 20,039 | 21,504 | 22,497 | 23,114 |
| Iron County | 5,916 | 6,861 | 6,153 | 6,730 | 6,533 | 7,830 | 8,714 | 10,049 | 9,933 | 10,261 | 8,306 | 6,616 |
| Jackson County | 20,449 | 19,100 | 16,588 | 16,831 | 15,325 | 15,151 | 16,073 | 16,599 | 16,468 | 17,746 | 17,075 | 17,466 |
| Jefferson County | 83,686 | 74,021 | 67,783 | 66,152 | 60,060 | 50,094 | 43,069 | 38,868 | 36,785 | 35,022 | 34,306 | 34,789 |
| Juneau County | 26,664 | 24,316 | 21,650 | 21,039 | 18,455 | 17,490 | 18,930 | 18,708 | 17,264 | 19,209 | 19,569 | 20,629 |
| Kenosha County | 166,426 | 149,577 | 128,181 | 123,137 | 117,917 | 100,615 | 75,238 | 63,505 | 63,277 | 51,284 | 32,929 | 21,707 |
| Kewaunee County | 20,574 | 20,187 | 18,878 | 19,539 | 18,961 | 18,282 | 17,366 | 16,680 | 16,037 | 16,091 | 16,784 | 17,212 |
| La Crosse County | 114,638 | 107,120 | 97,904 | 91,056 | 80,468 | 72,465 | 67,587 | 59,653 | 54,455 | 44,355 | 43,996 | 42,997 |
| Lafayette County | 16,836 | 16,137 | 16,076 | 17,412 | 17,456 | 18,142 | 18,137 | 18,695 | 18,649 | 20,002 | 20,075 | 20,959 |
| Langlade County | 19,977 | 20,740 | 19,505 | 19,978 | 19,220 | 19,916 | 21,975 | 23,227 | 21,544 | 21,471 | 17,062 | 12,553 |
| Lincoln County | 28,743 | 29,641 | 26,993 | 26,555 | 23,499 | 22,338 | 22,235 | 22,536 | 21,072 | 21,084 | 19,064 | 16,269 |
| Manitowoc County | 81,442 | 82,887 | 80,421 | 82,918 | 82,294 | 75,215 | 67,159 | 61,617 | 58,674 | 51,644 | 44,978 | 42,261 |
| Marathon County | 134,063 | 125,834 | 115,400 | 111,270 | 97,457 | 88,874 | 80,337 | 75,915 | 70,629 | 65,259 | 55,054 | 43,256 |
| Marinette County | 41,749 | 43,384 | 40,548 | 39,314 | 35,810 | 34,660 | 35,748 | 36,225 | 33,530 | 34,361 | 33,812 | 30,822 |
| Marquette County | 15,404 | 15,832 | 12,321 | 11,672 | 8,865 | 8,516 | 8,839 | 9,097 | 9,388 | 10,443 | 10,741 | 10,509 |
| Menominee County | 4,232 | 4,562 | 3,890 | 3,373 | 2,607 | --- | --- | --- | --- | --- | --- |  |
| Milwaukee County | 947,735 | 940,164 | 959,275 | 964,988 | 1,054,063 | 1,036,041 | 871,047 | 766,885 | 725,263 | 539,449 | 433,187 | 330,017 |
| Monroe County | 44,673 | 40,899 | 36,633 | 35,074 | 31,610 | 31,241 | 31,378 | 30,080 | 28,739 | 28,666 | 28,881 | 28,103 |
| Oconto County | 37,660 | 35,634 | 30,226 | 28,947 | 25,553 | 25,110 | 26,238 | 27,075 | 26,386 | 27,104 | 25,657 | 20,874 |
| Oneida County | 35,998 | 36,776 | 31,679 | 31,216 | 24,427 | 22,112 | 20,648 | 18,938 | 15,899 | 13,996 | 11,433 | 8,875 |
| Outagamie County | 176,695 | 160,971 | 140,510 | 128,799 | 119,356 | 101,794 | 81,722 | 70,032 | 62,790 | 55,113 | 49,102 | 46,247 |
| Ozaukee County | 86,395 | 82,317 | 72,831 | 66,981 | 54,421 | 38,441 | 23,361 | 18,985 | 17,394 | 16,335 | 17,123 | 16,363 |
| Pepin County | 7,469 | 7,213 | 7,107 | 7,477 | 7,319 | 7,332 | 7,462 | 7,897 | 7,450 | 7,481 | 7,577 | 7,905 |

Population of Wisconsin, by county, 1900-2010
Source: US Census Bureau (http://www.census.gov/population/cencounts/wi190090.txt)

| Year | 2010 | 2000 | 1990 | 1980 | 1970 | 1960 | 1950 | 1940 | 1930 | 1920 | 1910 | 1900 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pierce County | 41,019 | 36,804 | 32,765 | 31,149 | 26,652 | 22,503 | 21,448 | 21,471 | 21,043 | 21,663 | 22,079 | 23,943 |
| Polk County | 44,205 | 41,319 | 34,773 | 32,351 | 26,666 | 24,968 | 24,944 | 26,197 | 26,567 | 26,870 | 21,367 | 17,801 |
| Portage County | 70,019 | 67,182 | 61,405 | 57,420 | 47,541 | 36,964 | 34,858 | 35,800 | 33,827 | 33,649 | 30,945 | 29,483 |
| Price County | 14,159 | 15,822 | 15,600 | 15,788 | 14,520 | 14,370 | 16,344 | 18,467 | 17,284 | 18,517 | 13,795 | 9,106 |
| Racine County | 195,408 | 188,831 | 175,034 | 173,132 | 170,838 | 141,781 | 109,585 | 94,047 | 90,217 | 78,961 | 57,424 | 45,644 |
| Richland County | 18,021 | 17,924 | 17,521 | 17,476 | 17,079 | 17,684 | 19,245 | 20,381 | 19,525 | 19,823 | 18,809 | 19,483 |
| Rock County | 160,331 | 152,307 | 139,510 | 139,420 | 131,970 | 113,913 | 92,778 | 80,173 | 74,206 | 66,150 | 55,538 | 51,203 |
| Rusk County | 14,755 | 15,347 | 15,079 | 15,589 | 14,238 | 14,794 | 16,790 | 17,737 | 16,081 | 16,403 | 11,160 |  |
| St. Croix County | 84,345 | 63,155 | 50,251 | 43,262 | 34,354 | 29,164 | 25,905 | 24,842 | 25,455 | 26,106 | 25,910 | 26,830 |
| Sauk County | 61,976 | 55,225 | 46,975 | 43,469 | 39,057 | 36,179 | 38,120 | 33,700 | 32,030 | 32,548 | 32,869 | 33,006 |
| Sawyer County | 16,557 | 16,196 | 14,181 | 12,843 | 9,670 | 9,475 | 10,323 | 11,540 | 8,878 | 8,243 | 6,227 | 3,593 |
| Shawano County | 41,949 | 40,664 | 37,157 | 35,928 | 32,650 | 34,351 | 35,249 | 35,378 | 33,516 | 33,975 | 31,884 | 27,475 |
| Sheboygan County | 115,507 | 112,646 | 103,877 | 100,935 | 96,660 | 86,484 | 80,631 | 76,221 | 71,235 | 59,913 | 54,888 | 50,345 |
| Taylor County | 20,689 | 19,680 | 18,901 | 18,817 | 16,958 | 17,843 | 18,456 | 20,105 | 17,685 | 18,045 | 13,641 | 11,262 |
| Trempealeau County | 28,816 | 27,010 | 25,263 | 26,158 | 23,344 | 23,377 | 23,730 | 24,381 | 23,910 | 24,506 | 22,928 | 23,114 |
| Vernon County | 29,773 | 28,056 | 25,617 | 25,642 | 24,557 | 25,663 | 27,906 | 29,940 | 28,537 | 29,252 | 28,116 | 28,351 |
| Vilas County | 21,430 | 21,033 | 17,707 | 16,535 | 10,958 | 9,332 | 9,363 | 8,894 | 7,294 | 5,649 | 6,019 | 4,929 |
| Walworth County | 102,228 | 93,759 | 75,000 | 71,507 | 63,444 | 52,368 | 41,584 | 33,103 | 31,058 | 29,327 | 29,614 | 29,259 |
| Washburn County | 15,911 | 16,036 | 13,772 | 13,174 | 10,601 | 10,301 | 11,665 | 12,496 | 11,103 | 11,377 | 8,196 | 5,521 |
| Washington County | 131,887 | 117,493 | 95,328 | 84,848 | 63,839 | 46,119 | 33,902 | 28,430 | 26,551 | 25,713 | 23,784 | 23,589 |
| Waukesha County | 389,891 | 360,767 | 304,715 | 280,326 | 231,365 | 158,249 | 85,901 | 62,744 | 52,358 | 42,612 | 37,100 | 35,229 |
| Waupaca County | 52,410 | 51,731 | 46,104 | 42,831 | 37,780 | 35,340 | 35,056 | 34,614 | 33,513 | 34,200 | 32,782 | 31,615 |
| Waushara County | 24,496 | 23,154 | 19,385 | 18,526 | 14,795 | 13,497 | 13,920 | 14,268 | 14,427 | 16,712 | 18,886 | 15,972 |
| Winnebago County | 166,994 | 156,763 | 140,320 | 131,703 | 129,931 | 107,928 | 91,103 | 80,507 | 76,622 | 63,897 | 62,116 | 58,225 |
| Wood County | 74,749 | 75,555 | 73,605 | 72,799 | 65,362 | 59,105 | 50,500 | 44,465 | 37,865 | 34,643 | 30,583 | 25,865 |

A1. Your TA will provide you with a piece of graph paper. Use the population data table on the last two pages to graph (by hand!) the population history of your favorite Wisconsin county. If you don't have a favorite Wisconsin county, then pick Dane County (the county we are in). Assume the uncertainty for each population estimate is $+/-4 \%$, so make your error bars this long. Be neat and careful in your work.

## B. Fitting a curve to data

Sometimes you will be asked to make a "best fit" of the data you have gathered to a particular curve, such as a straight line or a parabola, predicted by theory (Appendix B contains a glossary of terms associated with error analysis).
Excel is good at accurately calculating best fits (it calls them "trendlines"), such as the best-fit straight line shown in the graph below.


B1. In the graph above, what is represented by the " $x$ " in the equation of the bestfit straight line?
B2. What is represented by the " $y$ "?

B3. Rewrite 3.99E+07 in familiar notation:


A statistically rigorous method of calculating best fits is "the method of least squares", and this is what Excel uses. In this class, whenever you prepare data to show to other people, make sure you use this method.
However, sometimes it is helpful to make a fit "by eye".
B4. Return to the graph you drew in A1. Use a ruler to draw the straight line fit by eye (depending on the county you chose, the straight line might fit the data very well or very poorly).

B5. As accurately as you can, write down the equation of the fit-by-eye line here: straight line fit by eye: $\mathrm{y}=$ $\qquad$ x + $\qquad$ ,
where " $y$ " represents $\qquad$ and " $x$ " represents $\qquad$
B6. Go to the computer, start a web browser, and download the file http://www.physics.wisc.edu/instructional/phys103/census2.xlsx, which contains the same population data that is printed in the lab manual.
B7. Use Excel to calculate the best-fit straight line for the population data of your favorite county, consulting Appendix C as necessary, and enter the parameters of Excel's best-fit straight line here:
Excel's best-fit straight line: y = $\qquad$ $\mathrm{x}+$ $\qquad$ .

B8. Compare the slopes of the two lines you found in B7 and B9: what is the percentage difference between them? (Hint: Calculate percentage difference as (SlopeFromB7-SlopeFromB5)/SlopeFromB7 x 100\%)
Percentage difference between slopes from best-fit line and fit-by-eye line : $\qquad$ \%

It's not necessary for the fit by eye to be precisely the same as the best fit, in order for it to be useful. One use of the fit by eye is to catch big mistakes in "best-fit" lines which have been calculated incorrectly (a typical example occurs when data entered into the computer contains an error, such as a misplaced decimal point).
B8. Suppose you are a physicist. For your job, you have to read a lot of physics papers, all of which contains graphs of data (similar to the graph on the previous page), but only some of which contain tables that present the same data (the papers have to be kept under a certain length, and this often prevents the inclusion of tables, which take up a lot of space). Also for your job, you sometimes have to write physics papers that present data using graphs and tables.

Try to think of one situation in which it is very useful to have:
The skill of creating a best-fit straight line to a series of data:

The skill of creating a fit by eye to a series of data:

Part C: Taking data on the computer


How to get the computer ready to take data:

1. Connect motion sensor to interface (yellow plug in "Digital Channel 1", black plug in "Digital Channel 2").
2. Turn on power strip.
3. Turn on power switch on back of interface.
4. Turn on computer.
5. Turn on monitor.
6. Go to https://www.physics.wisc.edu/instructional/phys103/ , download file MC1.ds, and launch it.

Computer is now ready to take data.
The motion sensor sends out sound pulses and listens for echoes. It then reports to the computer how long it took for the echoes to return. The computer knows the speed of sound, and calculates the distance to the echo-causing object. It then calculates the velocity from the differences between successive position measurements (see Appendix D for details).


On top of the motion sensor is a switch that tells it how hard to listen for echoes (that is, the switch sets the receiver amplification). Set the switch to the left, as shown in the picture (low-amplification setting). In this setting the motion sensor won't get distracted by weak echoes from the floor, walls, table, etc.

## How to take data with the motion sensor:

1. Click the "Start" button at the upper left of the computer screen. Sensor should start clicking. Move your hand in front of the sensor, coming closer and then moving further away. Wiggles should appear on the graph. Congratulations! That's your first data set.
2. Click "Stop" button to stop taking data.


The sound pulses emitted by the motion sensor are "directional" - they are restricted to a cone. The motion sensor is likely to receive echoes from objects within this cone.

C1. Tilt the motion sensor about $10^{\circ}$ above the horizontal, so that it can see echoes from the vane on the front of the cart, but not from the rest of the cart, as in the picture below.


C2. One person per group: while making sure that no part of your body is in between the motion sensor and the cart, take data with the motion sensor as you move the cart closer to, and further from, the motion sensor. Watch as the data points appear on the screen. Continue until all members of the group agree that the "wiggles" that appear on the screen are clearly correlated with the motion of the cart.

C3. Consider the following graph of position vs. time:


Discuss as a group what the corresponding velocity vs. time graph should look like. After agreement is reached, fill in the group's prediction here:


C4. Deputize one person to move the cart in such a way as to cause the position vs. time graph shown in C3 to appear on the top half of the computer screen. Verify that the velocity vs. time graph that appears on the bottom of the screen is in accordance with your prediction from C3.

Now designate members of your lab group to be "Person 1", "Person 2", and "Person 3", and do the following steps.

Person 1:
C5. Move the cart away from the motion sensor with constant velocity.
C6. Observe (with your lab group) the position and velocity graphs on the screen you obtained in C5. Highlight the parts of the velocity graph that were associated with the motion you performed. Ask your TA to verify the graphs are correct.

C7. Highlight the part of the position graph that corresponds to the time you moved the cart with constant velocity. Write down the times of the first highlighted data point and the last highlighted point: first highlighted data point: $\qquad$ s last highlighted data point $\qquad$ s

C8. What would the graphs look like if you were not moving the cart at a constant velocity, but instead, were moving it with constantly increasing velocity? Sketch on the graphs below.


Person 2:
C9. Verify that the answer to C 8 is correct: move the cart away from the motion sensor while increasing its velocity at a constant rate as you move it.

C10. Observe (with your lab group) the position and velocity graphs on the screen you obtained in C9. Highlight the parts of the graphs that were associated with the motion you performed. Call over your TA to verify the graphs are correct.

Person 3:
C11. Begin with the cart held in front of the motion sensor at a suitable distance. Move it toward to motion sensor with a constant velocity for some time, then move it away from the motion sensor with constant velocity, continuing for some distance past the starting point, and then stop.

C12. Observe (with your lab group) the position and velocity graphs on the screen you obtained in C11. Highlight the parts of the graphs that were associated with the motion you performed. Ask your TA to verify the graphs are correct.
(end of person-by-person instructions)
C13. If you put an object too close to the motion sensor, the motion sensor reading is not accurate. How close can you bring an object to the motion sensor, and still have the motion sensor read accurately?

C14. Likewise - how far away can you move an object, and still have the motion sensor read accurately?
furthest accurately-measurable distance: $\qquad$ m
C15. The motion sensor takes measurements a certain number of times per second. This is called the "sample rate". Based on the position graph, what is the sensor's sample rate?

When you make a measurement, ask yourself this question: if someone else makes the same measurement, how close do I expect it to be to mine? This question is the root of the notion of "accurate".

C16. How can you test the accuracy of the motion sensor? Plan how to: put an object about one meter away from the motion sensor; remove everything that might confuse the motion sensor, and lead to measurements of lower accuracy; compare the motion sensor's measurement to a measurement made by a different method that you know to be accurate.

C17. The best-case accuracy of the motion sensor's measurement of position is:

$$
\pm \quad \text { \% }
$$

C18. Figure out how to use DataStudio to display motion sensor data in the form of a table.

C19. Choose your favorite run and compare the position data shown in the table to the position data shown on the graph. Convince yourself that the table and the graph show the same data. Which contains more information, the table or the graph?

## D. Calculating fits and presenting motion sensor data using the computer

The main task in this part is to export the position data from step C6 into Excel, use Excel to find the best-fit straight line to a subset of the data points, plot the data and the fit, and present the data and fit to other people for review.

## How to Export DataStudio Data into Excel

1. Go to File pulldown menu in DataStudio and pick "Export Data...". "Export Data" pop-up window should appear.
2. Pick data set you want to export. Click "OK".
3. "Save as" pop-up window should come up. Choose the following:

Save in: Desktop.
Save as type: Text file (*.txt)
4. Double-click on Microsoft Excel icon on desktop to launch Excel.
5. Click on the Office Button in upper left corner.
6. Click on the Open icon
7. "Open" pop-up window should come up. Choose to:

Look in: Desktop
Files of type: text files
8. The file you just saved should be listed. Double-click it.
9. "Text import wizard step 1 of 3 " pop-up window appears. Choose original data type: delimited. Click Next.

1. "Text import wizard step 2 of 3 " pop-up window appears. Check Delimiters: Tab. Check to see if data lines up in columns OK. Move column breaks if necessary. Click Next when column breaks are OK.
2. "Text import wizard step 3 of 3 " pop-up window appears. Change column data format if necessary (if you're not sure of best column data format, try "general"). Click Finish.

Data should now be in Excel.

D1. In step C7 you highlighted a subset of the position data points. Identify these same data points in Excel.

D2. Fit a linear trendline to the data points you identified in D1 (see Appendix C, Physics Laboratory Excel Tutorial, for guidance). Display the equation of the fit.

D3. Go back to DataStudio, and make a linear fit to these same data points (you'll need to click the DataStudio fit button: $\qquad$

D4. Does the slope of the best-fit line calculated by Excel agree with the slope of the linear fit calculated by DataStudio? (If you can't get the two slopes to agree, call over your TA).

D5. Once you are confident Excel is calculating the fit correctly, make a nice graph of the data, using all the features described in Part A. Add the trendline from D2. Print out the graph.

D6. Exchange graphs with a neighboring lab group. Examine their graph with a critical eye: does this graph present everything you need to know to understand correctly the data presented? You should not have to "guess" about anything.

It is unfortunately very easy to omit important pieces of information, such as the units of measurement, without which a graph becomes incomprehensible. It's also sometimes difficult to resist the temptation to graph the data in a way that looks favorable to your conclusion, even though it is misleading.

Look carefully at the next graph you see in a newspaper or online. Do you understand exactly what is being graphed, or are important pieces of information missing? See the book "How to Lie With Statistics" by Darrell Huff for some examples of graphs that (intentionally or otherwise) present data in misleading ways.

## MC-2 MOTION ON AN INCLINE

What do we do in this lab: use the motion sensor to make quantitative measurements of the position, velocity, and acceleration of an object moving in one dimension under the influence of gravity.
In the first three parts (of five), the object in question is a cart that rolls up and down a ramp; in the fourth part, the object is a bouncing basketball. In the fifth part, two motion sensors will be used to measure the motion of the same object (a metal plate), but one motion sensor will be mounted on the table, and the other on the back of a rolling cart.
Why are we doing this lab: to determine whether the motion of objects is predictable. In particular, to see whether the rules asserted in the textbook and lecture regarding one-dimensional motion with constant acceleration apply to objects in real life. In addition, we'll look into the question of whether different observers, with different reference frames, can disagree concerning the motion of the same object.

Safety concerns: in part D, the stand holding the motion sensor sticks out over the edge of the table-be careful not to walk into it.

## You need to know:

Acceleration rate of change of velocity with time
Discrepancy an unexplained difference
Frame of reference point of view. Specifically, a point of view from which selfconsistent measurements of position, velocity, and acceleration can be made. In the lab we will use two frames of reference: the "lab frame", in which the observer is in the physics lab and the physics lab is stationary, and the "cart frame", in which the observer is on the cart and the cart is stationary.
Standard deviation sophisticated estimate of the uncertainty of a measurement
Uncertainty quantification of the experimenter's confidence in a measurement
Velocity rate of change of position with time

## Pre-lab questions:

1. Write down an example of a situation from daily life in which an object moves with a. constant position; b. constant velocity; and c. constant acceleration.
2. Write down an example from daily life in which an object moves with zero acceleration, but non-zero velocity.
3. Write down an example from daily life in which an object moves (for an instant) with zero velocity, but non-zero acceleration.
4. An object moves in one dimension with constant acceleration. What is the shape of the graph of the velocity of the object (vertical axis) vs. time (horizontal axis)
5. For the same object, what is the shape of the graph of the position of the object vs. time?
6. The triangle below has sides that are 5,12 , and 13 units long. What is the angle $\theta$ ?


12
7. Look up the speed of sound at standard pressure and temperature. How long does it take sound to travel a mile? How long does it take sound to travel a meter?
8. Some questions pertaining to "rate":
a. A car takes one hour to cover the distance between mile marker 240 and mile marker 300 on Interstate 94 . What is the average rate of change of position with time of the car, in units of miles per hour?
b. As, you know, the Earth rotates once per day. What is the rate of rotation of the Earth, in units of degrees/hour?
c. A motion sensor is set up to take a measurement every 0.05 seconds. What is the measurement rate of the motion sensor, in units of measurements/second?

## A: Motion on an incline: effect of angle

A1. Raise one end of the track so that the angle $\theta=5^{\circ}$ (see figure below).
A2. Connect the motion sensor: yellow plug into Digital Channel 1, black plug into Digital Channel 2.

A3. Tilt the motion sensor about $15^{\circ}$ above the horizontal, so that it can see echoes from the vane of the cart, but not from the track.


A4. From the Desktop, go to Pasco/Physics103 and launch the Pasco setup file "1-D Motion.cap".

A5. Take data. Start the cart at the bottom of the ramp, near the motion sensor. Give it a push, so that it rolls most of the way up the ramp, but doesn't hit the top.
Let the cart roll most of the way back down to the bottom of the ramp, but don't let it crash.

Make sure your hand and arm stay outside the region of sensitivity.
If you see any "jumps" in the data, you will have to retake the data; you may have to reposition the motion sensor.
A6. As you know, the acceleration of an object is the slope of the graph of its velocity as a function of time. Calculate the acceleration of the cart, while it is moving up the ramp, by using Capstone's "Fit" function to find the best-fit straight line to the graph of velocity as a function of time:

$$
\theta=5^{\circ}, \text { acceleration }=
$$

$\qquad$ meters/second ${ }^{2}$

A7. Repeat part A6 for $\theta=1^{\circ}$ :

$$
\theta=1^{\circ}, \text { acceleration }=\ldots \text { meters } / \text { second }{ }^{2}
$$

A8. Repeat part A6 again for $\theta=3^{\circ}$ :
$\theta=3^{\circ}$, measurement $1:$ acceleration $=$ $\qquad$ meters/second ${ }^{2}$

A9. Without deleting any data, repeat A8 four more times. Enter all five measurements of the acceleration of the cart while it is moving up the ramp in the table below:

| UP | Measurement 1 <br> $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | Measurement 2 <br> $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | Measurement 3 <br> $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | Measurement 4 <br> $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | Measurement 5 <br> $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Acceleration |  |  |  |  |  |

A10. Enter these five measurements in Excel, and use the Excel functions AVERAGE and STDEV to calculate the average and standard deviation of your measurements (see Appendix B, Error Analysis, especially the section "How do I determine the uncertainty of a measurement?"). Equate the uncertainty in your determination of the acceleration of the cart, $\delta a$, with the standard deviation of your measurements. Enter the results here:

$$
\begin{array}{cc}
\theta=3^{\circ}, \text { average acceleration } a= & \text { meters } / \text { second }^{2} \\
\theta=3^{\circ}, \text { uncertainty in acceleration } \delta a=+/- & \text { meters } / \text { second }^{2}
\end{array}
$$

A11. Use your results from A10 to calculate the fractional uncertainty $\delta a / a$ in the measurement:

$$
\text { fractional uncertainty } \delta a / a=\ldots \text { (dimensionless) }
$$

A12. Assume the fractional uncertainty in your results from A6 and A7 is the same as you found in A11. Using Excel, make a nice plot of acceleration (vertical axis) as a function of angle (horizontal axis). Include error bars on the accelerations.

A13. Exchange your plot from A12 with a neighboring lab group. Look over their plot as if you were considering it for publication in a journal. Does it contain everything that a reader who has not done this lab would need to know, in order to understand what is being presented? (See page MC-1-3.) If not, suggest the necessary changes.

## B. Motion on an incline: effect of mass

B1. Put the rectangular mass on the back of the cart, and repeat A8.
$\theta=3^{\circ}$, extra mass: acceleration $=$ $\qquad$ meters/second ${ }^{2}$

B2. Is the difference between the acceleration measured in B1, and the average acceleration found in A10, larger or smaller than the uncertainty found in A10?

## C. Motion on an incline: effect of friction

C1. Return to your data from steps A8 and A9. Now measure the acceleration of the cart when it is moving down the ramp, by finding the best-fit straight line to the velocity graph. Record your results in the table below.

| DOWN | Measurement 1 <br> $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | Measurement 2 <br> $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | Measurement 3 <br> $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | Measurement 4 <br> $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | Measurement 5 <br> $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Acceleration |  |  |  |  |  |

C2. Enter these five measurements in Excel, and repeat step A10. Enter the results here:

$$
\begin{array}{cc}
\theta=3^{\circ}, \text { average acceleration } a= & \text { meters } / \text { second }^{2} \\
\theta=3^{\circ}, \text { uncertainty in acceleration } \delta a=+/- & \text { meters } / \text { second }^{2}
\end{array}
$$

C3. Compare the measured average acceleration of the cart up the ramp (from A10) with the measured average acceleration of the cart down the ramp (from C2). Is the discrepancy between these two accelerations larger than the uncertainty in either?
(In tests done in 2016, the answer was found to be "yes". This discrepancy was attributed to the effect of friction, which will be studied later in the course).

C4. Make a careful sketch of the graph of position vs. time for one of the runs you took in A8 or A9. You will need to refer to this sketch in steps D1 and E5 below.

## D. Bouncing ball

Download and launch the Capstone file "Bouncing ball.cap" .


Mount the motion sensor on a vertical stand such that the sensor faces the ground, as in the picture.

Drop the basketball from below the sensor and record its motion in Capstone until it has bounced at least twice. If it moves sideways out of the beam of the motion sensor before it has bounced twice, start over.

D1: Compare the graphs obtained with your sketch from C4. Note down any similarities.

D2: Now imagine that you set this apparatus in a bus that is moving right with a suitable constant velocity that you decide. How would the motion of the ball appear to a person observing it from outside the bus? Draw the motion, as observed by this person, on the graph below. Assume that the floor of the bus is at $y=0$, and the ball is released at $x=0$ at height $y=h$.


D3. Assuming the bus moves with constant velocity, what is the acceleration of the ball?

D4. Will a physics student sitting inside the bus observe the same acceleration of the ball as a student standing outside the bus?

D5. A quarter of a second after the ball is released, what would be the velocity of the ball, as observed by a person sitting inside the bus?

D6. In the same situation, what would be the velocity of the ball, as observed by a person outside the bus, if the bus moves at $5 \mathrm{~m} / \mathrm{s}$ to the right?

## E: Relative motion

Return to the motion-on-an-incline setup. Connect the second motion sensor (yellow plug in Digital Channel 3, black plug in Digital Channel 4).

Mount the steel vane on the end of the track.
Put the second motion sensor on the cart and point it at the steel vane.
The first motion sensor should still be on the stand. Point it at the steel vane, and make sure it's at least 15 cm away from the steel vane.

See the picture below for the complete set up.


Launch the file 1Dmotion_relative.cap from Pasco/Physics103 on the desktop.
E1. Start taking data; give the cart a push so it rolls up the ramp, then back down.
E2. The graphs from the first motion sensor show the motion of the steel vane, as seen from which frame of reference?

E3. The graphs from the second motion sensor show the motion of the steel vane, as seen from which frame of reference?

E4. What was the frame of reference for the graphs you got in the "motion on an incline" part?

E5. Compare the graphs you got from the second motion sensor with the ones you got in the "motion on an incline" part for the same angle. Note both similarities and differences.

## F: Follow-up Questions

F1. Does the acceleration of the cart depend on its mass? (refer to your answer to B2)

F2. Does the acceleration of the cart depend on whether the cart is moving up the track or down the ramp? (refer to your answer to C3)

## MC-4 ACCELERATION IN FREE FALL

## What do we do in this lab?

In this lab you experimentally determine the acceleration of an object in free fall using several different methods, compare the results to find which has the least uncertainty, and estimate that uncertainty. This lab has three parts, plus a followup question:
(A "set of raw data" for this lab consists of a table of position measurements)
A. From a set of raw data, compute the average acceleration of the object first by fitting a straight line to the velocity, and then by computing the mean of the acceleration. Which has less uncertainty?
B. Take two more sets of raw data, and use the better method from part A to compute the average acceleration and uncertainty for each new data set. Is the discrepancy between the data sets larger, smaller, or about the same as the uncertainty in each data set?
C. Compare your results with other lab groups.
D. Follow-up question.

Safety concerns: none

## You need to know:

discrepancy an unexplained difference
free fall the state of moving freely under the influence of gravity alone mean arithmetic mean, also called 'average'
significant figure reliably-known digit
standard deviation sophisticated estimate of the uncertainty of a measurement
note 1: in this lab you won't be asked about the mathematical details of the calculation of mean or standard deviation;
note 2: this lab uses an Excel spreadsheet, which uses formulas. You won't be asked to enter any Excel formulas, although you will be asked to look at several Excel formulas, in order to determine how the value in a given cell is calculated (i.e. from which cells the data is coming).

## Pre-lab Questions

1. Make a sketch of the position of a freely-falling object vs. time.
2. The position of an object that falls freely from rest is proportional to what power of time $t$ ?

$$
\begin{array}{llll}
t^{0} & t^{1} & t^{2} & t^{3}
\end{array}
$$

3. Make a sketch of the speed of a freely-falling object vs. time.
4. The speed of an object that falls freely from rest is proportional to what power of time $t$ ?
5. Make a sketch of the acceleration of a freely-falling object vs. time.
6. The acceleration of an object that falls freely from rest is proportional to what power of time $t$ ?
```
t t tl}\mp@subsup{t}{}{0}\mp@subsup{t}{}{2}\quad\mp@subsup{t}{}{3
```

7. To three significant figures, what is the acceleration of an object in free fall near the surface of the Earth?

## A. Comparison of different methods of analyzing the same data

This lab uses a "spark timer" that makes sparks every $1 / 60$ of a second. Each spark leaves a black dot on the paper tape pulled through the spark gap.
Cut about 1.2 m of paper tape and attach the tape to the brass cylinder, as shown:


Fold paper several times, and insert into gap.
It takes two people to take a good set of raw data.

Have one person press and hold the button on the back of the spark timer that causes the sparks to occur.


Tighten screw.


Set the spark timer at least 1.2 m above the floor, in order that at least enough data points are taken.

Have another person hold the top of the paper tape directly above the spark gap. If this is done well, the tape does not drag against the spark gap as the cylinder falls.
These two people have to coordinate their efforts:
--tape holder says ‘OK, ready’
--button presser presses button, causing sparks to start;
--tape holder lets go of tape;
--cylinder hits floor
--button-presser releases button, causing sparks to stop.


The result of this should be a tape that looks Foam Cushion like this:


Inspect the tape to make sure the spark marks are visible and there are no extraneous marks. Call the big dot (many dots on top of each other) "dot 0 ".
Tape the tape to the table, so that it doesn't move.
Tape a meter stick to the table, as close as possible to the dots, so that 0 mm on the meter stick lines up with dot 1 .

Make sure that you have at least 25 visible dots (in addition to the big dot). It is helpful to circle and number them. Make sure you haven't missed any.
A1. Have one person measure the positions of the dots while a second person records them in column C of the Excel spreadsheet M4.xlsx.
The spreadsheet then uses the values of position in column C to calculate the velocity (column E) and the acceleration (column G). It takes two measurements of position to compute one measurement of velocity, and two measurements of velocity to compute one measurement of acceleration, as illustrated in the picture below.


A2. The average velocity of an object in free fall is speed $=$ (change in position)/(change in time), or $v=\Delta x / \Delta t$. Click on one of the velocity measurements in column E , and look at the formula that appears in the formula bar. What is the value of $\Delta t$ used in the calculation of velocity?
A3. The velocity given in cell E5 is calculated from the position of dots 1 and 2. What relation does the time given in cell D5 have to the times of dots 1 and 2?
A4. The value of the acceleration given in cell G8 is calculated from the positions of three dots: which ones?

The Excel functions AVERAGE and STDEV can be used to find the mean and standard deviation of a set of measurements.

A5. Copy here the mean (from G31) and the standard deviation (from G32) of your measurements of acceleration.
mean acceleration $\qquad$ $\mathrm{m} / \mathrm{s}^{2}$
standard deviation $\qquad$ $\mathrm{m} / \mathrm{s}^{2}$

The spreadsheet uses the Excel function LINEST to calculate the least-squares fit to a straight line, and to return the standard errors in the slope and y-intercept. These are recorded in cells I12, I13, J12, and J13.

A6. Copy here the acceleration and the standard error of acceleration from Excel's fit to velocity, but make sure you include the appropriate number of significant figures:
acceleration (from fit to velocity): $\qquad$ $\mathrm{m} / \mathrm{s}^{2}$ standard error $\qquad$ $\mathrm{m} / \mathrm{s}^{2}$

MC-4-4

A7. Which method of calculating the acceleration seems more trustworthy, fitting a straight line to the velocity, or calculating the mean of the acceleration?

We expect you will be convinced that fitting a straight line to the velocity is more trustworthy than finding the mean of the acceleration. If you're not convinced, call over your TA.

A8. For a normal distribution, approximately $70 \%$ of the measured values lie within one standard deviation of the mean. What percentage of the acceleration data (G5:G27) lie within one standard deviation (G32) of the mean (G31)?

A9. Suppose the paper tape dragged against the spark gap as the cylinder fell. What effect would this have on the acceleration that you measured?

## B. Discrepancies and calculated uncertainties

Take 2 more sets of raw data. If possible, have a different person fulfill each role each time (if there are three lab partners, then by the end of this part each person will have held the tape, pressed the button, and measured the positions of the marks).
B1. For each set of raw data, repeat step A6:
Raw data set \#2:
acceleration (from fit to velocity): $\qquad$ $\mathrm{m} / \mathrm{s}^{2}$ standard error $\qquad$ $\mathrm{m} / \mathrm{s}^{2}$

Raw data set \#3:
acceleration (from fit to velocity): $\qquad$ $\mathrm{m} / \mathrm{s}^{2}$ standard error $\qquad$ $\mathrm{m} / \mathrm{s}^{2}$

B2. Is the discrepancy between the values of the acceleration calculated from raw data sets \#1, \#2, and \#3 larger than, smaller than, or about the same as, the largest of the three uncertainties?
B3. Figure out how to use Excel to calculate the mean and standard deviation of your determination of the acceleration due to gravity. You will need to tell Excel the six numbers listed in steps A6 and B1. This will be your lab group's final determination of the acceleration due to gravity in this lab.
acceleration (final determination): $\qquad$ $\mathrm{m} / \mathrm{s}^{2}$ uncertainty in acceleration: $\qquad$ $\mathrm{m} / \mathrm{s}^{2}$

## C. Comparison with other lab groups

Tell your TA the results to question B3. Your TA will make a table on the board of the acceleration, and uncertainty of the acceleration, of an object in free fall, as
determined by each lab group. When all lab groups have reported, the TA will compute the acceleration of an object in free fall, and the uncertainty therein, as calculated by your section. Meanwhile, carry out the following instructions.

C1. Compare your result with the accepted value of the acceleration of a freefalling object from prelab question 7. Is the discrepancy between your result and the accepted value larger than, smaller than, or about the same as, the uncertainty in your result?

C2. Compare your results with at least one other lab group, and if possible, with the calculation of the section. Discuss with your lab partners what you would do to reduce the uncertainty in your determination of the acceleration, if you had an extra time to spend on this lab, such as "remeasure the position of dot 14 in raw data set \#3" or "try to find a better way to keep paper tape from dragging against spark gap". Summarize your discussion here:

## D. Follow-up question

D1. In our lab store-room, we have plastic cylinders which are the same size and shape as the brass cylinders you used in this lab, but which weigh much less than the brass cylinders. The force exerted by "air resistance", also known as air drag, on the two cylinders is the same (because they are the same size and shape), but the force exerted by gravity on the brass cylinder is much larger than the force exerted by gravity on the plastic cylinder (because the brass cylinder is much heavier).

Taking into account the effect of air resistance, if you drop a plastic cylinder and a brass cylinder from the same height, which will hit the ground first? Be prepared to justify your answer either with equations, or by analogy with something you have observed in real life, outside the physics lab.

## M-3 FORCE BALANCE

## What do we do in this lab?

In this lab you experimentally verify the vector addition of forces. This lab has two parts:
A. Verify the balance of horizontal forces acting on an object.
B. Verify the balance of vertical forces acting on an object.

Why are we doing this lab?
One of the most important areas of physics is the study of objects that are in static equilibrium-objects that are motionless, such as buildings and bridges. An object that is observed to be motionless has an acceleration of zero. Therefore, according to Newton's second law $\sum \overrightarrow{\boldsymbol{F}}=m \overrightarrow{\boldsymbol{a}}$, the forces on the object must sum to zero-they must balance. In this lab you will gain experience representing forces as arrows, and practice drawing free-body diagrams of objects in equilibrium.
Safety concerns: none

## You need to know:

equilibrium a condition in which all forces sum to zero
force a push or a pull
free-body diagram a diagram, showing all forces acting on a single object, in which the forces are represented as arrows
tension force transmitted by a rope, string, etc.
vector a quantity which has magnitude and direction, and can therefore be pictorially represented by an arrow
vector components the sides of a right-angled triangle that has the vector as hypotenuse
weight force due to gravity

## M-3 FORCE BALANCE

## Pre-lab Questions

1. The free-body diagram below shows three forces acting on an object:


The length of each line is proportional to the magnitude of the force it represents. Is this object in equilibrium?
2. Here is another free-body diagram, also showing three forces, all of which have the same magnitude, acting on an object. The angle between each pair of forces is $120^{\circ}$ :


Is this object in equilibrium?
3. Draw a free-body diagram of the following forces acting on a point mass at the origin (make sure the length of each arrow you draw is proportional to the magnitude of the force):
A force of 2.0 N , directed $45^{\circ}$ degrees counterclockwise as measured from the positive x axis;
A force of 2.0 N , directed $135^{\circ}$ degrees counterclockwise as measured from the positive x axis;

4. What is its magnitude and direction of the resultant force?
5. What is the magnitude and direction of the force that would be necessary to balance these two forces?

## A. Horizontal force balance

This lab uses a "force table" (see sketch below). There is a pin sticking up out of the center of the table, and a ring which surrounds the pin. Strings are attached to the ring, and run over pulleys to hangers, which have mass 50 g ; there are three hangars shown. The pulleys can be moved around the edge of the table. By positioning the pulleys properly, and adding the right amount of weights to the hangers, the ring can be centered on the pin without touching the pin. The ring is then in equilibrium, and the only forces on it in the horizontal direction are due to the tensions in the strings. The tension in each string is equal to the total weight hanging from it.


A1. Set the three pulleys at $0^{\circ}, 120^{\circ}$, and $240^{\circ}$, and add 200 g to each hangar. Is the ring in equilibrium (ie centered on the pin but not touching it)? (compare prelab question 2)

A2. Set one pulley at $45^{\circ}$, and another at $135^{\circ}$, and add 100 g to each of these two hangars. At what angle do you have to position the third pulley, and how much weight do you have to add to the third string, to bring the ring into equilibrium? (compare prelab question 5)

A3. Set up the weights and pulleys so that the following two forces act on the ring:

| Force | x-component | y-component |
| :--- | :--- | :--- |
| $\vec{F}_{1}$ | 0.838 N | 2.30 N |
| $\vec{F}_{2}$ | 1.26 N | -1.50 N |

Draw these two forces to scale on the diagram below (it may help to use a ruler, and let $1 \mathrm{~N}=1 \mathrm{~cm}$ ). Let the x axis be in the direction of $0^{\circ}$.


A4. What are the components of the third force that would be needed to bring the ring into equilibrium?

A5. Draw this force on the diagram above.
A6. Position the third pulley in location specified by your answer to A4 \& A5, and add the weight specified by your answer to A4 and A5. Does this bring the ring into equilibrium?

A7. Estimate the angular uncertainty associated with this apparatus as follows. Keep the weight hanging from the third pulley constant. How far can you move it clockwise and counterclockwise, and still keep the ring in equilibrium?
furthest clockwise: $\qquad$ degrees furthest counterclockwise: $\qquad$ degrees
A8. Estimate the uncertainty in magnitude associated with this apparatus as follows. Move the pulley to the middle of the range you found in A7. What are the largest and smallest weights you can hang from the string, and still keep the ring in equilibrium?
largest weight: $\qquad$ Newtons smallest weight: $\qquad$ Newtons

## B. Vertical Force balance



Set up your apparatus as shown in the sketch above.
Note that the hanger with mass $M$ should be approximately at the center of the set-up. This can be made easy by using strings of equal length on each side and having the pulleys fixed at the same height above the ground.

B1. Derive an equation for the tension $T$ in the string, in terms of $m, M$, and/or the acceleration due to gravity, $g$.

B2. Derive an equation for the angle $\theta$, in terms of $m, M$, and/or $g$.

B3. The apparatus include three hangers. The mass of each hanger is 50 g . Hang an empty hanger in the middle, so that $M=50 \mathrm{~g}$. Hang empty hangers on each side

## M-3 FORCE BALANCE

(so that $m=50 \mathrm{~g}$ also) and measure the angle $\theta$ indicated above. Also estimate the uncertainty in your measurement of $\theta$ :
measured value of $\theta=$ $\qquad$ degrees uncertainty in $\theta=$ $\qquad$ degrees

B4. From B1, what value of $\theta$ does your equation predict for these values of $m$ and $M$ ?
predicted value of $\theta=$ $\qquad$ degrees

B4. Do four more trials, one each for $m=100 \mathrm{~g}, 150 \mathrm{~g}, 200 \mathrm{~g}$, and 250 g .

B5. Tabulate your observations as follows:

| Trial <br> Number | Mass $m$ <br> on each <br> hanger <br> $(\mathrm{g})$ | Predicted <br> tension $T$ <br> in string <br> (N) | Predicted <br> value of $\theta$ <br> (degrees) | Measured <br> value of $\theta$ <br> (degrees) | Uncertainty in <br> measured value of $\theta$ <br> (degrees) |
| :--- | :---: | :---: | :--- | :--- | :--- |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  |  |  |  |  |
| 4 |  |  |  |  |  |
| 5 |  |  |  |  |  |

B5. What happens to the angle as the tension in the string increases?

B6. What should be the tension in the string to achieve $\theta=0$ ?

B7. Is this practical?

B8. In which trials (if any) is the discrepancy between the measured value of $\theta$ and the predicted value of $\theta$ larger than the uncertainty in the measured value of $\theta$ ?
$\qquad$

This sheet is the lab document your TA will use to "assess" your lab. It is to be turned in at the end of lab. This is a beta experiment and so only participation is required.
However you are expected to work through the lab and suggest improvements. The write up is definitely a work in progress.

Experimental set-up: The figure below sketches the basic layout for the lab. The set-up is a modified Atwood's machine. Both the small pulley and picket fence/photogate are precalibrated to measure the speed of the cord and that of the picket fence. The Pasco collision cart will run along the Pasco Track. There should be an S hook will allow the cord to connect the cart to the 50 gram hanger as it passes over the smart pulley. The cable from the smart pulley photogate should go to the first Pasco digital input and the cable from the picket fence photogate should go to the second Pasco digital input. It is important that the horizontal run of the cord is parallel to the Pasco track. Wooden block should have the wood face down.


Goal: The main goal of this lab is to gain experience with friction in three representations: constant, kinetic and static.

## Constant (or nearly constant friction):

The cart and the pulley are claimed to be nearly frictionless but this assertion needs to be tested. A simple light push of the free cart along the track should convince you that there is rolling friction and friction in the bearings. We will assume that these friction forces are constant.

In the first experiment you will determine the magnitude of the friction force. Your instructor will briefly show you the procedures for collecting and analyzing your data.

## Phase I:

In this experiment you will only examine the slope of the velocity versus time curve from the smart pulley output. The cart and the wooden block will have the same acceleration.

1) Measure the mass (and uncertainties) of the cart, the wooden block and the 50 gram hanger and enter the data on the next page.

Group member names
$\mathrm{M}_{\text {cart }}=$ $\qquad$ ; $\mathrm{M}_{\text {hanger }}=$ $\qquad$ ; $\mathrm{M}_{\text {block }}=$ $\qquad$
2) If only the 50 gram hanger is used to accelerate the cart, determine an equation in terms of the above masses and little $g$ which can be used to predict the acceleration of the cart if the hanger is allowed to drop down (if there were no friction).
3) Enter the predicted acceleration: $a_{1}=$ $\qquad$
4) Start the data recording and let the hanger drop freely for a distance of about 60 cm . You should observe that the velocity increases linearly. Stop the data acquisition.
5) Using the Pasco analysis software determine the acceleration of the cart.
6) Enter the calculated acceleration: $a_{1 c}=$
7) Do the two values for the acceleration agree? If things go well then you should observe a difference of about $25 \%$.
8) A reasonable assertion is that the friction force here, $f$, is a constant.
9) Determine a modified algebraic expression for the acceleration which includes a constant friction force.
10) Obtain a numerical value for the friction force, $f=$
11) Using the friction force in step 10 and, additionally, the assumption that you will increase the mass of the hanger by 200 grams, calculate a new predicted acceleration $\mathrm{a}_{2 \mathrm{c}}=$ $\qquad$
12) Now repeat steps 4 and 5 above and determine the acceleration experimentally $\mathrm{a}_{2}=$ $\qquad$
13) Do these two values for the acceleration agree (i.e., the difference should be less than $1 \%$ )? $\qquad$ . If not the repeat the experiment or ask your instructor for assistance.
$\qquad$
14) Now you will see if you can achieve consistency between your measurement of acceleration between the pulley photogate and the picket fence photogate. Move the cart so that the picket fence is just off to the side of the photogate (see sketch below). The photogate should be aligned so that the top picket (with a 1.0 cm repeat) is the one that is being recorded. Start the data recording, release the cart and stop things once the picket fence has moved completely past the photogate.

15) Determine the acceleration of the cart $1^{\text {st }}$ using pulley data and then the picket fence data. Enter the results: $a_{\mathrm{SP}}=$ $\qquad$ ; $\mathrm{a}_{\mathrm{PF}}=$ $\qquad$ . If things go well these measurements should be within $1 \%$ of each other.
16) Obtain an algebraic expression for the tension, $T$, in the cord using the masses and friction force.
17) Is the tension equal to $\left(\mathrm{m}_{\text {hanger }}+\mathrm{m}_{200 \mathrm{gm}}\right) \mathrm{g}$ ? What value would the acceleration of the cart have to be in order that the cord tension $T=\left(\mathrm{m}_{\text {hanger }}+\mathrm{m}_{200 \mathrm{gm}}\right) \mathrm{g}$ ? How could this happen?
$\qquad$
$\qquad$
$\qquad$
18) Compare the expected tension with that from the measured acceleration:
$T_{\text {calc }}=$ $\qquad$ ; $T_{\text {acc }}=$ $\qquad$
19) What was the magnitude of the static friction force on the wooden block?
$f_{\mathrm{WB}}=$ $\qquad$

Now you will move to the regime in which the block will slide relative to the cart when the system is accelerating. The cart and the wooden block will have different accelerations (if there is slippage).

1) Now increase the mass added to the hanger to 500 grams.
2) If there were no slippage between the block and the cart what would be the expected acceleration: $\mathrm{a}_{\text {IIe }}=$
3) Repeat step 14 of the previous section.
4) Determine the accelerations of the cart and the wooden block. Enter these results:
$\mathrm{a}_{\text {SP-II }}=$ $\qquad$ ; $\mathrm{a}_{\mathrm{PF}-\mathrm{II}}=$ $\qquad$ . Do these measurements agree will each other?
5) Now obtain two algebraic expressions, one from the block's acceleration and one from the cart's acceleration which will allow you to determine the coefficient of kinetic friction at the interface between the block and the cart.
6) Does the kinetic friction force by the cart on the block point towards the pulley? Y/N Does the kinetic friction force by the block on the cart point towards the pulley? Y/N Explain briefly in terms of the appropriate Newton's Law:
$\qquad$
$\qquad$
7) Enter the numerical experimental values for the coefficient of kinetic friction:
$\mu_{\mathrm{k} \text { from WB acceleration }}=$ $\qquad$ ; $\mu_{\mathrm{k} \text { from Cart acceleration }}=$
8) Does Newton's $3{ }^{\text {rd }}$ Law follow from this result? Explain.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Devise a procedure that will allow you to determine the coefficient of static friction. Write in the step in the space below.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
What is you value for the coefficient of static friction: ; $\mu_{\mathrm{S}}=$
How does it compare with the coefficient of kinetic friction?

## Final questions:

1) Did this lab help you better understand the role of friction in mechanical processes?
$\qquad$
$\qquad$
$\qquad$
2) Please make suggestions that would improve this lab (note that this write up itself is a work in progress): $\qquad$
$\qquad$
$\qquad$

## MC-7 SIMPLE PENDULUM

## What do we do in this lab?

In this lab you make careful measurements (to five significant figures!) of the period of a simple pendulum. You carry out three experiments:

1. For small oscillations, measure how the period $T$ of the pendulum depends on its length $L$, and compare it to the prediction $T=2 \pi \sqrt{L / g}$, using the accepted value of $g, g=9.803636 \mathrm{~m} / \mathrm{s}^{2}$ (see endnote).
2. For a specific length, measure how the period of the pendulum depends on the amplitude of the oscillations.
3. Use the measured period and length to calculate $g$.

## Why are we doing this lab?

A heavy object hanging from a string, and swinging back and forth, is an excellent example of an oscillating system which (we hope) you have seen already at some point in your life, outside of the physics lab. We are doing this lab in order that you can gain experience applying what you have learned in Physics 103 to a phenomenon with which you are already somewhat familiar.

Safety hazards: none

## You need to know:

amplitude the "size" of an oscillation
period the time it takes an oscillator to complete one oscillation
oscillation movement that repeats over and over

## Pre-lab Questions

1. Describe two occasions outside of Physics 103 on which you have seen a heavy object hanging from a string (or rope, or wire, or cable...) and swinging back and forth.
2. Which will complete its swing first, an object hanging from a short string, or the same object hanging from a long swing?
3. Draw free body diagrams of a pendulum
a. when it is at the top of its swing, and thus momentarily motionless;
b. at the middle of the swing, when it is directly under its support;
c. when it reaches the other end of its swing, and is again motionless.

## A. Dependence of Period on Length

A1. Turn the Pasco interface on, and connect the photogate to Digital Channel 1. Verify that the LED on the photogate lights up when the beam is interrupted.

A2. The pendulum bob is suspended by a "bifilar" (= "two-string") suspension in order that the ball swings in the same plane as the protractor.

The length $L$ of the pendulum is the distance from the bottom of the support to the center of ball, as shown in the diagram.

When setting $L$, make sure the ball remains centered under the protractor (this requires the two strings be the same length, as also shown in the diagram, lest the ball move to one side).


It is convenient to use a meter stick to set $L$; figure out how you can use it in such a way that you can set $L$ and center the ball at the same time.

Set $L$ to be 0.500 m , estimate $\delta L$, the uncertainty in $L$, and record it here.

$$
L=0.500 \mathrm{~m} \quad \delta L=\ldots m
$$

A3. Predict the period of pendulum using $T=2 \pi \sqrt{L / g}$ and $g=9.803636 \mathrm{~m} / \mathrm{s}^{2}$, and record it here to 5 significant figures:

$$
T_{\text {pred }}=\ldots \mathrm{s}
$$

A4. Adjust the photogate so that the center of the ball just interrupts the beam when the ball is hanging motionless. Rotating the photogate may help.

A5. Double-click PASCO/Physics103/Experiment M-7 (a table should appear).
A6. Set the pendulum swinging with an amplitude of $5^{\circ}$. Start data acquisition by clicking the Record icon . Let the bob swing for 17 periods, then click "Stop". Click on the statistics icon $\bar{\Sigma}$ and view the statistics the computer calculates. Record the measured mean period (call it $T_{\text {meas }}$ ) and the standard deviation ("Std. Dev"; call it $\sigma$ ) :

$$
T_{\text {meas }}=\ldots \mathrm{s}
$$

$\qquad$ s

A7. The standard deviation $\sigma$ characterizes the spread of a set of measurements. Let us equate it to the uncertainty $\delta T_{\text {meas }}$ in our measurement of the pendulum period. Record $\delta L, T_{\text {pred, }}$, and $T_{\text {meas }}$ and $\delta T_{\text {meas }}$ in the appropriate row of the table.

| $L(\mathrm{~m})$ | $\delta L(\mathrm{~m})$ | Predicted Period <br> $T_{\text {pred }}(\mathrm{s})$ | Measured Period <br> $T_{\text {meas }}(\mathrm{s})$ | Uncertainty <br> $\delta T_{\text {meas }}(\mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.700 |  |  |  |  |
| 0.600 |  |  |  |  |
| 0.500 |  |  |  |  |
| 0.400 |  |  |  |  |
| 0.300 |  |  |  |  |

A8. Repeat steps A6-A7 for $L=0.700 \mathrm{~m}, 0.600 \mathrm{~m}, 0.400 \mathrm{~m}$, and 0.300 m , and complete the table above. If you measure $T$ to be twice as big as it should be, you'll need either to more carefully align the photogate as per A4, or else use slightly larger-amplitude oscillations.

A9. Circle the $L$ for which the percentage uncertainty in $T_{\text {meas }}$ was the largest. What was that uncertainty? maximum percentage uncertainty in $T_{\text {meas }}$ :___ $\%$

A10. Is the discrepancy between prediction and measurement larger than the uncertainty in measurement?

The next few steps investigate what effect the uncertainty in setting the pendulum length has on the period.

A11. Here are some calculations done before the semester:

| $L(\mathrm{~m})$ | $\delta L(\mathrm{~m})$ | $2 \pi \sqrt{\frac{L}{g}}(\mathrm{~s})$ | $2 \pi \sqrt{\frac{L+\Delta L}{g}}(\mathrm{~s})$ | Uncertainty in $T$ <br> due to <br> uncertainty in $L$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.500 | 0.002 | 1.4190 | 1.4218 | 0.0028 s |
|  |  |  |  |  |

Repeat this calculation using your data from $L=0.500 \mathrm{~m}$ and enter in the bottom row of the table above.

Uncertainty in $T$ due to uncertainty in $L$ : $\qquad$ s

A12. Rewrite A11 as a percentage uncertainty in $T$ :
A13. What is the ratio (result of A12)/(result of A9)?
A14. Is the discrepancy between $T_{\text {pred }}$ and $T_{\text {meas }}$ larger than the uncertainty in $T$ due to uncertainty in $L$ ?

In tests done before the semester, the answers to A10 and A14 were "yes" and "no", respectively.

While the precision of the measurement of $T$ was found to be very high (about $0.01 \%$, or about 1 part in $10^{4}$ ), the accuracy of the experiment as a whole was considerably less (about $0.2 \%$, or 2 parts in $10^{3}$ ), due to the uncertainty in measuring $L$. Similar situations occur often in scientific research.

## B. Dependence of Period on Amplitude

The formula for the period given above, $T=2 \pi \sqrt{L / g}$, is based on the smallamplitude approximation, which is true when $\sin \theta \ll 1$. If one does not make the small-amplitude approximation, the formula for the period is:

$$
T=2 \pi \sqrt{L / g}\left(1+\frac{1}{4} \sin ^{2} \frac{\theta}{2}+\frac{9}{64} \sin ^{4} \frac{\theta}{2}+\cdots\right)
$$

Let's call the quantity in the parenthesis $f(\theta)$, so that $T=2 \pi \sqrt{L / g} f(\theta)$.
The function $f(\theta)$ is plotted, and tabulated for various values of $\theta$, below.


| $\theta$ (degrees) | $f(\theta)$ |
| :--- | :--- |
| 0 | 1 |
| 5 | 1.0005 |
| 10 | 1.0019 |
| 20 | 1.0077 |
| 30 | 1.0174 |
| 40 | 1.0313 |
| 50 | 1.0498 |

B1. Reset the pendulum length $L$ to 0.500 m . Record its uncertainty $\delta L$ :

$$
\delta L=\ldots \mathrm{m}
$$

B2. Compute $2 \pi \sqrt{L / g}$ using $g=9.803636 \mathrm{~m} / \mathrm{s}^{2}$ and $L=0.500 \mathrm{~m}$, and record it here:

$$
2 \pi \sqrt{L / g}=
$$ s

In the next step we will be measuring the period for different amplitudes, so it is important to measure the amplitude carefully. Here is how:

- close one eye and position the other eye so that when the pendulum reaches the top of its swing, the two strings of the bifilar support appear one behind the other;
- set the pendulum oscillating with a little larger amplitude than you want to measure, and focus your eye on the protractor reading of the desired amplitude;
- when you see the pendulum amplitude equal the desired amplitude, start recording data.
Choose one person to watch the pendulum and another person to press the button starting the recording of data (and rotate, so that everyone in the lab group gets some experience measuring the amplitude carefully).

B3. Complete the table below by measuring the period $T_{\text {meas }}$, and its uncertainty $\Delta T_{\text {meas }}$, for five values of $\theta$. For each value of $\theta$, estimate $\delta \theta$ (which depends on how well you were able to carry out the instructions given before this step), and copy $f(\theta)$ from the table above. Also compute the predicted period, taking into account the finite amplitude of the pendulum's oscillation, using the formula given above, $T_{p r e d}=2 \pi \sqrt{L / g} f(\theta)$.

| $\theta$ (degrees) | $\delta \theta$ (degrees) | $f(\theta)$ | $T_{\text {pred }}(\mathrm{s})$ | $T_{\text {meas }}(\mathrm{s})$ | $\delta T_{\text {meas }}(\mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 5 |  |  |  |  |  |
| 10 |  |  |  |  |  |
| 20 |  |  |  |  |  |
| 30 |  |  |  |  |  |
| 40 |  |  |  |  |  |

B4. By what percentage did the pendulum's measured period increase as the amplitude was increased from $5^{\circ}$ to $40^{\circ}$ ?

Percentage increase of measured period: $\qquad$ \%

B5. Is the discrepancy between $T_{\text {pred }}$ and $T_{\text {meas }}$ larger than the uncertainty $\Delta T_{\text {meas }}$ ?
The next few steps estimate the effect of the uncertainty in measuring $\theta$.
B6. Here are the results of some measurements done before the semester:

| $\theta$ <br> (degrees) | $\delta \theta$ <br> (degrees) | $2 \pi \sqrt{\frac{L}{g}} f(\theta)$ <br> (s) | $2 \pi \sqrt{\frac{L}{g}} f(\theta+\Delta \theta)$ <br> (s) | Uncertainty in $T$ <br> due to <br> uncertainty in $\theta$ |
| :---: | :---: | :---: | :---: | :---: |
| 5 | 1 | 1.4196 | 1.4197 | 0.0001 s |
| 40 | 2 | 1.4634 | 1.4674 | 0.0040 s |

Use these results to estimate your uncertainty in measuring $T$ due to $\delta \theta$. If your $\delta \theta$ differs from the pre-semester $\delta \theta$ given above, multiply the number in the last column by the appropriate ratio, and enter it here (example: if your estimate of $\delta \theta$ for $\theta=5^{\circ}$ was $2^{\circ}$, it would cause an uncertainty of 0.0002 s in $T$ ).

Uncertainty in $T$ due to $\delta \theta: \quad \theta=5^{\circ}: \delta T=$ $\qquad$ s $\quad \theta=40^{\circ}: \delta T=$ $\qquad$ s

B8. For $\theta=5^{\circ}$, which gives the larger contribution to your uncertainty in $T$ : the uncertainty in $L$ (from A11) or the uncertainty in $\theta$ ?

B9. For $\theta=40^{\circ}$ ?
In tests done before the semester, $\delta \theta$ was the limit to the accuracy of the experiment as a whole at $\theta=40^{\circ}$, but $\delta L$ was the limit at $\theta=5^{\circ}$.

## C. Measurement of $g$

Next (and last) we will use the data you took in part B for $5^{\circ}$ oscillations to measure a value for $g$, rather than using the accepted value.

C1. Transcribe $\delta L$ from step B1.

$$
L=0.500 \mathrm{~m}, \quad \delta L=\square \mathrm{m}
$$

C 2 . Transcribe $T_{\text {meas }}$ and $\delta T_{\text {meas }}$ for $\theta=5^{\circ}$ from the table in step B3:

$$
T_{\text {meas }}=\ldots \mathrm{s}, \quad \delta T_{\text {meas }}=\square \mathrm{s}
$$

C3. Discuss with your lab partners how to use the $L, T_{\text {meas }}$, and $f(\theta)$ to calculate $g$, and record the formula you come up with here, as well as the result of the calculation:

$$
g=\quad=\ldots \mathrm{m} / \mathrm{s}^{2}
$$

(formula goes here)

C 4 . Calculate the uncertainty $\delta g$ in $g$ using the following formula:

$$
\delta g=g \sqrt{(\delta L / L)^{2}+\left(2 \delta T_{\text {meas }} / T_{\text {meas }}\right)^{2}} \quad=\quad \mathrm{m} / \mathrm{s}^{2}
$$

C5. Is the discrepancy between your value of $g$ and the accepted value of $g$ larger than, smaller than, or about the same as, the uncertainty in your value of $g$ ?

Endnote: The UW Geophysics Department determined $g=9.803636 \pm 0.000001$ $\mathrm{m} / \mathrm{s}^{2}$ for the fourth floor of Sterling Hall; we extrapolate from Sterling to Chamberlin.

## M-5 CONSERVATION OF LINEAR MOMENTUM AND PROJECTILE MOTION

What do we do in this lab?
In this lab you use the laws of conservation of energy and conservation of momentum to solve a problem that is difficult to solve using Newton's $2^{\text {nd }}$ law.
You will use a spring gun that fires a ball, which embeds itself in a suspended metal cylinder, called a catcher. After the ball embeds itself in the catcher, the two together swing upward like a pendulum. The catcher is suspended from a protractor in such a way that the maximum angle of upward swing can be recorded. By measuring this maximum angle, you can work out the initial velocity of the ball. Knowing the initial velocity of the ball, you can calculate how far it would travel if the catcher were moved out of the way.


Why are we doing this lab?
To consolidate understanding of the laws of conservation of momentum and energy, which are of great practical importance.
Safety concerns: Don't fire the spring gun if someone is in front of it. Don't walk in front of someone who is about to fire the spring gun. When cocking the spring gun, be careful you don't slip.

## You need to know:

Gravitational potential energy change in potential energy due to change in height, $\Delta P E=m g \Delta y$
Kinetic energy energy associated with moving objects; $\mathrm{KE}=1 / 2 \mathrm{mv}^{2}$
Momentum the product of mass and velocity; $\overrightarrow{\boldsymbol{p}}=m \overrightarrow{\boldsymbol{v}}$

## Pre-lab Questions

Consider a ball of mass $m_{\text {ball }}$ that is fired horizontally from a spring gun a height $h$ above the floor, and lands on the floor a horizontal distance $R$ away.

1. Write an equation for time $t$ the ball takes to hit the floor, as a function of $h$ and the acceleration due to gravity $g$ (drawing a diagram may help).

$$
t=
$$

$\qquad$
2. Suppose you know the initial speed of the ball is $\mathrm{v}_{\mathrm{i}}$. Write an equation for the "range" $R$ in terms of $h, g$ and $\mathrm{v}_{\mathrm{i}}$.
3. Does the acceleration due to gravity have any effect on the horizontal component of the velocity?
4. The next 6 questions concern how to measure $v_{i}$ using the laws of conservation of energy and momentum. Consider the picture on the previous page: the ball fired by the spring gun is caught by the catcher. Ball and catcher then swing together, like a pendulum. The initial angle of the ball-and-catcher is $90^{\circ}$ (as shown in the picture). Let the final angle be called $\theta$. Let the length of the pendulum be called $L$. Write an expression for $\Delta h$, the height by which the ball-and-catcher rise, as a function of $L$ and $\theta$.

$$
\Delta h=
$$

$\qquad$
5. Let the mass of the catcher be $m_{\text {catcher }}$. What is the increase in gravitational potential energy of the ball and catcher, in terms of $m_{\text {catcher }}, m_{\text {ball }}, L, \theta$, and $g$ ?
$\Delta \mathrm{PE}=$ $\qquad$
6. What is the kinetic energy of the ball and catcher, just after impact, in terms of $m_{\text {catcher }}, m_{\text {ball }}, L, \theta$, and $g$ ? Assume that energy is conserved as the ball-catcher system swings up.

$$
\mathrm{KE}=
$$

$\qquad$
7. What is the momentum of the ball-catcher system, just after impact, in terms of $m_{\text {catcher }}, m_{\text {ball }}, L, \theta$, and $g$ ?

$$
\mathrm{p}_{\mathrm{f}}=
$$

$\qquad$
8. What is the momentum of the ball, just before impact, in terms of $m_{\text {catcher }}$, $m_{\text {ball }}, L, \theta$, and $g$ ? Assume that momentum is conserved during the impact.

$$
\mathrm{p}_{\mathrm{i}}=
$$

$\qquad$
9. What is $\mathrm{v}_{\mathrm{i}}$ in terms of $m_{\text {catcher }}, m_{\text {ball }}, L, \theta$, and $g$ ?

$$
\mathrm{v}_{\mathrm{i}}=
$$

$\qquad$

## A. Measurement of initial velocity of ball

A1. Set the apparatus near the edge of the table.
A2. Use the plumb bob mounted to the protractor to level the apparatus in the direction of travel of the ball.

A3. Use this plumb bob to level the apparatus in the direction perpendicular to the travel of the ball.

A4. Adjust the length of the four strings that hold the catcher so that it is directly lined up with the path of the ball.

A5. Fire the ball into the catcher a few times. Estimate how high the ball-catcher system rises. Put a piece of tape on the protractor at the approximate maximum angle. Focus your eyes on the tape and fire the ball again (this makes it easier to read the maximum angle).
A6. The picture on the first page was taken with the camera directly in front of the right-hand pair of strings holding the catcher. One appears to be in front of the other-you can't tell them apart. On the other hand, the left hand pair of strings do not appear to be lined up, and can be easily distinguished, because the camera is not directly in front of them (this phenomenon is called parallax).
If you want to read the maximum angle with the greatest possible accuracy, you should close one eye, and make sure the other is directly in front of one of the pairs of strings - which pair, the left-hand pair or the right-hand pair?

Practice reading the maximum angle to which the ball-catcher system rises.
A7. Measure the length $L$ of the ball-catcher pendulum. (Hint: It isn't the length of the strings)
$L=$ $\qquad$ m
A8. Measure the mass $m_{\text {catcher }}$ of the catcher.

$$
m_{\text {catche } r}=\ldots \mathrm{kg}
$$

A9. Measure the mass $m_{b a l l}$ of the ball.

$$
m_{b a l l}=\ldots \mathrm{kg}
$$

A10. Fire the ball ten times. Record the maximum angle each time.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |

A11. Compute (by using Excel) the mean and standard deviation of the maximum angle, based on these 10 measurements.

Mean $\theta$ : $\qquad$ degrees Standard deviation: $\qquad$ degrees

A12. Use the result of pre-lab question 9 to calculate the initial velocity of the ball:

$$
v_{\mathrm{i}}=
$$

$\qquad$ m/s

## B. Prediction and measurement of Range

B1. Remove the catcher, and hang the strings up out of the path of the ball.
B2. Measure the height $h$ above the floor at which the ball is fired.
$\qquad$
$h=$ m

B3. Use the result of pre-lab question 2 to predict the range of the ball.

$$
R=
$$

$\qquad$ m

B4. Use the plumb bob to determine which spot on the floor is directly below the point at which the ball leaves the gun.

B5. Use the results of B3 and B4 to predict where on the floor the ball will land. Tape a piece of paper to the floor centered on this spot.
B6. Fire the gun. Does the ball land on the piece of paper?

B7. If the ball does not land on the piece of paper, recheck your calculations. If no errors are found, call over your TA to consult.

## C Follow-up Questions

In this lab, you assumed that momentum was conserved during the collision (prelab question 8) and that energy was conserved as the ball and catcher swung after the collision (pre-lab question 6). Let us see whether energy was conserved during the collision. Use your values from A8, A9, and A12.
C 1 . Compute the momentum of the ball before the collision:

$$
\mathrm{p}_{\mathrm{i}}=\ldots \mathrm{kg} \mathrm{~m} / \mathrm{s}
$$

C 2 . Compute the kinetic energy of the ball before the collision:

$$
\mathrm{KE}_{\mathrm{b}}=\ldots \mathrm{kg} \mathrm{~m}^{2} / \mathrm{s}^{2}
$$

C3. Assuming momentum is conserved, compute the velocity of the ball-catcher system just after the collision:

$$
\mathrm{v}_{\mathrm{f}}=\ldots \mathrm{m} / \mathrm{s}
$$

C 4 . Use your result from C3 to compute the kinetic energy of the ball-catcher system just after the collision:

$$
\mathrm{KE}_{\mathrm{bc}}=\ldots \mathrm{kg} \mathrm{~m}^{2} / \mathrm{s}^{2}
$$

C5. What is the ratio of the kinetic energy immediately after and immediately before the collision:

$$
\mathrm{KE}_{\mathrm{bc}} / \mathrm{KE}_{\mathrm{b}}=
$$

$\qquad$

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## Physics 103 <br> Lab MC-11: Colliding Carts

What do we do in this lab?
In this lab you measure the energy and momenta of two carts before and after they collide, and determine whether energy and momentum were conserved during the collision. You are asked to go to some trouble to properly account for the effects of friction.

The carts are constructed in such a way that two types of collision can be arranged: one in which they carts stick together after the collision, and one in which they move apart.


Why are we doing this lab?
The two most important conservation laws in mechanics are the law of conservation of energy and the law of conservation of momentum. There are many problems that are much easier to solve by application of one or both of these conservation laws than by any other way. However, application of these conservation laws in real-world situations is often complicated by the presence of friction.

Safety concerns: none

## You need to know:

elastic adjective meaning "energy conserving"
inelastic adjective meaning "energy non-conserving"
kinetic energy energy associated with moving objects; K.E. $=1 / 2 \mathrm{mv}^{2}$
momentum the product of mass and velocity; $\overrightarrow{\boldsymbol{p}}=m \overrightarrow{\boldsymbol{v}}$

## Pre-lab questions:

1. Which adjective means "a process during which energy is conserved"?
2. You throw a Frisbee to a passing skateboarder, and it is caught. Which of the following is true, concerning the "collision" between Frisbee and skateboarder:
a. kinetic energy is conserved, momentum is not conserved;
b. kinetic energy is not conserved, momentum is conserved;
c. both kinetic energy and momentum are conserved.
3. During pre-game warm-ups two basketball players take shots at the same moment. The basketballs collide in midair, and go off in different directions. During the collision between the basketballs:
a. kinetic energy is conserved, momentum is not conserved;
b. kinetic energy is not conserved, momentum is conserved;
c. both kinetic energy and momentum are conserved.
4. On an icy winter's afternoon, a car going 10 miles an hour goes into a skid and crashes into a car that is initially motionless. The two cars slide together an additional 20 feet after impact, and then come to a stop. What is wrong with the following chain of reasoning:
a. before the collision, the first car has momentum and the second car does not;
b. the total momentum of the two cars before the collision is equal to the momentum of the first car, and is therefore not zero;
c. after the collision, the total momentum of the two cars is zero, because they are both motionless;
d. therefore, momentum is not conserved during the collision.
5. Suppose you are sitting in a car moving 55 miles/hour. Which would be worse:
a. to collide head-on with an identical car travelling 55 miles/hour in the opposite direction;
b. to collide with a brick wall.
6. Cart 1 (mass $\mathrm{m}_{1}$, velocity $\mathrm{v}_{1}$ ) crashes into cart 2 (mass $\mathrm{m}_{2}$ ), which is initially motionless, and they stick together. Write an equation for the final velocity $\mathrm{v}_{\mathrm{f}}$ of the two carts.
7. Write down the equation for the position of an object undergoing one-dimensional motion with constant acceleration.

## A. Making observations with your eyes (and ears)

A1. Set the motion sensors about 15 cm from the ends of the track, as in this picture:


A2. Connect the left-hand sensor to channels 1 and 2 of the interface (yellow plug in channel 1), and the right-hand sensor to channels 3 and 4 (yellow plug in channel 3).
A3. Use the level, and the screw foot under one end of the track, to level the track. How do you know if the track is level? Devise an experiment that shows the track is level, carry it out to verify the track is level, and write down here what you did:

A4. Without taking data, make happen a gentle collision of type A (see picture on first page of lab write-up). Watch (and listen!) to what happens. This collision is inelastic.
A5. Now make happen a gentle collision of type B. Watch and listen (if there is a 'crash' sound, the collision wasn't gentle enough). This collision is elastic.

Now we will inspect some raw data taken by the computer using the setup shown above.

## B. Using the computer to take data

Below are a series of screenshots showing data taken from a single collision.


## Position as a function of time

B1. To the nearest 0.05 second, when did this collision occur? $\qquad$ s

B2. Did the carts stick together after the collision?
yes
no
B3. Cart 2 after the collision was observed (by eye) to move in the same direction as Cart 1 had been moving before the collision, yet the slope of the position vs. time graph for Cart 2 after the collision is negative, while the slope of the position vs. time graph for Cart 1 before the collision is positive. How can this be?

B4. What was the speed of Cart 1 , including units, just before the collision? $\qquad$
B5. What was the speed of Cart 2, including units, just after the collision? $\qquad$


## Velocity as a function of time

B6. There is a relation between the velocity of an object, and the graph of its position vs. time. What is that relation?

B7. At 0.8 seconds, Cart 1 was at rest on one position on the track, and at 2.0 seconds it was at rest at a different position on the track. What is the relation between the distance between those two positions, and the graph of the velocity of Cart 1 as a function of time?


B8. What is the relation between the velocity of an object, and its momentum?
B9. We tell you in class that momentum is conserved during all collisions, but it seems plain that Cart 2 has less momentum coming out of the collision than Cart 1 had going into it. What causes this apparent loss of momentum?

B10. Keeping in mind that friction is continuously taking away the momentum of moving carts, would you agree, or disagree, that the above graph supports the contention that momentum was conserved during this particular collision? Cite specific features of the above graph to support your opinion.


## Energy as a function of time

B11. What kind of energy is plotted on the graph above, kinetic or potential?
B12. What is the relation between kinetic energy and velocity?
B13. Again keeping in mind that friction is continuously taking away the velocity of moving carts, would you agree, or not, that the above graph supports the contention that energy was conserved during this particular collision? (Again, cite specific features of the graph to support your opinion.)

B14. What kind of collision is this, elastic or inelastic?
Below is another set of screenshots, taken from a different collision.


## Position as a function of time

MC-11 Colliding Carts


## Velocity as a function of time



Momentum as a function of time


## Energy as a function of time

B15. To within the nearest 0.2 seconds, at what time did the collision occur?
B16. Did the carts stick together after the collision, or not?
B17. Keeping in mind friction-is momentum conserved? Cite specific graph features.

B18. Keeping in mind friction-is energy conserved? Cite specific graph features.

B19. What kind of collision is this, elastic or inelastic?

## C. Quantitative measurements of a collision in which the carts stick together

C1. On the computer desktop, go to PASCO/Physics103 and launch MC11.cap . The screen should show the graphs presented in steps B1-B14.

C2. Get rid of fits:
--click on a data point to make that dataset active;
--click the Curve-fit button $\%$ and uncheck "Linear".
C3. Delete annotations:
--mouse over border of annotation;
--right click;
--click "Delete Text Annotation Tool".
C4. Put the North/North cart in the middle of the track, and put the steel vane on the end closest to the right-hand motion sensor.
C5. Put the North/South cart at the left-hand end of the track, orient it so that an inelastic collision will occur, and put the steel vane on the end closest to the left-hand motion sensor.

C6. Tile the motion sensors so that they point about $5^{\circ}$ above the horizontal.
C7. Take data; give the North/South cart a moderate push towards the initially motionless North/North cart. The "push" should be brief, so that the North/South cart coasts for a while before the collision.

C8. If necessary, click the Run Select button $\Delta$ and deselect unwanted runs.
C9. Inspect the data to make sure there are no sudden jumps, isolated data points, measurements that contradict what your eyes tell you, or other spurious data. If there are, realign the motion sensor and take data for another collision. Repeat this process until you trust that the motion sensor is returning trustworthy position measurements.

When you have successfully completed step C8, take a few moments to discuss with your lab partners the graphs on of momentum and energy on your screen. Is momentum conserved? Is energy conserved?

Once you have done this, move on to the next part.

## D. Estimation of uncertainties in velocity

Recall that the motion sensor measures the position of an object. From this, it calculates the velocity. The computer, into which we programmed the mass of the carts, then calculates the momentum and energy, as indeed you could do if you had the same information. In this section we will use only the motion sensor's measurement of position, and use the computer's graphs of velocity, momentum and energy only as a check that our calculations have not gone astray.
D1. Measure the mass of the North/North cart; estimate the uncertainty in your determination of mass; record these below, and in the table on the next page.
mass of North/North Cart: $\qquad$ kg uncertainty: $\qquad$ kg
D2. Measure the mass of the North/South cart; estimate the uncertainty in your determination of mass; record these below, and in the table on the next page.
mass of North/South Cart: $\qquad$ kg uncertainty: $\qquad$ kg

Now we will determine the velocities of both carts before and after the collision, as well as the attendant uncertainties.

D3. Go to the "position as a function of time" window, and click the autoscale button
in order to see all the position measurements.

D4. Mouse over the vertical axis until the cursor looks like this: | , then click the |
| :---: | mouse and drag to adjust the vertical scale. Similarly with the horizontal axis.

Move the mouse until the cursor looks like a little hand, and click and drag to pan the plot.
Use these two actions to make the data taken when the carts are moving fill the whole screen.
D5. Click on a data point from the North/South cart to make the dataset active.
D6. Click the Highlight Selected Data button $: \therefore$ and click and drag the highlight box that appears until it includes about 10-20 position measurements of the North/South cart (i.e. the cart that was moving before the collision) immediately before the collision. They should all lie close to a straight line at some non-zero angle to the horizontal.
D7. Click on the Curve-fit button $\%$ and check "Linear". Verify that the fit that appears on the screen goes through all the points you selected.
D8. The coefficient $m$ given in the fit box is the velocity of the cart before the collision; also given is the uncertainty of $m$. Estimate the slope (rise/run) of the graph and compare to $m$; if $m$ is wrong (which it sometimes is), "nudge" the boundaries of the highlighted area; this should make $m$ jump to the correct value. Record $m$ and its uncertainty in the table below.

D9. Write down the time of the first highlighted point, and the last highlighted point:
Time of first highlighted point: $\qquad$ s time of last highlighted point: $\qquad$ s

D10. Compute the time of measurement $\mathbf{1}$ as the average of these (also record in table):
Time of measurement 1 : $\qquad$ s

D11. Repeat steps D6-D10 to measure the velocity of the North/South cart after the collision, and record this and the time of measurement 2 in the table.

D12. Since the North/North and North/South carts were stuck together after the collision, assume their velocities were the same. Copy the appropriate data into the appropriate cells.

D13. Calculate the percentage uncertainty in each measurement, and record this in the table.

D14. Estimate the actual time of collision, and enter this in the table.

|  | Before Collision |  |  |  | Time of <br> Collision: | After Collision <br> Time of measurement 1: |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mass <br> (kg) | Velocity <br> $(\mathrm{m} / \mathrm{s})$ | Uncertainty <br> in velocity <br> $(\mathrm{m} / \mathrm{s})$ | Percentage <br> uncertainty <br> $\%$ |  | Velocity <br> $(\mathrm{m} / \mathrm{s})$ | Uncertainty <br> in velocity <br> $(\mathrm{m} / \mathrm{s})$ | Percentage <br> uncertainty <br> $\%$ |
| North- <br> North <br> Cart |  | 0 | 0 | 0 |  |  |  |  |
| North- <br> South <br> Cart |  |  |  |  |  |  |  |  |

## E. Accounting for friction

We will now account for friction.
E1. Go to the "Velocity as a function of time" window, and make a linear fit to the velocity of the moving cart immediately before the collision. The coefficient $m$ is the deceleration of the North/South cart due to friction (make sure $m$ is right!): record it here:
deceleration of North/South cart before collision due to friction: $\qquad$ $\mathrm{m} / \mathrm{s}^{2}$

E2. How much time elapsed between the time of measurement $\mathbf{1}$ of velocity in D10, and the time of collision (D14)?

Elapsed time: $\qquad$ s

E3. Multiply E1 and E2 to get the velocity decrement of the North/South cart due to friction, after the velocity measurement and before the collision itself:
velocity decrement: $\qquad$ $\mathrm{m} / \mathrm{s}$

E4. Subtract this from your result from D8. This is your best estimate of the velocity of the North/South cart in the instant before the collision:

Velocity of the North/South cart at the instant before the collision: $\qquad$ $\mathrm{m} / \mathrm{s}$

E5. Now make a linear fit to the velocity of the North/South cart immediately after the collision. Use as many data points as you can.
deceleration of North/South cart after collision due to friction: $\qquad$ $\mathrm{m} / \mathrm{s}^{2}$

Since the North/South and North/North carts were moving together after the collision, we will use the same value of the deceleration for both. Repeat steps E2-E4 to get the best estimate of the velocity of the two carts at the instant after the collision: time between time of collision (D14) and time of measurement 2 (D11): $\qquad$ s

Velocity decrement of both carts after collision, due to friction: $\qquad$ m/s

Velocity of both carts at the instant after the collision: $\qquad$ m/s

E6. Enter the velocities from E4 and E5 in the table below. Copy the uncertainties from the previous table. Recalculate the percentage uncertainty and enter in table.

|  |  | The Instant Before Collision |  |  |  | The Instant After Collision |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mass <br> $(\mathrm{kg})$ | Velocity <br> $(\mathrm{m} / \mathrm{s})$ | Uncertainty <br> in velocity <br> $(\mathrm{m} / \mathrm{s})$ | Percentage <br> uncertainty <br> $\%$ |  | Velocity <br> $(\mathrm{m} / \mathrm{s})$ | Uncertainty <br> in velocity <br> $(\mathrm{m} / \mathrm{s})$ | Percentage <br> uncertainty <br> $\%$ |
| North- <br> North Cart |  | 0 | 0 | 0 |  |  |  |  |
| North- <br> South Cart |  |  |  |  |  |  |  |  |

We have now accounted for the effect of friction.

## F. Is energy conserved?

Kinetic energy is $E=1 / 2 m v^{2}$.
Calculate the percentage uncertainty in energy for each cart as:
<percentage uncertainty in energy> $=2$ <percentage uncertainy in velocity> (this is true as long as the measurement of mass is much more accurate than the measurement of velocity).

Work backwards from the percentage uncertainty in energy to get the uncertainty in energy.

F1. Use the above strategies to complete the first two rows in the table below.

|  | The Instant Before Collision |  |  |  | The Instant After Collision |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Energy <br> (J) | Uncertainty <br> in Energy <br> (J) | Percentage <br> uncertainty <br> $\%$ |  | Energy <br> (J) | Uncertainty <br> in Energy <br> (J) | Percentage <br> uncertainty <br> $\%$ |
| North- <br> North <br> Cart | 0 | 0 | 0 |  |  |  |  |
| North- <br> South <br> Cart |  |  |  |  |  |  |  |
| TOTAL |  |  |  |  |  |  |  |

F2. Calculate the total energy before and after the collision by adding the energy of each cart.

F3. Compare your results for the energies of each cart to what is shown on the "Energy as a function of time" graph: are your results consistent?

F4. Calculate the uncertainty in total energy as follows:
--let $\Delta E_{1}=$ uncertainty in energy of Cart 1 ;
-- let $\Delta E_{2}=$ uncertainty in energy of Cart 2;
-- let $\Delta E=$ uncertainty in total energy;
then: $\Delta E=\sqrt{\Delta E_{1}^{2}+\Delta E_{2}^{2}}$
After filling in the final row to the table, answer the following question.

F5. Is the discrepancy between the total energy the instant before the collision and the total energy the instant after the collision considerably larger than the uncertainties in either?

Yes
No

If you answered yes, that is equivalent to saying "according to our measurements, energy was not conserved in the collision".

F6. What fraction of the initial energy was lost in the collision?

## MC-9 ANGULAR ACCELERATION OF A FLYWHEEL

What do we do in this lab?
You conduct three experiments using a flywheel (a rotating disk). Each involves measuring the angular speed of the flywheel using a slotted disk and photogate.

In the first experiment you give the flywheel a good spin, and then (carefully) drop a ring onto it, so that the ring spins with the flywheel. What is the relation between the initial and final angular speeds of the flywheel?

In the second experiment (see picture) you wind a string around a spool connected to the flywheel, run it over a pulley, and tie it to a weight. As the weight falls, the flywheel spins faster and faster. What is the relation between the mass of the hanging weight and the angular acceleration of the flywheel?
In the third experiment (no string or weight) you give the flywheel a good spin and measure its angular speed as frictional forces cause it
 to slow down.

## Why are we doing this lab?

To gain practical experience with the two most important principles governing rotational motion, Conservation of Angular Momentum ( $\mathrm{L}_{\text {init }}=\mathrm{L}_{\text {final }}$ ) and Newton's $2^{\text {nd }}$ Law written in terms of angular quantities ( $\Sigma \tau=I \alpha$ ).

Safety concerns: none

## You need to know:

angular acceleration $\alpha$; rate of increase of angular speed, $\alpha=\Delta \omega / \Delta t$.
angular momentum $L$; product of moment of inertia and angular speed, $L=I \omega$.
lever arm $r \sin \theta$; perpendicular distance between axis of rotation and line of action of force (see diagram).

moment of inertia $I$; resistance to angular acceleration; an
object's moment of inertia is $I=\sum m r^{2}$, where $m$ is the mass
of each component of the object, and $r$ is the distance between that component and the axis of rotation ( $\sum$ means "sum").
torque $\tau$; that which causes angular acceleration $(\alpha=\tau / I)$. The torque exerted on an object by a force $F$ is the product of $F$ and the lever arm $\tau=r F \sin \theta$.

## Pre-lab questions

1. Have you ever seen (either in person, or in a video) a figure skater spin really fast by pulling her arms in after she has started a spin at a moderate speed?
2. What law of physics governs this phenomenon?
(questions 3-10 pertain to the first diagram on the previous page)
3. Draw a free-body diagram of the flywheel, as seen from above. Assume there is no friction.
4. Draw a free-body diagram of the weight, as seen from the side, showing all the forces on it.
5. Identify the force that appears in both diagrams.
6. What is the relationship between the torque on the flywheel and the tension in the string?
7. What is the relationship between the angular acceleration of the flywheel and the linear acceleration of the weight?
8. What is the formula for the moment of inertia of a solid disk?
9. Based on the answers to pre-lab questions 3-8, write an equation that predicts the angular acceleration of the flywheel, given various masses and radiuses (you'll also need to use $g$ in the equation).
10. Now suppose that there is friction between the flywheel and the stand that holds the flywheel, so that if you give the flywheel a spin, it will eventually come to a stop. As the weight drops, will the angular acceleration of the flywheel be larger than, or smaller than, that predicted by the equation in prelab question 9 ?

## A. Conservation of Angular Momentum

## How to read calipers



This caliper reads 0.03


This caliper reads 9.13
(arrows show which lines are used to read the last digit)
A1. Measure the mass $\mathrm{M}_{\mathrm{F}}$ and the radius $\mathrm{R}_{\mathrm{F}}$ of the flywheel.

$$
M_{F}=\ldots \mathrm{kg} \quad \mathrm{R}_{\mathrm{F}}=
$$

$\qquad$ m
A2. Calculate the moment of inertia $I_{F}$ of the flywheel.

$$
\mathrm{I}_{\mathrm{f}}=\ldots \mathrm{kg} \mathrm{~m}^{2}
$$

A3. Measure the mass $M_{R}$, the inner radius $R_{\text {inner }}$, and the outer radius $R_{\text {outer }}$ of the ring.
$\mathrm{M}_{\mathrm{R}}=$ $\qquad$ $\mathrm{kg} \quad \mathrm{R}_{\text {inner }}=$ $\qquad$ $\mathrm{m} \quad \mathrm{R}_{\text {outer }}=$ $\qquad$ m
A4. Discuss with your lab partners the best formula to use to calculate $I_{\text {ring }}$, the moment of inertia of the ring, and record the formula and the numerical result.

$$
\mathrm{I}_{\text {ring }}=\quad=\ldots \mathrm{kg} \mathrm{~m}^{2}
$$

## MC-9 ANGULAR ACCELERATION OF A FLYWHEEL

A5. Set up the photogate as shown in the picture on the first page, connect it to input channel 1 of the interface, and run DataStudio setup file MC9.ds .

A6. Start taking data. Give the flywheel a moderate spin. Carefully drop the ring onto the flywheel, neither from too high (so that it crashes) nor too low (so that your fingers rub against it after it rests on the flywheel). Verify that the angular speed of the flywheel suddenly decreases when the ring is dropped on it.

A7. What is the angular speed of the flywheel measured to be at the last moment before the ring is dropped on it?

$$
\omega_{\text {before }}=
$$

$\qquad$ radians/sec
A8. Use conservation of angular momentum to write an equation that predicts $\omega_{\text {after }}$, the angular speed of the flywheel immediately after the ring has landed on it, in terms of $\omega_{\text {before }}$ and any of the quantities you measured in A1 and A3. Record the equation and the predicted numerical value of $\omega_{\text {after }}$.
predicted $\omega_{\text {after }}=$
$=$ $\qquad$ radians/sec
A9. What was the angular speed of the flywheel measured to be immediately after the ring has landed on it?
measured $\omega_{\text {after }}=$ $\qquad$ radians/sec
A10. What is the discrepancy, in percent, between your predicted and measured values for $\omega_{\text {after }}$ ?

## B. Angular Acceleration Due to Torque

B1. Measure the radius of the largest tier of the spool using the calipers.
Radius of spool:

$$
\mathrm{r}_{\text {spool }}=
$$

$\qquad$ m

B2. Add 200 g to the hanger, and find the mass of the hanger + weight (should be about 250 g ).

$$
\mathrm{m}_{\text {hanging }}=\ldots \mathrm{kg}
$$

B3. Use the measurements from B1 and B2, together with your result from A2, and the equation from pre-lab question 9 , to predict the angular acceleration of the flywheel.

$$
\text { predicted } \alpha=
$$

$\qquad$ $\mathrm{rad} / \mathrm{sec}^{2}$
B4. Cut a piece of string about 1 m long. Tie one end to the largest-radius segment of the spool (the one on the bottom). Tie the other end to the hanger. Make sure the string is short enough that the hangar doesn't hit the floor.
B5. Wind the string around the spool, bring the hanger as high as possible, and let it go. Allow the hanger to go through at least 3 cycles of fall-rise. Use the
photogate and DataStudio to measure the angular velocity of the flywheel for 3 falls and 3 rises.
B6. Compute the angular acceleration for each of the 3 falls and rises by fitting a straight line to the angular velocity.

|  | $\alpha_{\text {fall }}$ <br> $\left(\mathrm{rad} / \mathrm{sec}^{2}\right)$ |
| :--- | :---: |
| first |  |
| second |  |
| third |  |


|  | $\alpha_{\text {rise }}$ <br> $\left(\mathrm{rad} / \mathrm{sec}^{2}\right)$ |
| :--- | :---: |
| first |  |
| second |  |
| third |  |

B7. Compute the mean $\alpha_{\text {fall }}$ and the mean $\alpha_{\text {rise }}$. Estimate the uncertainty in each. mean $\alpha_{\text {fall }}$ : $\qquad$ $\mathrm{rad} / \mathrm{sec}^{2} \quad$ uncertainty in mean $\alpha_{\text {fall }}$ : $\qquad$ $\mathrm{rad} / \mathrm{sec}^{2}$
mean $\alpha_{\text {rise }}$ $\qquad$ $\mathrm{rad} / \mathrm{sec}^{2}$ uncertainty in mean $\alpha_{\text {rise }}$ :___ $\mathrm{rad} / \mathrm{sec}^{2}$ B8. Declare that the "measured" angular acceleration is the mean of the absolute values of $\alpha_{\text {fall }}$ and $\alpha_{\text {rise }}$. What is it?

Measured $\alpha$ : $\qquad$ $\mathrm{rad} / \mathrm{sec}^{2}$

B9. Is the discrepancy between the measured $\alpha$ from B8 and the predicted $\alpha$ from B3 larger than, smaller than, or about the same as, the discrepancy between the absolute value of $\alpha_{\text {fall }}$ and the absolute value of $\alpha_{\text {rise }}$ ?

B10. Compare the absolute value of $\alpha_{\text {fall }}$ with the absolute value of $\alpha_{\text {rise }}$. Is the discrepancy between the mean $\alpha_{\text {fall }}$ and the mean $\alpha_{\text {rise }}$ larger than the uncertainties in both, smaller than the uncertainties in both, or larger than the uncertainty in one, and smaller than the uncertainty in the other?

B11. In data taken before the beginning of class, it was found that $\alpha_{\text {rise }}$ was larger than $\alpha_{\text {fall }}$, and the discrepancy between the two was larger than the uncertainties in both. Is there any effect that we have neglected that could make $\alpha_{\text {rise }}$ larger than it would otherwise be, and $\alpha_{\text {fall }}$ smaller than it would otherwise be; and if sowhat is this effect?

## C. Friction

C1. Untie the string from the flywheel.

C2. Give the flywheel a good spin and take data with DataStudio. Allow the flywheel to spin freely for at least one minute while graphing the angular velocity. C3. Fit a straight line to the angular velocity, thus measuring the angular acceleration. What is it?

$$
\alpha_{\text {friction }}=\ldots \mathrm{rad} / \mathrm{sec}^{2}
$$

C4. Compute the torque necessary to cause this angular acceleration.

$$
\tau_{\text {friction }}=
$$ Nm

C5. Is this torque sufficient to explain the discrepancy between $\alpha_{\text {fall }}$ and $\alpha_{\text {rise }}$ noted in B11?

C6. In Part B, which is "right", the prediction B3, or the measurement B8? Choose one person on your team to argue that the prediction is "right", and another person to argue that the measurement is "right". Summarize the chief arguments used by each person.

In real life, no one knows which is right, the prediction or the measurement. Physicists who are good at making predictions are called "theorists" and physicists who are good at making measurements are called "experimentalists".

## M-13 Buoyancy

What do we do in this lab?
You measure the buoyant force on four small objects, and use it to calculate the density of each object.

Why are we doing this lab?
To make the buoyant force seem less mysterious by making quantitative measurements of it.

Safety concerns: one of the small objects is made of lead. Please wash your hands after handling it.

## You need to know:

Buoyant force upward force exerted on an object totally or partially submerged in a fluid; equal in magnitude to the weight of the fluid displaced by the object.

## Pre-lab questions



1. Draw a free-body diagram of a ball hanging from a string (left-hand picture).
2. Draw a free-body diagram of a ball hanging from a string and immersed in a fluid (right-hand picture above).
3. Is the magnitude of the tension force the same in the two diagrams?
4. Assuming the ball is completely submerged, does the buoyant force depend on the density of the ball?
5. If the ball is lifted out of the water, by how much will the scale reading change?

## How to calibrate the scale

Make sure both pans are empty. Observe the pointer (lower left) - it should be directly in front of the " 0 " mark. If it isn't, adjust the calibration knob (lower right).


Make sure the scale is calibrated before you make each measurement.

## A. Measurement of Buoyant Force

A1. Fill the plastic beaker with about 700 ml of water. Place the beaker on the left-hand pan of the scale. Place slotted weights on the right-hand pan until the scale nearly balances.

A2. Carefully adjust the sliding weights on the scale until the scale exactly balances. Record the weight of beaker+water here:
weight of beaker plus water: N
A3. Tie the aluminum ball (it's the shiniest of the four balls) to the stand, as shown. Submerge the ball in the water. Adjust the sliding weight until the scale exactly balances again. By how much did the scale reading increase?
increase in scale reading: $\qquad$ kg

A4. Estimate the uncertainty in your measurement in A3.
uncertainty in increase in scale reading: $\qquad$ kg

A5. Use your measurements in A3 and A4 to determine the buoyant force on the ball, and its uncertainty.

Buoyant force: $\qquad$ N uncertainty in buoyant force: $\qquad$ N

## B. Determination of Volume

B1. The density of water is $1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}$. Use your result from A5 to calculate the volume of water displaced by the ball, and its uncertainty. Use $g=9.81 \mathrm{~m} / \mathrm{s}^{2}$.
Volume of displaced water: $\qquad$ $\mathrm{m}^{3}$ uncertainty : $\qquad$ $\mathrm{m}^{3}$
B2. Since the aluminum ball was completely submerged, the volume from B1 is equal to the volume of the aluminum ball. Calculate the percentage uncertainty in this determination of the volume of the aluminum ball (that is, percentage uncertainty $=100 \% *$ uncertainty in volume of ball/volume of ball).
Volume of ball: $\qquad$ $\mathrm{m}^{3}$ percentage uncertainty in volume of ball: $\qquad$ \%

Here's an alternate method to find the volume of the ball without using buoyancy.
B3. Use the calipers to measure the diameter of the ball, and its uncertainty.

Diameter of ball: $\qquad$ m uncertainty in diameter of ball: $\qquad$ m

B4. Calculate the percentage uncertainty in your determination of the diameter of the ball (ie $100 \%$ * uncertainty in diameter of ball/diameter of ball).
percentage uncertainty in diameter of ball: $\qquad$
B5. Calculate the volume of the ball from your measurement of its diameter in B3, assuming the ball is a perfect sphere. Estimate the percentage uncertainty in the volume by multiplying your result from B4 by 3 .
volume of ball: $\qquad$ $\mathrm{m}^{3}$ percentage uncertainty in volume of ball: $\qquad$ \%

B6. Compare B2 and B5. Which is more accurate, the volume calculated from the buoyant force, or the volume calculated from the caliper measurement?

$$
\text { buoyant force } \quad \text { caliper measurement }
$$

## C. Determination of density of aluminum ball

C1. Carefully measure the mass of the aluminum ball using the scale. Note the uncertainty, and compute the percentage uncertainty.
mass of ball: $\qquad$ kg uncertainty in mass of ball: $\qquad$ kg
percentage uncertainty in mass of ball: $\qquad$ \%

C 2 . Which do you know more accurately: the mass of the ball or the volume of the ball?

C3. Compute the density of the ball using your results from B2 and C1. Let the percentage uncertainty in the density be the percentage uncertainty in the least accurately known quantity (mass or volume). Compute the uncertainty in the density of the ball.
density of ball: $\qquad$ $\mathrm{kg} / \mathrm{m}^{3}$ uncertainty in density of ball: $\qquad$ $\mathrm{kg} / \mathrm{m}^{3}$

C4. Look up the accepted value for the density of aluminum and record it here.
$\qquad$ $\mathrm{kg} / \mathrm{m}^{3}$

C5. Is the discrepancy between the accepted value for the density of aluminum and your value for the density of the ball larger than, smaller than, or about the same as, the uncertainty in your determination of the density of the ball?

## D. Determination of density of other balls

D1. Following steps similar to those outlined above, experimentally determine the density of the lead ball (it's the heaviest of the four balls).

$$
\text { density of lead ball:__ } \mathrm{kg} / \mathrm{m}^{3}
$$

uncertainty in density of lead ball: $\qquad$
D2. Look up the accepted value for the density of lead. accepted density of lead: $\qquad$ $\mathrm{kg} / \mathrm{m}^{3}$

D3. Is the discrepancy between the accepted value for the density of lead and your value for the density of the lead ball, larger than, smaller than, or about the same as, the uncertainty in your determination of the density of the lead ball?

D4. Repeat the same steps for the "mystery" ball (ie the remaining ball that is not made out of wood).
density of mystery ball: $\qquad$ $\mathrm{kg} / \mathrm{m}^{3}$
uncertainty in density of mystery ball:___ $\quad \mathrm{kg} / \mathrm{m}^{3}$
D5. Do a web search on "density of elements". Is the density of the mystery ball consistent with the density of any element? (If it is, record the element or elements here).

D6. The wood ball floats on top of the water. Modify the steps above as necessary to determine the density of the wood ball.
density of wood ball:__ $\mathrm{kg} / \mathrm{m}^{3}$
uncertainty in density of wood ball: ___ $\mathrm{kg} / \mathrm{m}^{3}$
D7. Do a web search on "density of wood" and record at least one type of wood that has a density consistent with your result from D6.

D8. Every once in a while, one of the wood balls becomes "waterlogged"-does that mean its density has increased, or decreased, compared to dry wood?

## M-13 BUOYANCY

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## HC-1 IDEAL GAS LAW

## What do we do in this lab?

This lab has two parts. In the first part you measure the change in the pressure of a fixed amount of air as the volume of the air is changed but the temperature is held constant. In the second part you measure the change in the pressure of a fixed amount of air as the temperature is changed but the volume is held constant.

## Why are we doing this lab?

The ideal gas law $P V=N k_{B} T$ ( $P=$ absolute pressure of ideal gas; $V=$ volume of ideal gas; $N=$ number of molecules of ideal gas; $k_{B}=$ Boltzmann's constant $=1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K} ; T=$ absolute temperature of ideal gas) contains a terrific amount of information, and can be used to solve many problems encountered in everyday life that involve gases. We're doing this lab in order to gain experience relating physical situations to the variables in the ideal gas law. This lab also gives you the chance to work with liquid nitrogen, which many students find enjoyable (although the rules for safe handling must be strictly observed).

Safety concerns: The liquid nitrogen, while not toxic, is cold enough ( $\mathrm{T}=-196 \mathrm{C}=-321 \mathrm{~F}$ ) to freeze your skin on contact and cause serious injury.
DO NOT PUT LIQUID NITROGEN INTO A CONTAINER AND THEN SEAL THE CONTAINER. The liquid nitrogen will evaporate and the container will explode. Anyone found doing this will be removed from lab and referred to the Dean of Students for disciplinary action.

The hot water can scald you.

## You need to know:

Absolute pressure force per unit area
Absolute temperature scale temperature scale in which the lowest possible temperature is zero degrees. Example: the Kelvin temperature scale.
Gauge pressure pressure compared to ambient atmospheric pressure. Example: If the air in a room is at Standard Temperature and Pressure (STP), the absolute pressure of the air is about $1 \times 10^{5} \mathrm{~Pa}$ and gauge pressure is 0 Pa .
Ideal gas a substance composed of molecules that don't interact with each other.
Systematic error an error that is not determined by chance but is introduced by an inaccuracy inherent in the system. Example: if your watch is 5 minutes slow, you'll be 5 minutes late to all your appointments.

## Pre-lab Questions

1. Write the ideal gas equation:
2. This equation applies to ideal gases. Which of the following are (approximately) ideal gases? (circle all that are)
The air you are breathing the air in a bicycle tire the water in Lake Mendota the helium in a party balloon the blood in your veins a cube of solid aluminum liquid nitrogen automobile exhaust cheese steam gasoline
3. Rearrange the ideal gas equation so that pressure on the left and everything else on the right.
4. In order for pressure to be a function of volume, and nothing other than volume, what must remain constant?
5. If you make a graph of the equation in question 3, with pressure plotted on the vertical axis, what function of the volume must you plot on the horizontal axis in order to get a straight line?
6. Does this straight line go through the origin?
7. If an ideal gas is sealed in a container, and the temperature of the container is held constant, while the volume is cut in half, what happens to the absolute pressure?
8. Assume N and V are fixed. If you make a graph with absolute pressure of an ideal gas plotted on the vertical axis, and the absolute temperature plotted on the horizontal axis, do you get a straight line?
9. Does this straight line go through the origin?
10. If an ideal gas is sealed in a container, and the volume of the container is held fixed, while the temperature is reduced by a factor of 5 , what happens to the absolute pressure?
11. Is the Celsius temperature scale an absolute temperature scale?

HC-1-2

## A. Change in volume at fixed temperature

A1. Make sure the plunger on the syringe is all the way out; then carefully connect the syringe to the pressure sensor (see diagram): insert the twist-lock connector on the syringe into the socket on the sensor and twist it gently and slightly clockwise. You'll feel a small click when it snaps into place.


A2. Start the Capstone setup file HC1.cap .
A3. Take data. Push in the plunger of the syringe, and watch on the screen as the pressure changes. Does the pressure go up or down?

A4. Does DataStudio tell you the gauge pressure, or the absolute pressure?

A5. Which is needed for the ideal gas law, gauge pressure or absolute pressure?

A6. Can you feel the increase in pressure-does the plunger resist your efforts to push it in more and more as it is pushed further and further in?

A7. If you don't feel any resistance, or if the feel a resistance that fades away after a while, what might that indicate?

A8. The pressure units displayed on the pressure readout, and the volume units printed on the syringe - are either of these suitable for use with the ideal gas law?

A9. If you want to check your measurements against the prediction of the ideal gas law, what else do you need to know besides the volume and pressure of the gas in the syringe?

A10. Measure the temperature of the lab in ${ }^{\circ} \mathrm{C}$, then convert to K : $\qquad$ ${ }^{\circ} \mathrm{C}=$ $\qquad$ K

A11. Fill in the table below, with measurements taken at five different volumes:

| Temeperature in lab: ${ }^{\circ} \mathrm{C}=\mathrm{K}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Volume |  | Absolute Pressure |  |
| Data Point | $(\mathrm{cc})$ | $\left(\mathrm{m}^{3}\right)$ | kPa | Pa |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |

A12. (Recall your answer to prelab question 5). Make a graph of this data in Excel. Show pressure on the vertical axis, and whatever is needed on the horizontal axis so that the data will fall on a straight line if it obeys the ideal gas. Show your TA the graph.

A13. Add a Linear Trendline to the chart (Layout $>$ Trendline $>$ Linear Trendline). Display the trendine equation on the chart (Layout>Trendline>Linear Trendline>More Trendline Options...). What is the slope of the trendline?
slope of trendline $=$ $\qquad$ $\mathrm{Pam}{ }^{3}$

A14. Use your result from A12, along with your measurement of the absolute temperature of the lab, and Boltzmann's constant, to estimate the number of molecules of gas in the syringe.

> number of molecules in syringe:
$\qquad$
A15. Extend the trendline towards the origin (More Trendine Options.../Forecast Backwards: put in a number equal to your smallest value of $1 / \mathrm{V}$ ). Does the line go through the origin?

A16. The volume of air being compressed is slightly larger than the volume shown on the syringe, because there's some air in the tube connecting the syringe to the pressure sensor, as well as some air in the pressure sensor. Write two sentences: one describing the systematic error this introduces into your measurement of the volume, and one stating whether this error could explain the deviation of your trendline from the origin.

## B. Change of temperature at fixed volume

B1. Is the small stainless steel can (volume: 98 cc ) at approximately room temperature? (If not, consult with your TA).
B2. Fill the steam generator with water and start heating it now (you'll be needing boiling water in a few minutes, in part B4). Monitor the water level and make sure enough remains to immerse the can.

B3. Disconnect the syringe from the pressure sensor and connect the can in its place. Always support the tube leading to the can with the edge of a container so that it never hangs unsupported from the pressure sensor (to do this you may need to adjust the height of the pressure sensor on the stand).

Note: in the last part of the following step you will immerse the can in liquid nitrogen. When you are ready to do this, fill your dewar only about $2 / 3$ full of liquid nitrogen, and be prepared for the liquid nitrogen to boil violently for a while when the can is introduced.

B4. Subject the can to the following four conditions in the order given in the table below (that is, do the liquid nitrogen last). Measure the temperature with the Fluke digital thermometer, making sure that the temperature probe is in good contact with the can. Measure the pressure with Capstone. Record the results in the table:

|  | Volume of can: 98 cc |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Condition: | Temperature | Absolute <br> Temperature | Absolute Pressure |  |
|  | $\left({ }^{\circ} \mathrm{C}\right)$ | $(\mathrm{K})$ | $(\mathrm{kPa})$ | $(\mathrm{Pa})$ |
| Immersed in <br> room <br> temperature air |  |  |  |  |
| Immersed in <br> boiling water |  |  |  |  |
| Immersed in <br> ice water |  |  |  |  |
| Immersed in <br> liquid nitrogen |  |  |  |  |

B5. Make a graph of this data using Excel, showing the absolute pressure on the vertical axis, and the absolute temperature on the horizontal axis. Add a trendline. Use the slope of the trendline, the volume of the can, and Boltzmann's constant to estimate the number of molecules in the can.
number of molecules in can:

B6. How does this compare to your answer in A14?

B7. Extend the trendline in the direction of the origin. Does the trendline pass close to the origin?

## HC-3 LATENT HEAT AND SPECIFIC HEAT

## What do we do in this lab?

This lab has two parts. In the first part, you submerge an aluminum cylinder in liquid nitrogen, and measure how much heat is loses as it cools from room temperature (about $22^{\circ} \mathrm{C}$ ) to liquid nitrogen temperature (about $-196^{\circ} \mathrm{C}$ ). In the second part, you submerge the aluminum cylinder in room-temperature water, and measure how much heat it gains as it warms back up to (almost) room temperature. You expect your two measurements to be about the same.
In the first part, what you actually measure is the mass of nitrogen that is boiled away while the cylinder is cooling. Then you can use the latent heat of vaporization of liquid nitrogen $(47.8 \mathrm{cal} / \mathrm{g})$ to calculate the amount of heat the liquid nitrogen must have absorbed, and therefore the amount of heat the aluminum cylinder must have lost.
In the second part, what you actually measure is the decrease in temperature of the water bath in which the aluminum cylinder is submerged. Then you can use the specific heat of water $\left(1.00 \mathrm{cal} / \mathrm{g}^{\circ} \mathrm{C}\right)$ to calculate the amount of heat lost by the water, and therefore the amount of heat the aluminum cylinder must have gained.

Why are we doing this lab?
To apply two important principles that you have learned regarding thermal proceses (latent heat; specific heat).

## Safety hazards:

The liquid nitrogen, while not toxic, is cold enough ( $\mathrm{T}=-196 \mathrm{C}=-321 \mathrm{~F}$ ) to freeze your skin on contact and cause serious injury.

## DO NOT PUT LIQUID NITROGEN INTO A CONTAINER AND THEN

 SEAL THE CONTAINER. The liquid nitrogen will evaporate and the container will explode. Anyone found doing this will be removed from lab and referred to the Dean of Students for disciplinary action.
## You need to know:

Calorimeter water-filled vessel, designed to minimize the transfer of heat between its inside and its outside. Used to measure the heat generated or absorbed during processes taking place inside it.
Dewar container for storing very cold liquids (named after its inventor, James Dewar).
Latent heat of vaporization amount of heat necessary to change a unit mass of a substance from liquid to gas, without raising its temperature.
Specific heat amount of heat necessary to raise the temperature of a unit mass of a substance by one degree.

## Pre-lab Questions

1. Heat capacities, and heats of vaporization, are sometimes given in units of $\mathrm{cal} / \mathrm{g}$ and sometimes in units of $\mathrm{J} / \mathrm{kg}$. What is the conversion factor between these units?
2. Suppose you decide to boil some water on a gas-burning stove. Natural gas provides about $1.2 \times 10^{4} \mathrm{cal} / \mathrm{g}$ of heat. You fill a pot with 1 liter ( $=1000 \mathrm{~g}$ ) of water from the faucet $\left(\mathrm{T}=10^{\circ} \mathrm{C}\right)$. How many grams of natural gas does it take to raise the water to boiling temperature ( $\mathrm{T}=100^{\circ} \mathrm{C}$ )?
3. How many additional grams of natural gas would be needed to boil all the water away?
4. You submerge an aluminum block, initially at room temperature, in liquid nitrogen. The liquid nitrogen begins to boil violently at the moment the block is submerged. The boiling continues for 165 seconds, and then suddenly stops. Why does the boiling suddenly stop?
5. During the above process, a mass $m$ of liquid nitrogen boiled away. The latent heat of vaporization of nitrogen is $L$. How much heat $Q$ was transferred from the block to the nitrogen?
6. The block is now taken out of the liquid nitrogen, and put into a calorimeter containing a mass $M$ of water. The temperature of the water is initially $T_{\text {room }}$. The temperature of the water falls quickly to $T_{f}$. Estimate the amount of heat transferred to the block from the water (let $c_{w}$ stand for the specific heat of water).
7. The estimate from question 6 can be made more accurate. The water is contained in a can of mass $M_{\text {can }}$. The temperature of the can is always exactly the same as the temperature of the water. The can is made of a substance that has a heat capacity $c_{c a n}$. Use this information to make a more accurate estimate of the amount of heat transferred to the block.
8. Let the mass of the block be $m_{b}$, and the temperature at which liquid nitrogen boils be $T_{L N 2}$. Estimate the heat capacity of aluminum.

## A. Cooling the block: latent heat of vaporization of nitrogen

A1. Fill the stainless steel Dewar about $2 / 3$ full of liquid nitrogen.
A2. Tie a string around the aluminum cylinder that is just long enough to allow you to submerge it in the liquid nitrogen, and get it out again.
A3. Put the Dewar, the cylinder, and the string on the left-hand pan of the balance.

A4. Put enough slotted masses on the right-hand pan of the balance so that the sliding weights have to be set at about 180 g in order for the balance to be in balance (the reason for this will become apparent in the next few steps).


A5. (Do not put the cylinder into the liquid nitrogen during this step). Using the stopwatch, measure the mass of the Dewar, cylinder, and string every sixty seconds for the next ten minutes. Try to measure the mass to the nearest 0.1 g . This will require you to delicately move the small sliding mass ten times. (Read step A6 while you are completing this table)

| Time (s) | Mass (g) |
| :--- | :--- |
| 0 |  |
| 60 |  |
| 120 |  |
| 180 |  |
| 240 |  |
| 300 |  |
| 360 |  |
| 420 |  |
| 480 |  |
| 540 |  |
| 600 |  |

A6. (when you are done with A5) Predict what the mass will be at 720 seconds. Set the sliding mass to this value. Observe the balance carefully. When the mass reaches this value, submerge the cylinder in the nitrogen (be prepared for the nitrogen to boil vigorously when you do this). Record the time at which the cylinder was submerged to the nearest second, and record the mass of the Dewar, cylinder, and string at the moment of submersion to the nearest 0.1 g .
time cylinder was submerged: $\qquad$ s
mass of Dewar, cylinder, and string at the moment of submersion: $\qquad$ g

A7. Carefully observe the boiling of the nitrogen, and note the time the boiling stops, and the mass of the Dewar, cylinder, and string at the moment the boiling stops.
time boiling stopped: $\qquad$ s
mass of Dewar, cylinder, and string at the moment boiling stopped: $\qquad$ g

A8. Use the masses you recorded in A6 and A7 to calculate the amount of nitrogen that boiled away, while the vigorous boiling was occurring:
mass of nitrogen that boiled away: $\qquad$ g

A9. Use the times you recorded in A6 and A7 to calculate the duration of the boiling.
duration of boiling: S

A10. Use your results from A5 to estimate how much liquid nitrogen would have boiled away during this time if you hadn't put the aluminum cylinder in the nitrogen.
mass of nitrogen that would have boiled away anyway: g
A11. Use your results from A8 and A10 to calculate the mass of nitrogen that boiled away because you put the aluminum cylinder into the nitrogen
mass of nitrogen that boiled away due to cylinder: $\qquad$

A12. The latent heat of vaporization of nitrogen is accepted to be $47.8 \mathrm{kcal} / \mathrm{kg}$. Use this and your result from A11 to calculate the amount of heat gained by the nitrogen while the cylinder was cooling down:
heat gained by nitrogen: $\qquad$ cal

## B. Warming the block: specific heat of water

B1. Fill the beaker with at least 100 ml of water.
B2. Based on your result from A12, how much heat would have to be given to the block in order to warm it back up to room temperature?
heat needed to re-warm block: $\qquad$ cal
B3. If this amount of heat were removed from 100 g of water, by how many degrees would the water cool down?
estimated temperature decrement of water: $\qquad$ ${ }^{\circ} \mathrm{C}$


The above picture shows the parts of your calorimeter. Item 1. Thermal jacket; item 2: chrome-plated brass can; item 3: thermal isolator; item 4: cover.
B4. Measure the mass $m_{c a n}$ of the chrome-plated brass can.

$$
m_{c a n}=
$$

$\qquad$
B5. (recalling pre-lab question 7) Refine your estimate from B3 by noting that in the actual calorimeter you have, the water will be contained in the chrome-plated brass can. Assume the water and the can always have the same temperature, and that the can is made of pure brass. Pure brass has a specific heat of 0.092 $\mathrm{cal} / \mathrm{g}^{\circ} \mathrm{C}$. Suppose the amount of heat from B2 is removed from the water-can "system". What is the estimated temperature decrement of the system?
estimated temperature decrement of water-can system: $\qquad$ ${ }^{\circ} \mathrm{C}$

B6 Measure the mass of the beaker of water. Pour approximately 100 g of water into the chrome-plated brass can. Measure the mass of the beaker again. How many grams of water were added to the can?
mass of beaker (before): $\qquad$ g
mass of beaker (after): $\qquad$ g
mass of water added to can: g

B7. Insert the temperature probe into the thermometer, turn on the thermometer, and wait one minute for the thermometer temperature to stabilize. If the temperature continues to fluctuate, call over your TA.

B8. Place the thermal isolator in the bottom of the thermal jacket. Place the chrome-plated brass can on top of the thermal isolator. Place the cover on top of the thermal jacket. Insert the thermometer into the small hole in the cover. Wait one minute for the temperature to stabilize.
B9. Reset the watch. Take the aluminum cylinder from out of the liquid nitrogen, and start the watch when you put it into the water. Measure the temperature of the water every 15 seconds until it stops decreasing, and has not decreased for one minute. Fill out the table below:

| time $(\mathrm{s})$ | $\mathrm{T}\left({ }^{\circ} \mathrm{C}\right)$ | time $(\mathrm{s})$ | $\mathrm{T}\left({ }^{\circ} \mathrm{C}\right)$ | time $(\mathrm{s})$ | $\mathrm{T}\left({ }^{\circ} \mathrm{C}\right)$ |  | time $(\mathrm{s})$ | $\mathrm{T}\left({ }^{\circ} \mathrm{C}\right)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 |  | 90 |  | 180 |  |  | 270 |  |
| 15 |  | 105 |  | 195 |  | 285 |  |  |
| 30 |  | 120 |  | 210 |  | 300 |  |  |
| 45 |  | 135 |  | 225 |  | 315 |  |  |
| 60 |  | 150 |  | 240 |  |  | 330 |  |
| 75 |  | 165 |  | 255 |  | 345 |  |  |

B10. Record the initial and minimum temperatures of the water:
initial water temperature: $\qquad$ ${ }^{\circ} \mathrm{C}$ minimum water temperature: $\qquad$ ${ }^{\circ} \mathrm{C}$
B11. What was the measured temperature decrement of the water-can system? measured temperature decrement of water-can system: $\qquad$ ${ }^{\circ} \mathrm{C}$

B12. Estimate the amount of heat that was removed from the water-can system during this process:
amount of heat removed from water-can system: $\qquad$ cal

B13. Compare this to your result from A12. What is the discrepancy, in percent, between your results from A12 and B12?

## SC-1 STANDING WAVES ON A STRING

What do we do in this lab?
In this lab you do two experiments related to standing waves on a string. In the first experiment you measure the frequencies of some of the standing waves that fit on string of a particular length and tension. In the second experiment, you investigate how the frequency of the $\mathrm{n}=3$ harmonic changes when you change the tension of a string of fixed length.
Why are we doing this lab?
Waves are everywhere one looks in nature (surface waves on water; sound waves in the air, for example). Many of us enjoy listening to (or producing) standing waves on the strings of musical instruments such as violins, pianos, and guitars. Some people think that the most fundamental theory of physics is string theory the theory of standing waves on strings.
Safety concerns: none.

## You should know:

amplitude maximum distance of a wave from the average position
antinode location of maximum motion of a standing wave ("A" on diagram at right).
frequency number of complete cycles per unit of time.
harmonic mode of vibration. The $\mathrm{n}=1, \mathrm{n}=2$, and $n=3$ harmonics are shown at right.
node location of minimum motion of a
 standing wave ("N" on diagram at right).
wavelength distance between two successive points on a wave that behave identically.


Equipment for Lab SC-1: (from left) clamp for speaker, table clamp and pulley, bridge, loudspeaker, string, power amplifier and cable.

## Pre-lab questions

1. Refer to the sketch on the previous page. Assume the string has length $L$. What are the wavelengths of the three harmonics shown in the sketch?

| harmonic | wavelength of harmonic |
| :---: | :--- |
| $\mathrm{n}=1$ | $\lambda_{1}=$ |
| $\mathrm{n}=2$ | $\lambda_{2}=$ |
| $\mathrm{n}=3$ | $\lambda_{3}=$ |

2. Consider standing waves that are excited on a string of length $L$, with both ends fixed. Cross out all of the following statements that are not true:
The wavelength of the standing wave is equal to the distance between a node and the nearest antinode.

The wavelength of a standing wave is equal to the distance between one node and the next node.

The wavelength of a standing wave is equal to twice the distance between a node and the nearest antinode.
The wavelength of a standing wave is equal to twice the distance between one node and the next node.
3. Given the length of a string is $L$ and the total mass of the string is $m$, what is the $\mu$, mass per unit length of the string?
4. Suppose you tie one end of a string to a wall, run the string over a pulley, and hang a mass $M$ from the other end. What is the tension $F$ in the string?
5. Suppose you know the length $L$, the mass per unit length $\mu$, and the tension $F$ of a string. What are the natural frequencies of vibration, $f_{n}$, of the string?
6. For the string in question 5 -suppose you measure the frequencies of the six lowest harmonics, $\left(f_{1}, f_{2}, f_{3}, f_{4}, f_{5}\right.$, and $f_{6}$, corresponding to $n=1, n=2, n=3$, $\mathrm{n}=4, \mathrm{n}=5$, and $\mathrm{n}=6$ ), and plot these frequencies on the vertical axis, and the corresponding n on the horizontal axis. What is the shape of the curve?
7. Again for the string in question 5-suppose you have a collection of 5 different masses, and you hang them, one by one, from the string. For each mass you measure the frequency of the third harmonic. If you plot the frequency of the third harmonic on the vertical axis, and the tension on the horizontal axis, what will be the shape of the resulting curve?
8. What function of the tension would you need to plot on the horizontal axis in order to get a straight line?
9. What would be the slope of this straight line, in terms of $L$ and $\mu$ ?

## A. Harmonics of a string under fixed tension

A1. Cut a piece of string about 2 m long. Measure the mass per unit length of the string using the Ohaus Precision electronic balance.
mass per unit length of string, from balance: $\qquad$ $\mathrm{kg} / \mathrm{m}$

A2. (see photo below) Tie one end of the string to the blue metal support; depress the spring and thread the string through the slot in the white plastic pin sticking up from the speaker. Avoid pressing or pulling too hard on the white plastic pin, as this could damage the speaker.


A3. Run the string over the bridge and pulley, and tie the free end of the string to the mass hanger, as shown in the diagram below. Make sure the top of the pulley is below the top of the bridge. Measure the distance $L$ between the white plastic pin and the bridge.

$L=$ $\qquad$ m

A4. Add 200 g to the mass hanger, so that the total mass hanging from the string is 250 g .

A5. Tape pieces of paper to the table between the speaker and the bridge (this uniform background should make the vibrating string easier to see).

A6. Connect the power amplifier to the speaker using the red and black cables, and connect the signal interface to the power amplifier using the grey cable.

A7. Connect the power amplifier to the power strip; make sure the switch on the back of the power amplifier is on (light on front should light up).
A8. Start the DataStudio setup file SC1.cap. In this lab you'll use Capstone to control the amplitude and frequency of the vibration of the string, but you won't take data with it.

A9. On the screen you should see a dialog box "Signal Generator", set to produce a sine wave with a frequency of 60 Hz and an amplitude of 2 Volts. Click the "On" button, and the string should start vibrating.

Lower the amplitude to 0 Volts whenever you're not making measurements, in order to minimize unnecessary noise.
To avoid damaging the speaker: Don't exceed 2 Volts. Use only sine waves.
If a speaker makes a rattling sound, or if the red light on front of amplifier comes on, turn down the amplitude and call over your TA.

A10. Estimate the frequency of the $\mathrm{n}=1$ harmonic (wavelength $=2 L$ ) in the following way. Start with a frequency below the frequency of the harmonic. Raise the frequency in steps of 1 Hz . Note the highest frequency that is definitely below the harmonic frequency, and the lowest frequency that is definitely above the harmonic frequency. Take the average of these two and call it the harmonic frequency.

$$
\text { frequency of } n=1 \text { harmonic : }
$$

$\qquad$ Hz

A11. Predict the frequencies of the $\mathrm{n}=1$ through $\mathrm{n}=6$ harmonics using your answer from prelab question 5, and then measure them. Complete this table:

| Harmonic | Predicted <br> Frequency (Hz) | Measured <br> Frequency (Hz) |
| :---: | :---: | :---: |
| $\mathrm{n}=1$ |  |  |
| $\mathrm{n}=2$ |  |  |
| $\mathrm{n}=3$ |  |  |
| $\mathrm{n}=4$ |  |  |
| $\mathrm{n}=5$ |  |  |
| $\mathrm{n}=6$ |  |  |

A12. (recall prelab question 6) Make a graph of measured frequency (vertical axis) as a function of harmonic (horizontal axis). Is this data well-fit by a straight line?

## B. Frequency of a particular harmonic as tension is changed

B1. Experimentally measure the frequency of the $\mathrm{n}=3$ harmonic for the five masses given below, and complete the table:

| Mass (g) | Tension (N) | Frequency of $\mathrm{n}=3$ harmonic (Hz) |
| :---: | :--- | :--- |
| 50 |  |  |
| 150 |  |  |
| 250 |  |  |
| 350 |  |  |
| 550 |  |  |

B2. Make a graph of frequency (vertical axis) as a function of tension (horizontal axis). Is this data well fit by a straight line?

B3. (recall answer to pre-lab question 8) What function of the tension should you plot on the horizontal axis in order to make the data fall on a straight line?

B4. Make the plot in B3, fit a trendline to it, and record the slope of the trendline, and the appropriate units.
Slope of trendline: $\qquad$ units of slope of trendline: $\qquad$
B5. Use your result from prelab question 9 to calculate the mass per unit length of the string.
mass per unit length, from third harmonics $=$ $\qquad$ $\mathrm{kg} / \mathrm{m}$
B6. What is the discrepancy, in percent, between this measurement and your measurement from A1?

## C. Follow-up questions

C1. What harmonic was most easily seen and had the least uncertainty of frequency?

C2. For stringed instruments, the strings that are tuned to low frequencies are large diameter, while those for high frequencies are much thinner. Why?

SC-1 STANDING WAVES ON A STRING

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## Appendix A: Significant Figures

The number of significant figures tells the reader the precision of a measurement.

| Measurement | Number of <br> Significant Figures |
| :--- | :---: |
| 22.74 | 4 |
| 91.5 | 3 |
| 7.50 | 3 |
| 7.5 | 2 |
| 12.25345 | 7 |

Always express your measurements in scientific notation. The fact that a measurement like 0.05 m has one significant figure is made more clear if it is written $5 \times 10^{-2} \mathrm{~m}$.

## How do I use significant figures in calculations?

Addition and subtraction: The result should be given to as many decimal places as the term that is given to the smallest number of decimal places.

## Examples

Addition
6.242
$+4.23$
$+0.013$
10.485
10.49

Subtraction
5.875
-3.34
2.535
2.54

The uncertain figures in each number are shown in bold-faced type. Note that when it is necessary to decrease the number of significant figures in the answer, if the last digit is $0,1,2,3$, or 4 , you round down, and if it is $5,6,7,8$, or 9 , you round up.

Multiplication and division: The result should be given to as many significant figures as the term that is given to the smallest number of significant figures.

The basis behind this rule is that the least accurately known term in the product will dominate the accuracy of the answer.

## Examples

## Multiplication

| 15.84 | 17.27 |
| :--- | :--- |
| $\times 2.5$ | $\mathbf{x 4 . 0}$ |
| $\mathbf{3 9 . 6 0 0}$ | 69.080 |
| 40 | 69 |

## Division

$$
2691 \div 23=117
$$

$$
1.2 \times 10^{2}
$$

## PRACTICE EXERCISES

1. Determine the number of significant figures of the quantities in the following table:

| Length <br> (centimeters) | Number <br> Significant Figures |
| :--- | :--- |
| 17.87 |  |
| 0.4730 |  |
| 17.9 |  |
| 0.473 |  |
| 18 |  |
| 0.47 |  |
| $1.34 \times 10^{2}$ |  |
| $2.567 \times 10^{5}$ |  |
| $2.0 \times 10^{10}$ |  |
| 1.001 |  |
| 1.000 |  |
| 1 |  |
| 1000 | 1001 |

2. Add: 121.3 to $6.7 \times 102$ :
[Answer: $121.3+6.7 \times 102=7.9 \times 10^{2}$ ]
3. Multiply: 34.2 and $1.5 \times 10^{4}$
[Answer: $34.2 \times 1.5 \times 104=5.1 \times 10^{5}$ ]

## Appendix B: Error Analysis

This appendix contains a glossary of terms used in the lab manual that are related to error analysis, followed by some examples.
accuracy proximity to the true value
average deviation simple estimate of the uncertainty of a set of measurements
best fit (noun) or best-fit (adjective): the result of a rigorous mathematical calculation matching parameters given by an equation to data measured in an experiment.
discrepancy an unexplained difference
error bars graphical indication of the uncertainty of a measurement
fit "by eye" the result of an graph-based attempt to match, without the use of calculations, parameters associated with an equation to data measured in an experiment.
linear fit best-fit straight line (term used by DataStudio)
mean arithmetic mean, also called "average"
method of "least squares" mathematical technique for calculating a best fit precision range in values of the same measurement performed many times significant figure reliably-known digit
standard deviation sophisticated estimate of the uncertainty of a measurement trendline name given by Excel to the best fit calculated by Excel uncertainty quantification of the experimenter's confidence in a measurement

The accuracy of a measurement is the difference between the measurement and the true value of the thing being measured. Unfortunately, it's usually impossible to know the true value of the thing being measured.

Therefore one makes the same measurement many times. The precision of a measurement is the range of the values of repeated measurements. Unfortunately, one's measurement technique may be erroneous (example: not all car odometers are calibrated accurately), and lead to the same wrong result over and over again.

One is thus driven to speak of uncertainty. This is a description of your confidence in your measurement. Whenever you make a measurement, state the uncertainty of the measurement as well. If you plot the measurement on a graph, make the length of the error bar to be equal to the uncertainty.

## How do I determine the uncertainty of a measurement?

If time or experimental constraints make repeated measurements impossible, the best you can do is estimate the uncertainty. Guess how much different the result might be if you were to take the measurement again: take this difference as your uncertainty.
If estimating the uncertainty is not good enough for your situation, make several measurements and equate the uncertainty to the average deviation of those measurements, as follows:

1. Find the average of all your measurements;
2. Find the absolute value of the difference of each measurement from the average (its deviation);
3. Find the average of all the deviations by adding them up and dividing by the number of measurements.

If Excel is available, use the function "STDEV" to calculate the standard deviation of the measurements, and use this as your uncertainty.
Example: measuring the mass of a penny
Student A measures the mass of the penny to be 3.110 g , estimates the uncertainty as 0.005 g , and writes: mass of penny $=3.110 \pm 0.005 \mathrm{~g}$.

Students B, C, D, E, F, and G each measure the mass of the same penny, and get:
Mass (grams) : 3.110, 3.125, 3.120, 3.126, 3.122, 3.120
Average: 3.1205 g ; round up to 3.121 g
The deviations are: $0.011 \mathrm{~g}, 0.004 \mathrm{~g}, 0.001 \mathrm{~g}, 0.005 \mathrm{~g}, 0.001 \mathrm{~g}, 0.001 \mathrm{~g}$
Sum of deviations: 0.023 g ; average deviation: $(0.023 \mathrm{~g}) / 6=0.004 \mathrm{~g}$
Thus these students write: mass of penny $=3.121 \pm 0.004 \mathrm{~g}$
Later the students enter the six measurements into Excel, which calculates the standard deviation of the six measurements to be 0.006 g . The students thus revise their estimates of the uncertainty: mass of penny $=3.121 \pm 0.006 \mathrm{~g}$.

## How can I do calculations with values that have uncertainty?

A simple way to estimate uncertainties is to find the largest possible uncertainty the calculation could yield. This will always overestimate the uncertainty of your calculation, but an overestimate is better than no estimate. The method for performing arithmetic operations on quantities with uncertainties is illustrated in the following examples:

Addition:

$$
(3.131 \pm 0.008 \mathrm{~g})+(3.121 \pm 0.004 \mathrm{~g})=?
$$

First find the sum of the values: $3.131 \mathrm{~g}+3.121 \mathrm{~g}=6.252 \mathrm{~g}$
Next find the largest possible value: $3.139 \mathrm{~g}+3.125 \mathrm{~g}=6.264 \mathrm{~g}$
The uncertainty is the difference between the two: $6.264 \mathrm{~g}-6.252 \mathrm{~g}=0.012 \mathrm{~g}$
Answer: $6.252 \pm 0.012 \mathrm{~g}$.
Note: This uncertainty can be found by simply adding the individual uncertainties: $0.004 g+0.008 g=0.012 g$

## Subtraction:

$$
(3.131 \pm 0.008 \mathrm{~g})-(3.121 \pm 0.004 \mathrm{~g})=?
$$

First find the difference of the values: $3.131 \mathrm{~g}-3.121 \mathrm{~g}=0.010 \mathrm{~g}$
Next find the largest possible value: $3.139 \mathrm{~g}-3.117 \mathrm{~g}=0.022 \mathrm{~g}$
The uncertainty is the difference between the two: $0.022 \mathrm{~g}-0.010 \mathrm{~g}=0.012 \mathrm{~g}$
Answer: 0.010 $\pm \mathbf{0 . 0 1 2} \mathbf{g}$.
Note: This uncertainty can be found by simply adding the individual uncertainties:
$0.004 \mathrm{~g}+0.008 \mathrm{~g}=0.012 \mathrm{~g}$

## Multiplication:

$$
(3.131 \pm 0.013 \mathrm{~g}) \times(6.1 \pm 0.2 \mathrm{~cm})=?
$$

First find the product of the values: $3.131 \mathrm{~g} \times 6.1 \mathrm{~cm}=19.1 \mathrm{~g}-\mathrm{cm}$
Next find the largest possible value: $3.144 \mathrm{~g} \mathrm{x} 6.3 \mathrm{~cm}=19.8 \mathrm{~g}-\mathrm{cm}$
The uncertainty is the difference between the two: $19.8 \mathrm{~g}-\mathrm{cm}-19.1 \mathrm{~g}-\mathrm{cm}=0.7$

$$
\mathrm{g}-\mathrm{cm}
$$

Answer: $19.1 \pm 0.7 \mathrm{~g}-\mathrm{cm}$.
Note: The percentage uncertainty in the answer is the sum of the individual percentage uncertainties:
$(0.013 \div 3.131 \times 100 \%)+(0.2 \div 6.1 \times 100 \%)+(0.7 \div 19.1 \times 100 \%)$

## Division:

$$
(3.131 \pm 0.008 \mathrm{~g}) \div(3.121 \pm 0.004 \mathrm{~g})=?
$$

First divide the values: $3.131 \mathrm{~g} \div 3.121 \mathrm{~g}=1.0032$
Next find the largest possible value: $3.139 \mathrm{~g} \div 3.117 \mathrm{~g}=1.0071$
Notice that the largest possible value for the numerator and the smallest possible value for the denominator gives the largest result.
The uncertainty is the difference between the two:
$1.0071-1.0032=0.0039$
Answer: $1.003 \pm 0.004$
Note: The percentage uncertainty in the answer is the sum of the individual percentage uncertainties:
$(0.008 \div 3.131 \times 100 \%)+(0.004 \div 3.121 \times 100 \%)+(0.0039 \div 1.0032 \times 100 \%)$

## PRACTICE EXERCISES:

1. Consider the following results for different experiments. Determine if they agree with the accepted result listed to the right. Also calculate the percentage uncertainty for each result.
a) $\mathrm{g}=10.4 \pm 1.1 \mathrm{~m} / \mathrm{s}^{2}$

$$
\mathrm{g}=9.8 \mathrm{~m} / \mathrm{s}^{2}
$$

b) $\mathrm{T}=1.5 \pm 0.1 \mathrm{sec}$
$\mathrm{T}=1.1 \mathrm{sec}$
c) $\mathrm{k}=1368 \pm 45 \mathrm{~N} / \mathrm{m}$

$$
\mathrm{k}=1300 \pm 50 \mathrm{~N} / \mathrm{m}
$$

Answers: a) Yes, 11\%; b) No, 7\%; c) Yes, 3.3\%
2. The area of a rectangular metal plate was found by measuring its length and its width. The length was found to be $5.37 \pm 0.05 \mathrm{~cm}$. The width was found to be $3.42 \pm 0.02 \mathrm{~cm}$. What are the area and the uncertainty?
Answer: $18.4 \pm 0.3 \mathrm{~cm}^{2}$
3. Each member of your lab group weighs the cart and two mass sets twice. The following table shows this data. Calculate the total mass of the cart with each set of masses and for the two sets of masses combined.

| Cart (grams) | Mass set 1 (grams) | Mass set 2 (grams) |
| :---: | :---: | :---: |
| 201.3 | 98.7 | 95.6 |
| 201.5 | 98.8 | 95.3 |
| 202.3 | 96.9 | 96.4 |
| 202.1 | 97.1 | 96.2 |
| 199.8 | 98.4 | 95.8 |
| 200.0 | 98.6 | 95.6 |

## Answers:

Cart and set $1: 299.3 \pm 1.6 \mathrm{~g}$.
Cart and set 2: $297.0 \pm 1.2 \mathrm{~g}$.
Cart and both sets: $395.1 \pm 1.9 \mathrm{~g}$.

# Appendix C: Physics Laboratory Excel Tutorial 

(text adapted from Clemson University Physics and Astronomy Excel Tutorial)

## Graphing Data and Curve Fitting



In this tutorial on graphing, we will examine data taken from an experiment in which the circumferences and radii of several circular objects were measured. The data is displayed on the screen shot to the left. Note that the error in the measurement of the radius is not given; suppose that from your knowledge of radius measurement techniques, you estimate it at $+/-7 \%$.

Of course, the equation associated with this data is $\mathrm{C}=2 \pi \mathrm{r}$, or the circumference of the circle is equal to two times pi times the circle's radius. In this "experiment," the circumferences and radii are measured. We hope to be able to determine the value of $\pi$ by fitting a straight line to the data.

It is our firm belief (although not necessarily the belief of everyone at this university) that beginning laboratory students should learn to plot their data by hand before using a computer application to learn this task. Nonetheless, here we show how to use Excel to plot the data and fit a curve to the data.

## How to Plot the Data

1. Enter the data onto the worksheet as shown in the above screenshot
2. Click on an empty cell.
3. Click on the "Insert" tab at the top of the screen.
4. Click on "Scatter".
5. Click on "Scatter with only Markers"
6. A completely blank chart should now appear. Position the cursor over the blank chart, right-click, and choose "Select Data".
7. Click on "Add" under "Legend Entries (Series). This will cause value boxes,

8. Click on the Collapse Dialog button 医罦 at the right end of the "Series X Values" box. This will temporarily shrink the dialog window so you can highlight the x -values that you wish plotted on the horizontal axis.
9. When the dialog box shrinks, you can use the mouse to highlight the $x$-values that will be plotted along the horizontal axis (in this case the measured radii). Highlight values only, not title.

10. When finished click the Expand Dialog button which will return the dialog window to maximum size.
11. Repeat steps 8 and 9 for the "Series Y Values", which will tell the computer where along the vertical axis to place the data points.
12. Click "OK". A graph should now be visible.
13. Click "OK" again. The graph should no longer be hidden. However, it lacks titles for the x and y axes, as well as for the entire graph.
14. Double click somewhere in the chart area. The buttons on the top should change.
15. Click on "Layout 1" in the Chart Layout area

16. You should now be able to type titles for the x axis, the y axis, and the entire graph. Select a title so it looks like this chart title. What you type appears in the formula bar at the top, and will appear on the graph when you hit enter. Your graph should now look like this:


## Making the Graph Look Nice

Now that you have plotted the data, take a minute or two to put the finishing touches on the graph.

1. Include units on each axis title: double-click on each axis title and edit.
2. Delete the legend box ${ }^{\star \text { Series1 }}$ by right-clicking on it and choosing "Delete".
3. Add appropriate symbols. For example, change "pi" to " $\pi$ " by double clicking on the graph title, deleting the " i ", clicking and dragging so the " p " is highlighted, placing the mouse over the " p " and right-clicking, choosing "Font...", and under "Latin text font:" choosing "symbol". Then click "OK".
4. Add error bars.
a. Select the "Layout" tab at the top of the screen, click the "Error bars" pull down menu, and click "More error bars options...". The "Format error bars" window should appear.
b. Click "custom" and "Specify Value". Error Bars" window should appear.
Error Value" means how far above each the error bar should extend, and Error Value" means how far below each the error bar should extend. Since (in


The "Custom
"Positive data point
"Negative data point this example) we measured the error in the circumference, use the Collapse Dialog Button, and highlight the values in column C. Do this for both Positive Error Value and Negative Error Value. Then hit OK.
c. Note that Excel has chosen horizontal error bars, which you now have to fix. Position the cursor over a horizontal error bar, and leave it there for a moment till you see "Series 1 X Error Bars" appear on the screen. Then right-click. Choose "Format Error Bars...". The "Format Error Bars" window should appear again, only now it should describe "Horizontal Error Bars". In this example, we have estimated the error in measuring the radius at $+/-7 \%$, so click "percentage" and then enter 7.0. Hit Close. You are finished adding error bars to your graph, which should now look like this:


Sometimes, after you make a nice-looking graph of your data, you want to make a fit to the data and plot it as well. If you want to fit a straight line to the data, you need
to do a "linear fit". Follow the steps below to make a linear fit to a dataset (Excel can also do other types of fit, besides linear fits).

## Fitting the Data, and Plotting the Fit

1. Choose the "Layout" tab at the top of the screen.
2. Click "trendline".
3. Select "more trendline options". The "Format Trendline" window should appear.
4. Choose "linear", and click "Display Equation on chart" and "Display Rsquared value on chart". Then click close.
5. Move the equation and R-squared value where you want them by clicking and dragging.
6. The equation mentions " $y$ " and " $x$ " but we are trying to relate "circumference" and "radius". Make the equation easier to understand by:
a. Editing the x axis label to be radius $\mathrm{r}(\mathrm{cm})$ and the y axis label to be circumference C (cm)
b. Editing the equation by changing " $y$ " to " $C$ " and " $x$ " to " $r$ ".

By doing these things, you are in essence telling your TA that you really do understand what was actually measured, and also that you know what you're doing using Excel to make plots instead of plotting things by hand.

The final result of your graph should look something like this:


Now that you have finished making the graph, the real thinking begins! A good graph will help you make sense of your data, and perhaps relate the data to your expectations. The slope of the linear fit is 6.179 , and the $y$ intercept is 0.2327 . What values would you have expected for the slope and $y$-intercept of the linear fit to this data?

# Appendix D: What do the Velocity and Acceleration "Measured" by the PASCO Motion Sensor Really Mean? 

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Abstract: We deduce the algorithm by which the DataStudio software, used with the PASCO Motion Sensor, calculates velocity and acceleration from measurements of position. The motion sensor measures the distances to objects the same way a bat does: it periodically emits sound pulses, and listens for the echo. Knowing the velocity of sound, it can calculate the distance to the object which made the echo using $\mathrm{D}=\mathrm{v} \mathrm{t}$, where D is the distance to the object, v is the velocity of sound, and t is one-half the time delay between the pulse and the echo. But how can the motion sensor calculate the velocity and acceleration of the object from the time delay? We conclude the following algorithm is used:

$$
v_{n+1 / 2}=\frac{x_{n+2}-x_{n-1}}{t_{n+2}-t_{n-1}} \quad a_{n}=\frac{v_{n+1 / 2}-v_{n-1 / 2}}{t_{n+1 / 2}-t_{n-1 / 2}}
$$



The PASCO Motion Sensor

## DATA

Here is a screen shot of some data from the PASCO motion sensor:


Note that the first position data point is at 0.0 seconds, while the first velocity point is not until 0.3 seconds, and the first acceleration point is not until 0.4 seconds. Further note that the position measurements are taken at multiples of 0.2 seconds (ie $0.2,0.4$, $0.6,0.8 \ldots$...) while the velocity data points occur half-way between (ie $0.3,0.5,0.7,0.9 \ldots$ ). The acceleration data points are at the same times as the position measurements (except for 0.0 and 0.2 ). Finally, note that the position measurements for $0.2,0.4$, and 0.6 seconds are identical, while the velocities given for 0.3 and 0.5 seconds are decidedly negative.

## Hypothetical algorithm

We believe that DataStudio calculates velocity and acceleration from position in the following way. Let the first measurement $x_{0}$ be at time $t_{0}$ and subsequent position measurements $\mathrm{x}_{\mathrm{n}}$ be at times $\mathrm{t}_{\mathrm{n}}=\mathrm{t}_{0}+\mathrm{n} * \mathrm{dt}$, where dt is the time between measurements (for the data shown, collected at 5 samples/second sampling frequency, $\mathrm{dt}=0.2$ seconds). In this case the velocity points are given by:

$$
v_{n+1 / 2}=\frac{x_{n+2}-x_{n-1}}{t_{n+2}-t_{n-1}}
$$

As an example, we believe the first velocity point, that appears midway between the second and third position points, would be calculated by putting $\mathrm{n}=2$ in the above formula-that is by measuring the slope between the first and fourth position measurements and ignoring the second and third position measurements.

We further believe that DataStudio calculates acceleration using this algorithm:

$$
a_{n}=\frac{v_{n+1 / 2}-v_{n-1 / 2}}{t_{n+1 / 2}-t_{n-1 / 2}}
$$

That is, DataStudio calculates acceleration as the slope between successive velocity measurements.

## Verification

We exported the DataStudio position, velocity, acceleration, and time data into Excel. Then we created formulae for the hypothetical velocity and acceleration data corresponding to the two algorithms given above, and compared the values computed by Excel to values given by DataStudio. All relevant values are given in the table on the next page.

The first and second columns are the time and position as given by DataStudio. The difference between the hypothetical velocity (third column) and the DataStudio velocity (sixth column) is given in the fourth column. Note the difference is close to zero. The DataStudio acceleration is given in the eighth column and the hypothetical acceleration is given in the ninth column. The difference, given in the tenth column, is also close to zero.

Note that the expected position data point at 5.97 seconds is missing (it should fall between the shaded time measurements in the first column). This results in noticeable discrepancies in two acceleration data points (shaded). Also note the discrepancies in the final position and velocity data points (also shaded). These are due to a truncated data set-we only exported the data shown on the graph into Excel, while the actual DataStudio data set continues on. Thus all discrepancies are easily accounted for, and we believe our hypotheses are correct.

## Appendix D: PASCO Motion Sensor Velocity and Acceleration "Measurements"






[^0]:    * A "C" in the experiment number means the lab is computerized.

