

Physics 247 Lab Manual

Mechanics, Heat, Sound/Waves

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NOTE: M=Mechanics, H=Heat, S=Sound/Waves

Introduction

General Instructions and Helpful Hints

Physics is an experimental science. In this laboratory, we hope you gain a realistic feeling for the experimental origins, and limitations, of physical concepts; an awareness of experimental errors, of ways to minimize them and how to estimate the reliability of the result in an experiment; and an appreciation of the need for keeping clear and accurate records of experimental investigations.

Physics is also a social activity. We expect you will find it useful to discuss all phases of the experiment with your lab partners. During set-up, establish the purpose of each piece of apparatus; during data-taking, cooperate to choose who does what, and critique the data as it is taken (is it as expected?); during analysis, perform calculations independently, and then compare results to check each others' work; when drawing conclusions, debate the reliability of the results, and what may fairly be concluded from them.

Maintaining a clearly written laboratory notebook is crucial. This lab notebook, at a minimum, should contain the following:

1. *Heading of the Experiment:* Copy from the manual the number and name of the experiment.
Include both the current date and the name(s) of your partner(s).
2. *Original data:* Original data must always be recorded directly into your notebook as they are gathered. "Original data" are the actual readings you have taken. All partners should record all data, so that in case of doubt, the partners' lab notebooks can be compared to each other. Arrange data in tabular form when appropriate. *A phrase or sentence introducing each table is essential for making sense out of the notebook record after the passage of time.*
3. *Housekeeping deletions:* You may think that a notebook combining all work would soon become quite a mess and have a proliferation of erroneous and superseded material. Indeed it might, but you can improve matters greatly with a little house-keeping work every hour or so. Just draw a box around any erroneous or unnecessary material and hatch three or four parallel diagonal lines across this box. (This way you can come back and rescue the deleted calculations later if you should discover that the first idea was right after all. It occasionally happens.) Append a note to the margin of box explaining to yourself what was wrong.

We expect you to keep up your notes as you go along. Don't take your notebook home to "write it up" – you probably have more important things to do than making a beautiful notebook. (Instructors may permit occasional exceptions if they are satisfied that you have a good enough reason.)

4. *Remarks and sketches:* When possible, make simple, diagrammatic sketches (rather than "pictorial" sketches" of apparatus. *A phrase or sentence introducing each calculation is essential for making sense out of the notebook record after the passage of time.* When a useful result occurs at any stage, describe it with at least a word or phrase.
5. *Graphs:* There are three appropriate methods:

- A. Affix furnished graph paper in your notebook with transparent tape.
- B. Affix a computer generated graph paper in your notebook with transparent tape.
- C. Mark out and plot a simple graph directly in your notebook.

Show points as dots, circles, or crosses, i.e., \cdot , \circ , or \times . Instead of connecting points by straight lines, draw a smooth curve which may actually miss most of the points but which shows the functional relationship between the plotted quantities. Fasten directly into the notebook any original data in graphic form (such as the spark tapes of Experiment M4).

6. *Units, coordinate labels:* Whenever you write down a physical quantity, *always* write down its units. Likewise, whenever you make a graph, always label the *abscissa* (horizontal coordinate) and *ordinate* (vertical coordinate), so that the reader (who might be yourself at a later time) knows what has been graphed.
7. *Final data, results and conclusions:* At the end of an experiment some written comments and a neat summary of data and results will make your notebook more meaningful to both you and your instructor. The conclusions must be faithful to the data. It is often helpful to formulate conclusion using phrases such as “the discrepancy between our measurements and the theoretical prediction was larger than the uncertainty in our measurements.”

COMPLETION OF WORK

Plan your work so that you can complete calculations, graphing and miscellaneous discussions before you leave the laboratory. Your instructor will check each completed lab report and will usually write down some comments, suggestions or questions in your notebook. Approach your instructor the way you would approach a senior colleague, who can help deepen your understanding and “feel” for the subject.

Errors and Uncertainties

Reliability estimates of measurements greatly enhance their value. Thus, saying that the average diameter of a cylinder is 10.00 ± 0.02 mm tells much more than the statement that the cylinder is a centimeter in diameter. The reliability of a single measurement (such as the diameter of a cylinder) depends on many factors:

FIRST, are **actual variations** of the quantity being measured, e.g. the diameter of a cylinder may actually be different in different places. You must then specify where the measurement was made; or if one wants the diameter in order to calculate the volume, first find the average diameter by means of a number of measurements at carefully selected places. Then the scatter of the measurements will give a first estimate of the reliability of the average diameter.

SECOND, the micrometer *caliper used may itself* be in error. The errors thus introduced will of course not lie equally on both sides of the true value so that averaging a large number of readings is no help. To eliminate (or at least reduce) such errors, we calibrate the measuring instrument: in the case of the micrometer caliper by taking the zero error (the reading when the jaws are closed) and the readings on selected precision gauges of dimensions approximately equal to those of the cylinder to be measured. We call such errors **systematic**, and these cause errors in accuracy.

THIRD, Another type of **systematic** error can occur in the measurement of a cylinder: The micrometer will always measure the largest diameter between its jaws; hence if there are small bumps or depressions on the cylinder, the average of a large number of measurements will not give the true average diameter but a quantity somewhat larger. (This error can of course be reduced by making the jaws of the caliper smaller in cross section.)

FINALLY, if one measures something of definite size with a calibrated instrument, one's measurements will vary. For example, the reading of the micrometer caliper may vary because one can't close it with the same force every time. Also the observer's estimate of the fraction of the smallest division varies from trial to trial. Hence the average of a number of these measurements should be closer to the true value than any one measurement.

Characterization of Uncertainties

The experimenter cannot immediately determine the cause of the variation of a set of measurements. A simpler task is to characterize this variation.

Consider the data below. Column 1 represents 10 readings of the diameter of a cylinder taken at one place, so that variations in the cylinder do not come into consideration. The mean of these 10 readings turns out to be 9.9427 mm. Column 2 gives the magnitude (absolute) of each reading's deviation from the mean.

Measurements	Deviation from Mean
9.943 mm	0.0003
9.942	0.0007
9.944	0.0013
9.941	0.0017
9.943	0.0003
9.943	0.0003
9.945	0.0023
9.943	0.0003
9.941	0.0017
9.942	0.0007

How should one state the results of these measurements in a scientific fashion?

A quick way to estimate the reliability of a set of measurements is to calculate the **average deviation** from the mean. Expressed algebraically, the mean is $\bar{x} = \sum x_i/n$, and the average deviation from the mean is $(\sum |x_i - \bar{x}|)/n$, where n is the number of measurements taken and x_i is the i^{th} measurement.

A more formal measure of the reliability of a set of measurements is the **standard deviation** σ (also known as the “root mean square deviation”). You calculate σ as

$$\sigma = \sqrt{\frac{1}{(n-1)} \sum_{i=1}^n (x_i - \bar{x})^2}$$

Note that if you make one measurement of some quantity x , then $n = 1$ and $\bar{x} = x_1$ in the above equation: the standard deviation of your measurement is $\sqrt{0/0}$, which is undefined. You don't know how reliable your measurement is.

Assuming you make more than one measurement, σ tells you the typical deviation from the mean you will find for an individual measurement. It depends on your experimental technique. If you make more measurements, σ cannot be expected to change much.

However, the more measurements you make, the more accurately you can expect to know the value of the mean. The uncertainty of the value of the mean is called **the standard deviation of the mean** σ_μ , and in general it is less than σ :

$$\sigma_\mu = \text{standard deviation of the mean} = \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{n}} \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$$

The process of taking more measurements in order to minimize σ_μ is often referred to as “gathering statistics”.

One states one's results as **mean \pm standard deviation of the mean**. From the data above, the diameter of the cylinder was measured to be 9.9427 ± 0.0004 mm.

Error Distribution and Error Bars

For a wide variety of measurements it may be assumed that the error distribution is “normal”: the probability of a given error ϵ is proportional to $e^{-\epsilon^2}$. If the error distribution is normal, then on average 68% of a large number of measurements will lie closer than σ to the true value.

The purpose of the error bars shown on a graph in a technical report is as follows: if the reader attempts to reproduce the results in the graph using the procedure described in the report, the reader should expect his or her results to have a 50% chance of falling within the range indicated by the error bars.

If the error distribution is normal, the error bars should be of length $\pm 0.6745\sigma$.

For a clear discussion of the theory of errors, as it relates to experimental measurements in the physical sciences, see P.R. Bevington and D.K. Robinson, *Data Reduction and Error Analysis for the Physical Sciences*, 3rd ed., McGraw Hill 2003, p. 11.

Propagation of Uncertainties

Let $R = f(x, y, z)$ be a result R which depends on measurements of three different quantities x , y , and z . The uncertainty ΔR in R which results from an uncertainty Δx in the measurement of x is then

$$\Delta R = \frac{\partial f}{\partial x} \Delta x ,$$

and the fractional uncertainty in R is

$$\frac{\Delta R}{R} = \frac{\partial f}{f} \Delta x .$$

In most experimental situations, the errors are uncorrelated and have a normal distribution. In this case the uncertainties add in quadrature (the square root of the sum of the squares):

$$\frac{\Delta R}{R} = \sqrt{\left(\frac{\partial f}{\partial x} \Delta x\right)^2 + \left(\frac{\partial f}{\partial y} \Delta y\right)^2 + \left(\frac{\partial f}{\partial z} \Delta z\right)^2} .$$

Some examples:

A.) $R = x + y$. If errors in x and y each have a normal or Gaussian distribution and are independent, they combine in quadrature:

$$\Delta R = \sqrt{\Delta x^2 + \Delta y^2} .$$

Note that if $R = x - y$, then $\Delta R/R$ can become very large if x is nearly equal to y . Hence avoid, if possible, designing an experiment where one measures two large quantities and takes their difference to obtain the desired quantity.

B.) $R = xy$. If the measurement errors have a normal distribution and are independent, the relative errors combine in quadrature:

$$\frac{\Delta R}{R} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2} .$$

Note the same result occurs for $R = x/y$.

C.) Consider the density of a solid (Exp. M1):

$$\rho = \frac{m}{\pi r^2 L}$$

where m = mass, r = radius, L = length, are the three measured quantities and ρ = density. Hence

$$\frac{\partial \rho}{\partial m} = \frac{1}{\pi r^2 L} \quad \frac{\partial \rho}{\partial r} = \frac{-2m}{\pi r^3 L} \quad \frac{\partial \rho}{\partial L} = \frac{-m}{\pi r^2 L^2} .$$

If the errors have normal distribution and are uncorrelated, then

$$\frac{\Delta \rho}{\rho} = \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(2\frac{\Delta r}{r}\right)^2 + \left(\frac{\Delta L}{L}\right)^2}$$

Significant Figures

Suppose you have measured the diameter of a circular disc and wish to compute its area $A = \pi d^2/4 = \pi r^2$. Let the average value of the diameter be 24.326 ± 0.003 mm; dividing d by 2 to get r we obtain 12.163 ± 0.0015 mm with a relative error $\frac{\Delta r}{r}$ of $\frac{0.0015}{12} = 0.00012$. Squaring r (using a calculator) we have $r^2 = 147.938569$, with a relative error $2\Delta r/r = 0.00024$, or an absolute error in r^2 of $0.00024 \times 147.93 \dots = 0.036 \approx 0.04$. Thus we can write $r^2 = 147.94 \pm 0.04$, any additional places in r^2 being unreliable. Hence for this example the first five figures are called *significant*.

Now in computing the area $A = \pi r^2$ how many digits of π must be used? A pocket calculator with $\pi = 3.141592654$ gives

$$A = \pi r^2 = \pi \times (147.94 \pm 0.04) = 464.77 \pm 0.11 \text{ mm}^2$$

Note that $\frac{\Delta A}{A} = 2\frac{\Delta r}{r} = 0.00024$. Note also that the same answer results from $\pi = 3.1416$, but that $\pi = 3.142$ gives $A = 464.83 \pm 0.11 \text{ mm}^2$ which differs from the correct value by 0.06 mm^2 , an amount comparable to the estimated uncertainty.

A good rule is to use one more digit in constants than is available in your measurements, and to save one more digit in computations than the number of significant figures in the data. When you use a calculator you usually get many more digits than you need. Therefore **at the end**, be sure to **round off** the final answer to display the correct number of **significant figures**.

SAMPLE QUESTIONS

1. How many significant figures are there in the following numbers?
 - (a) 976.45
 - (b) 4.000
 - (c) 10

2. Round off each of the following numbers to three significant figures.
 - (a) 4.455
 - (b) 4.6675
 - (c) 2.045

3. A function has the relationship $Z(A, B) = A + B^3$ where A and B are found to have uncertainties of $\pm\Delta A$ and $\pm\Delta B$ respectively. Find ΔZ in term of A , B and the respective uncertainties assuming the errors are uncorrelated.

4. What happens to σ , the standard deviation, as you make more and more measurements? what happens to $\bar{\sigma}$, the standard deviation of the mean?
 - (a) They both remain same
 - (b) They both decrease
 - (c) σ increases and $\bar{\sigma}$ decreases
 - (d) σ approaches a constant and $\bar{\sigma}$ decreases

M-1 Errors & Motion

M-1a Measurement and Error

OBJECTIVES:

The major objectives of M-1a are to develop a basic understanding of what it means to make an experimental measurement, and to provide a methodology for assessing random and systematic errors in this measurement process. In addition this lab will introduce you to the PASCO interface hardware and software.

THEORY:

How long is one second? The formal definition—the second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the Cesium-133 atom—is unhelpful for everyday affairs. In this lab you will test your ability to internalize the one second time interval by flicking your finger back and forth through an infrared beam sensor once per second. The duration of each full cycle (back and forth, approximately 2 seconds) will be simultaneously recorded, plotted and tabulated by the PASCO interface software.

Your goal in this experiment is to learn to distinguish systematic and random errors, and assess the size of the systematic and random errors in your data set.

SYSTEMATIC ERRORS: These are errors which affect the *accuracy* of a measurement. Typically they are reproducible so that they always affect the data in the same way. For instance, if a clock runs slowly, its measurement of some duration will be less than the true duration.

RANDOM ERRORS: These are errors which affect the *precision* of a measurement. A process itself may have a random component (as in radioactive decay) or the measurement technique may introduce noise that causes the readings to fluctuate. If many measurements are made, a statistical analysis will reduce the uncertainty from random errors by averaging.

APPARATUS:

- ⇒ A PASCO photogate and stand: This device emits a narrow infrared beam in the gap and occluding the beam prevents it from reaching a photodetector. When the beam is interrupted the red LED should become lit.
- ⇒ A PASCO 850 Universal Interface monitors the photogate output *vs* time and can be configured to tabulate, plot and analyze this data.

PROCEDURE:

To configure the experiment you should refer to Fig. 1 below. Adjust the photogate so that one member can easily and repetitively flick his/her finger through the gap. Insert the phone-jack cable from the photogate into the DIGITAL INPUT #1 socket.

To launch the Pasco interface software, Capstone, you will need to click on the “Experiment M-1a” file in the PASCO/Physics201 folder on the desktop. Fig. 2 below gives a good idea of how the display should appear.

You will note that a “dummy” first data set already exists on start-up showing a typical data run. In the table you can view all of the data points and the statistical analysis, including mean and standard deviation. In addition there should be a plot of this data and a histogram.

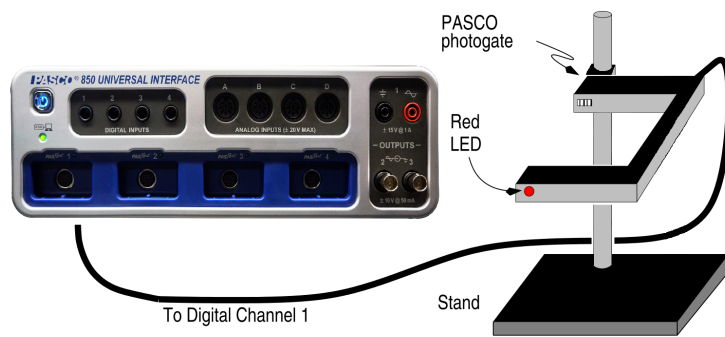





Figure 1: A schematic of the M-1a components and layout.

SUGGESTED PROCEDURE:

1. Start recording data by clicking on the “Record” icon  and practice flicking a finger back and forth so that a two second interval appears in the window. The Record icon has now changed to the “Stop” icon ; click the Stop icon when done. Since the program started with one set of data, this will produce a second set.
2. Each run gets its own data set in the “Data Summary”  tab under the **Tools** Palette. (If there are any data sets in existence you will *not* be able to reconfigure the interface parameters or sensor inputs.) A data set can be deleted by selecting the data set and clicking the red X to the right of the name. You can select which data set is displayed on a particular graph by clicking on the Data Summary icon that appears on that graph.

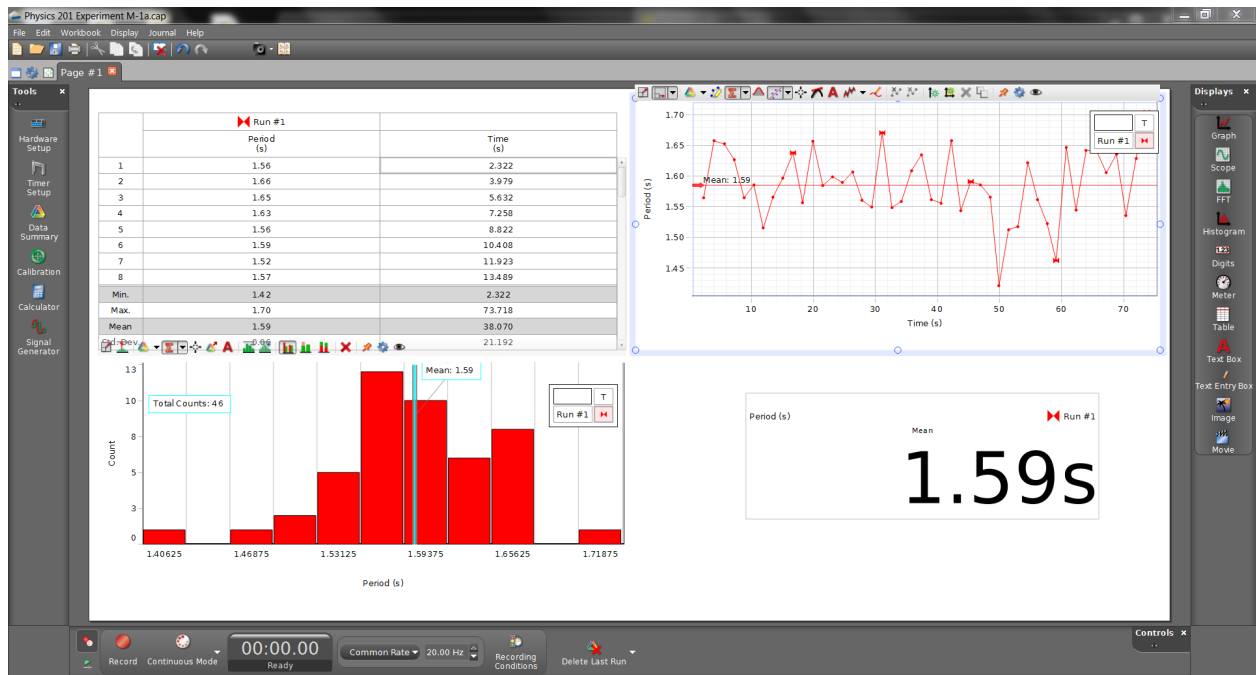



Figure 2: The PASCO Capstone display window

- Once you are comfortable with the procedure then click on the Record icon, cycle a finger back and forth over fifty times, then click on the Stop icon. DO NOT watch the time display while you do this, since you want to find out how accurately and precisely you can reproduce a time interval of 2 s using only your mind.
- What is the mean time per cycle? What is the standard deviation? The mean \bar{t} and standard deviation σ are given by:

$$\bar{t} = \sum_{i=1}^N t_i / N \quad \text{and} \quad \sigma = \sqrt{\sum_{i=1}^N (t_i - \bar{t})^2 / [N - 1]} .$$

Click on the Statistics icon  to ask PASCO to calculate the mean and standard deviation of the data. Is your mean suggestive of a systematic error?

- Look at the **Histogram** of the data. Does your data qualitatively give the appearance of a normal distribution (a Gaussian bell curve)?
- For analyzing and quantifying random errors, you need to assess how a data set is distributed about the mean. The standard deviation σ is one common calculation that does this. In the case of a normal distribution approximately 68% of the data points fall within $\pm 1\sigma$ of the mean (90% within $\pm 2\sigma$). Is your data consistent with this attribute?
- Assessing the possibility of systematic behavior is somewhat more subtle. In general σ is a measure of how much a *single* measurement fluctuates from the mean. In this run you have made fifty presumed identical measurements. A better estimate of how well you have really determined the mean is to calculate the *standard deviation of the mean* $\bar{\sigma} = \sigma / \sqrt{N}$. After recording $\bar{\sigma}$ in your lab book, can you now observe any evidence that there is a systemic error in your data? Answer this same question with respect to the first “sample” data set.
- OPTIONAL: Systematic errors can sometimes drift over time. In the best-case scenario they drift up and down so that they average out to zero. (Clearly it would be better if they could be eliminated entirely.) With respect to \bar{t} and $\bar{\sigma}$ for the first 25 and second 25 cycles do you observe any systematic trends? Click on the Data Highlight icon  . Dragging the mouse will highlight a subset of the data. The mean, σ , and other statistical attributes will appear at the bottom of the table.

M-1b Errors and the Density of a Solid

OBJECTIVES:

To learn about systematic and random errors; to understand significant figures; to estimate the reliability of one's measurements; to propagate uncertainties; and to calculate the reliability of the final result.

NOTE: This experiment illustrates the earlier section on Errors and Uncertainties. The accuracy of your estimate of reliability will show whether you have mastered the material in this section.

APPARATUS:

Metal cylinders of varying sizes; micrometer; vernier calipers; precision gauge blocks; precision balance.

PRECAUTIONS:

Avoid dropping the metal cylinders, or deforming them in any other way.

Avoid damage to the precision screw of the micrometer by turning only the ratchet knob to open or close the jaws.

The caliper has a lock which lets you preserve a reading—be sure to disengage the caliper lock before using.

Improper weighing procedures may damage the precision balance. Consult your instructor if in doubt.

In handling the gauge blocks avoid touching the polished surfaces since body acids are corrosive.

INTRODUCTION:

First read the material on Errors and Uncertainties and Significant Figures in the lab manual Introduction. Since density is the mass per unit volume, you must measure the mass (on a balance), and compute the volume ($h\pi r^2 = h\pi d^2/4$) from measurements of the cylinder's diameter d and height h . Any one of three length-measuring devices may be used. These include a micrometer, a vernier caliper and/or a simple metric ruler. The micrometer will permit the highest precision measurements but using one can be cumbersome, especially when reading the vernier scale. All methods will demonstrate the aforementioned objectives. Your instructor will give you guidance in choosing an appropriate measurement device.

THE MICROMETER:

Record the serial number of your micrometer. Then familiarize yourself with the operation of the caliper and the reading of the scales: work through Appendix A on the Micrometer. Note that use of the ratchet knob in closing the jaws insures the same pressure on the measured object each time. Always estimate tenths of the smallest division. Some micrometers have verniers to assist the estimation.

THE VERNIER CALIPER:

The vernier scale was invented in 1631 by Pierre Vernier. Work through Appendix A for practice reading a vernier scale. Experiment with one of the large verniers in the lab until you are confident you can use it correctly.

Precautions on use of the calipers:

1. Unclamp **both** top thumbscrews to permit moving caliper jaws.
2. Open caliper to within a few mm of the dimension being measured.
3. Close right thumbscrew to lock position of lower horizontal knurled cylinder, which executes fine motions of caliper jaw. **Never over tighten!**

CALIBRATION:

1. Zero Error: For those using the calipers: wipe the caliper jaws with cleaning paper. Close the jaws and make a reading. Record the reading, separate the jaws, close them again, and make another reading. Continue in this way until you have recorded five readings. The mean of these readings is the *zero error*. The variation of the readings gives you an estimate of the reproducibility of caliper measurements.

For those using the micrometers: follow the same procedure above; refer to Appendix A for instruction on how to read the zero error.

2. Measure all four calibration gauge blocks (6, 12, 18 and 24 mm): Set the gauge blocks on end, well-in from the edge of the table, and thus freeing both hands to handle the caliper. Record the actual (uncorrected) reading. A single measurement of each block will suffice.
3. Plot a correction curve for your measuring device: plot errors as ordinates and nominal blocks sizes (0, 6, 12, 18, and 24 mm) as abscissa. Normally the correction will not vary from block to block by more than 0.003 mm (for the micrometer). If it is larger, consult your instructor.

DENSITY DETERMINATION:

1. Make five measurements (should be in millimeters) of the height and five of the diameter. Since our object is to determine the volume of the cylinder, distribute your measurements so as to get an appropriate average length and average diameter. Avoid any small projections which would result in a misleading measurement. If not possible to avoid, estimate their importance to the result. Record actual readings and indicate, in your lab book, how you distributed them.
2. Calculate the average length, average diameters and the respective standard deviation.
3. Use your correction curve to correct these average readings. If you were to use the uncorrected values, how much relative error would this introduce?
4. Weigh the cylinder twice on the electronic balance; estimate to 0.1 mg.

5. From the average dimensions and the mass, calculate the volume and density. Make a quantitative (refer to the Propagation of Uncertainties section on page 7) estimate of the uncertainty in the density. The sample worksheet asks for both the maximum and minimum values. You should use, as your starting point:

$$\text{Density}(\rho \pm \Delta\rho) = \rho(h \pm \Delta h, d \pm \Delta d, m \pm \Delta m) = \frac{m}{\pi(d/2)^2 h}$$

6. Compare the density with the tabulated value. Tabulated values are averages over samples whose densities vary slightly depending upon how the material was cast and worked; also on impurity concentrations.
7. Summarize the data and results in your notebook. Also record your result on the blackboard. Is the distribution of blackboard values reasonable, i.e. is it a “normal” distribution (refer to the section on page 6)?

To test how accurately you can estimate a fraction of a division, estimate the fractions on the vernier caliper before reading the vernier. Record both your estimate and the vernier reading.

Link to the world’s premier institution devoted to improving the accuracy of measurements:

National Institute of Science & Technology at <http://www.nist.gov/>

Experiment 1b Worksheet

1.

CALIBRATION TABLE		
Micrometer Serial # =	_____	
MEASUREMENTS		
Zero gap 1 =	_____	± _____
2 =	_____	
3 =	_____	
4 =	_____	
5 =	_____	
Mean ± σ =	_____	± _____
Gauge block 6 mm	_____	
12 mm	_____	
18 mm	_____	
24 mm	_____	

2. Make a plot of Gauge block thickness (x-axis) versus Micrometer reading. Next fit the data to a line; this generates a correction curve where
 Corrected value = actual × slope + intercept
 slope = _____
 intercept = _____

3. Now for the unknown value:

CYLINDER	Height (± _____)	Width (± _____)	Mass (± _____)
1=	_____	_____	_____
2=	_____	_____	_____
3=	_____	_____	_____
4=	_____	_____	_____
5=	_____	_____	_____
Mean ± σ =	_____ ± _____	_____ ± _____	_____ ± _____
Corrected Mean ± σ =	_____ ± _____	_____ ± _____	_____ ± _____
Density =	_____		
Max. Density =	_____		
Min. Density =	_____		

4. Final result, density = _____ ± _____
5. Cut and tape this into your lab notebook.
6. Answer the following questions in your notebook.
 A. Identify two sources of systematic error and give their magnitude.
 B. Identify two examples of random error and give their magnitude.

M-1c Motion, Velocity and Acceleration

OBJECTIVES:

The two objectives of this exploratory computer lab are: become familiar with the PASCO interface hardware and software; develop an intuition for Newtonian mechanics by experimenting with 1-D motion.

THEORY:

The motion of an object is described by indicating its distances x_1 and x_2 from a fixed reference point at two different times t_1 and t_2 . From the change in position between these two times one calculates the average *velocity* (remember velocity is a vector, and specifies direction) for the time interval:

$$\text{average velocity} \equiv \bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \text{ m/s}$$

The *acceleration* of an object may be calculated by finding its velocity v_1 and v_2 at two different times t_1 and t_2 . From the change in velocity between t_1 and t_2 one calculates the average acceleration for the time interval:

$$\text{average acceleration} \equiv \bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \text{ m/s}^2$$

FUNDAMENTAL CONCEPTS OF LINEAR MOTION:

1. The equation that describes the motion of an object that moves with *constant velocity* is: $x = A + B \cdot t$.

If you make a plot of x versus t , you find that it describes a straight line. The letter A indicates the position of the object at time $t = 0$. The letter B is the slope of the line, and is equal to the velocity of the object. So we can rewrite this $x = x_0 + v \cdot t$.

2. The equation that describes the motion of an object that moves with *constant acceleration* is: $x = x_0 + v_0 \cdot t + \frac{1}{2}at^2$ and $v = v_0 + at$.

So we can rewrite this as: $x = A + Bt + Ct^2$ and $v = B + 2Ct$. A indicates the position of the object at time $t = 0$. B is the the velocity of the object at time $t = 0$, and is the slope of the graph at this time. C is equal to half the acceleration.

PRECAUTIONS:

In order for the motion sensor to work properly it must be pointed in such away that it “sees” the vane, and doesn’t “see” the front of the cart; that means that it must be pointed slightly upwards. The motion sensor tends to “see” the closest reflecting surface. In addition the minimum range is approximately 20 cm.

Make sure you do not drop the carts or allow them to roll off the table. This damages the bearings and drastically increases the friction.

APPARATUS

- ⇒ A PASCO motion sensor; this device emits a series of short pulses of sound, and receives a series of echoes reflected from a nearby object. The length of the time interval between the emission and the reception of the sound pulse depends on the distance to the reflecting object.
This method of locating an object is the same as the one used by bats to find flying insects or by navy ships to locate submarines.
- ⇒ A PASCO Interface; this converts the time interval between the emission and reception of the sound pulse to digital form, i.e., numbers that can be stored by the computer. Connect yellow plug to digital channel 1.
- ⇒ PASCO dynamic track with magnetic bumpers; cart with reflecting vane; meter stick; one or two steel blocks.

PROCEDURE:

Your instructor will demonstrate how to configure the experiment. To initiate the PASCO interface software you will need to double click the “Experiment M-1c” file in the PASCO/Physics201 folder on the desktop. Fig. 1 below shows how the display should appear. All three measured quantities, position, velocity, and acceleration, are displayed simultaneously. Since velocity is determined from the position data and acceleration from the velocity the “scatter” in the data will become progressively more pronounced.

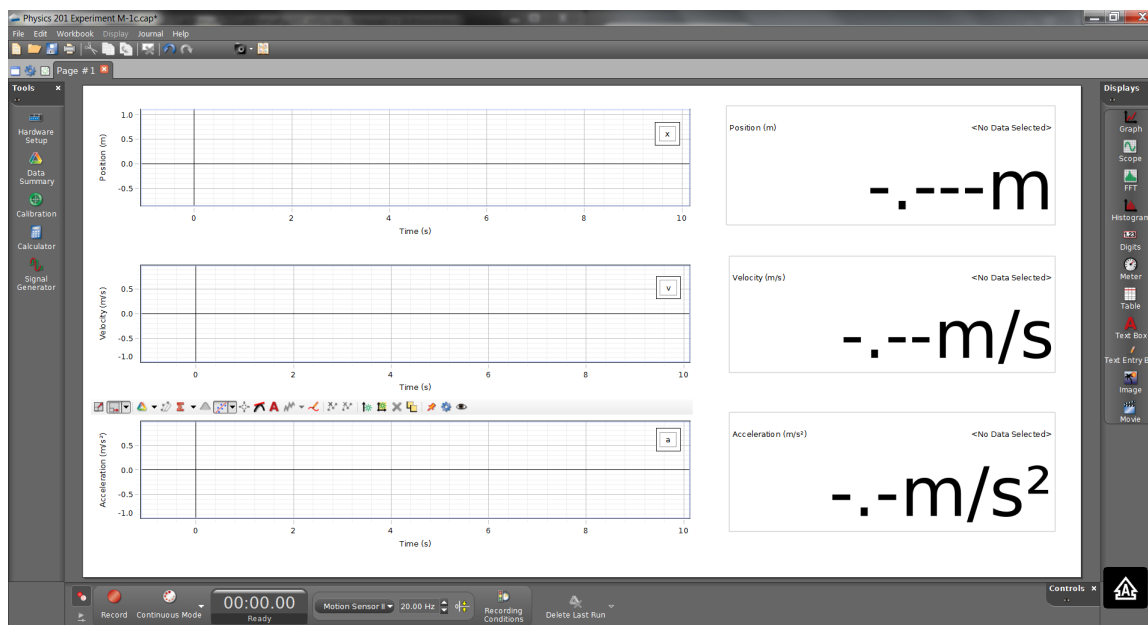











Figure 1: The PASCO Capstone display format

EXPERIMENT I: BASIC OPERATION AND MOTION SENSOR CALIBRATION:

1. To start the data acquisition click on the Record icon . To stop it click on the same icon which will now become the Stop icon . Each run gets its own data set in the Data Summary  tab under the **Tools** Palette. If there are any data sets

in existence you will *not* be able to reconfigure the interface parameters or sensor inputs, unless you clear (delete) all data.

2. With the data acquisition started move the cart *to* and *fro* and watch the position, velocity and acceleration displays.
3. Stop the data acquisition.
4. Click on the Zoom-to-Fit icon  on one of the plots. Click on the Data Highlight icon  and the Zoom-to-Fit icon  to zoom in to particular regions of interest. Use Ctrl-Z to get back to the original plot area. Click on the Crosshair icon  and move about within a plot window. Click on the Statistics icon  to bring up statistics for a given set of data. Make sure that all members of the group have an opportunity to test these components. It will facilitate the rest of the course if the basic operations on the software interface are understood by everyone!
5. Delete the data set by clicking on the red 'X' next to RUN #1 item in the Data Summary  tab under the **Tools** Palette.
6. Start the data acquisition and find the closest distance to the motion sensor at which it still functions. This value is supposed to be close to 20 *cm*. If it is much larger, re-aim the motion sensor.
7. Configure the distances so that when the cart nearly touches the near magnetic bumper the motion sensor still records accurately.
8. Measure the position at two distances approximately 0.8 *m* apart and compare the printed centimeter scale with the motion sensor readout. By how much do the readings differ?

EXPERIMENT II: INCLINED PLANE AND MOTION:

1. Raise the side of the track closest to the position sensor using one of the supplied blocks.
2. Find an appropriate release point that allows the cart to roll down the track slowly enough that it bounces off the magnetic bumper *without* striking it.
3. Click the Record button and release the cart, letting it bounce three or four times, and then click the Stop button.
4. Qualitatively describe the shape of the three curves: position, velocity and acceleration and discuss how they evolve with time.
5. Obtain a hard copy of this data by pressing **CTRL-p**.
6. Label/identify the various key features in the various curves by writing directly on the hard copies. Paste the graphs in your note book.

QUESTIONS: (to be discussed as a group)



1. Does the velocity increase or decrease linearly with time when the cart is sliding up/down the track?

2. When the velocity is close to zero can you observe any nonlinear oscillations in the data? Can you think of a reason for any deviations from linearity?
3. Why does the cart bounce less high with each subsequent bounce?
4. Is the collision with the magnetic bumper or the residual friction in the bearing the most likely source of loss?
5. Can you think of a method using your data to determine which of these two proposed mechanisms is the most likely culprit?

EXPERIMENT III: MOVING THE CART BY HAND:

1. Remove the block so that the track is no longer raised.
2. Start a new data set (by clicking the Record button) and try to move the cart back and forth at constant velocity using your hand from the side.
3. Using the cross hair estimate the velocity in either m/s or cm/s .
4. Repeat step 2, but now try to obtain (for a limited time) a constant acceleration.

EXPERIMENT IV: ACCELERATION AT g ($9.8 m/s^2$):

1. Have one member of the group stand *carefully* on a chair and hold the position sensor facing downward above a second member's head, while the second member holds a notebook above his or her head.
2. Start a new data set and have the student holding the notebook jump up and down a few times.
3. Stop the recording and determine whether free-fall yields a constant acceleration close to the accepted value.
4. Try to measure the acceleration by making fits to data. First use the Data Highlight icon  to choose a range of data. Then click on the Fit icon  to get at the pull-down menu. You have three choices: fit a quadratic function to the position data, fit a straight line to the velocity data, or find the mean of the acceleration. Which method seems least accurate? Which seems most accurate?

M-4 Acceleration in Free Fall

OBJECTIVE: To measure g , the acceleration of gravity; to propagate uncertainties.

APPARATUS:

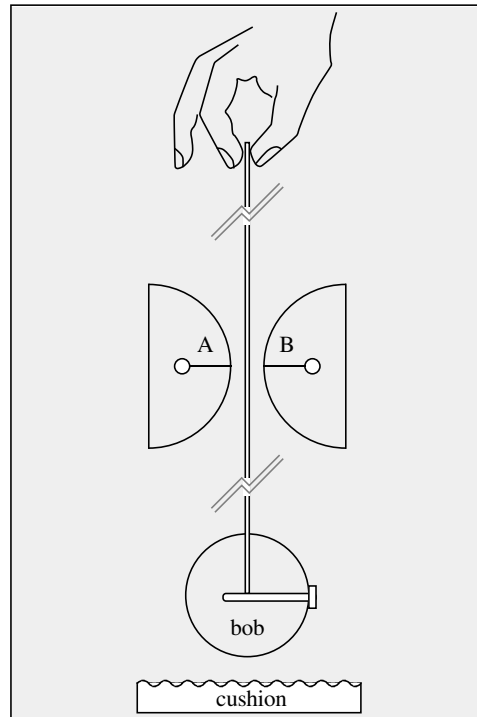
Free fall equipment: cylindrical bobs (identical except in mass) which attach to paper tape for recording spark positions; spark timer giving sparks every 1/60 second; cushion; non-streamline bobs to study air resistance (by adding a front plate).

INTRODUCTION:

As shown in the figure below, the spark timer causes sparks to jump from sharp point A (flush with the convex surface of the plastic insulator) through the falling vertical paper tape to the opposite sharp point B flush with the other insulator surface. As the bob (plus paper tape) undergoes free fall, sparks from A to B mark the paper tape's position every 1/60 second. These data give the bob's acceleration in free fall, g , if air resistance is negligible. Air resistance will be considered later.

EXPERIMENTAL SUGGESTIONS:

1. Position spark timer chassis near table edge and with the convex surfaces (A and B) extended beyond table edge. Put cushion on floor directly under A and B.
2. Select the heaviest bob (no front plate). Insert one end of about a meter of paper tape between the two halves of the cylindrical bob and fasten together with thumb screw.
3. Insert paper tape between A and B. Hold the tape end high enough (vertically) above A and B that the bob just touches below A and B (and is centered). Start the spark and immediately release tape plus bob. Discard any part of tape which fell through the spark gap after bob hit the cushion.



4. Fasten sparked tape to table top with masking tape. Place a meter stick on its side (on top of tape) so that ends of the mm graduations touch the dot track on the tape; this minimizes any parallax error (see Appendix B).
5. **Ignore** the first spark dot; then mark and measure the position of **every other** of the first 24 dots, thus using 1/30 s as the time interval instead of 1/60 s. **Estimate the dot positions to 0.1 mm, and assume this is your uncertainty, $\delta_r = 0.1$ mm. Don't move the meter stick between readings!** Tabulate as in sample below.
6. Check the measurement set by remeasuring the tape **after moving the meter stick so that the recorded dot positions will be different**. The differences should be the same. How close are they?

EXAMPLE OF TABULAR FORM FOR DATA:

Dot i	Time $t(i)$ sec	Position $r(i)$ cm	Average velocity $\bar{v}(i)$ cm/sec	Average acceleration $\bar{a}(i)$ cm/sec ²
	$\delta_t =$	$\delta_r =$	$\delta_v =$	$\delta_a =$
0	$t(0)$	a	—	—
1	$t(1)$	b	$b - a$	$(c - b) - (b - a)$
2	$t(2)$	c	$c - b$	$(d - c) - (c - b)$
3	$t(3)$	d	$d - c$	$(e - d) - (d - c)$
4	$t(4)$	e	$e - d$	$(f - e) - (e - d)$
5	$t(5)$	f	$f - e$	$(g - f) - (f - e)$
6	$t(6)$	g	$g - f$	$(h - g) - (g - f)$
7	$t(7)$	h	$h - g$	$(i - h) - (h - g)$
8	$t(8)$	i	$i - h$	$(j - i) - (i - h)$
9	$t(9)$	j	$j - i$	$(k - j) - (j - i)$
10	$t(10)$	k	$k - j$	$(l - k) - (k - j)$
11	$t(11)$	l	$l - k$	$(m - l) - (l - k)$
\vdots	\vdots	\vdots	\vdots	\vdots

SUGGESTIONS ON HANDLING DATA:

1. Analysis may be conveniently done using Excel.
2. Record your estimates of the uncertainties in measurement: δ_t , the uncertainty in the times of the sparks; δ_r , the uncertainty in the positions of the dots on the paper.
3. Let $t = 0$ represent where your readings start (the actual starting value is arbitrary). Tabulate the readings, $r(i)$, on the meter stick at the end of each time interval as in column 3 above.
4. Calculate the average velocity, \bar{v} in each interval; then calculate the average acceleration, \bar{a} during each interval, as shown in the table.
5. Calculate the uncertainty in each measurement of \bar{v} and \bar{a} by using the appropriate technique described in “Propagation of Uncertainties” on page 7, and record it in your table.
6. Define $\langle \bar{a} \rangle$ to be the average of all the readings in the “average acceleration” column. Find $\langle \bar{a} \rangle$. This “average of the average” is more accurate than fluctuations in the \bar{a} column might indicate. Explain why. (Hint: Are the values independent? See suggestion 8, below.)
7. With 1/30 s as the time interval, plot r vs time, \bar{v} vs time, and \bar{a} vs time. Note that for \bar{v} the value is the average of v between to adjacent rows, i and $i + 1$, and therefore equals the **instantaneous** v at the *middle* of the interval.
8. Plot velocity as a function of time. Fit a straight line to the plot; equate the slope of the fit to the acceleration, and the uncertainty in the slope to the uncertainty in the acceleration. This approach is a better way of using all the data than the above numerical one since averaging the \bar{a} column involves summing the \bar{a} column and in such a sum all readings except $(b - a)$ and $(m - l)$ drop out!

ERROR ANALYSIS:

Spark timing errors are negligible. However, spark position errors are not. The spark in the spark timer does not follow the same path every time. Assume the deviation of the path of a particular spark from the mean spark path is characterized by $\sigma = 0.5$ mm. Thus the computed value of $(b - a)$ would be subject to an error ~ 0.7 mm. This is in excess of the 0.1 mm measurement uncertainty.

However, successive measurements of v are correlated: since $v(1) \propto b - a$ and $v(2) \propto c - b$, a fluctuation in b will always push $v(1)$ and $v(2)$ apart. This in turn means the uncertainty calculated for the slope of \bar{v} will be larger than it really is.

Calculate the slope using only every other $v(i)$ because $v(1)$, $v(3)$, $v(5)$ etc. use independent data. Estimate the uncertainty in the slope. Compare to the uncertainty in the slope calculated in step 8 above.

Calculate g , the free-fall acceleration of the bob, using whichever method gives the smallest uncertainty, and record both g and the uncertainty.

QUESTION:

The local value of g has been measured to be 9.803636 ± 0.000001 m/s².¹ Is the discrepancy between this value of g and your value of g larger or smaller than the uncertainty in your value of g ?

OPTIONAL EXPERIMENTS

A. EQUIVALENCE OF GRAVITATIONAL AND INERTIAL MASS:

Galileo showed (crudely) that the acceleration of falling bodies was independent of the mass. Check Galileo's experiment by measuring the free-fall acceleration of the light plastic bob (identical in size and shape to the heavy metal bob), and comparing it to the free-fall acceleration of the heavy metal bob.

Is the difference between the two measurements of the free-fall acceleration larger than the uncertainties in both, or smaller than the uncertainty in either, or somewhere in between?

B. EFFECT OF AIR RESISTANCE:

The effect of air resistance is small for streamlined objects at low velocities, but can become large for non-streamlined objects, such as a parachute. To quantify it, first make both bobs non-streamlined by inserting into the bottom of the bob the banana plug holding a small flat plate. Then repeat the experiment for the non-streamlined bobs. Since the force of air resistance, $f(v)$, is a function of only velocity if the size, shape and roughness are the same, then $f(v)$ on the two bobs will be almost the same since their velocities are similar. The net force on the falling body is then $F = mg - f(v)$. Hence

$$a = \frac{F}{m} = \frac{mg - f(v)}{m} = g - \frac{f(v)}{m}$$

¹Wollard and Rose, "International Gravity Measurements", UW Geophysical and Polar Research Center, (1963) p. 211 and p. 236.

and

$$\langle a \rangle \approx g - \left(\frac{1}{m} \right) \langle f(v) \rangle .$$

Thus if we measure $\langle a \rangle$ for bobs identical except in mass and plot $\langle a \rangle$ against $1/m$, we should obtain a straight line. The slope of this line is the force $\langle f(v) \rangle$ due to air resistance, and the extrapolation of the line to $(1/m) = 0$ gives g .

To improve the accuracy of your measurements of $\langle f(v) \rangle$ and g , pool your data with other lab groups.

C. MEASUREMENT OF REACTION TIME BY FREE FALL:

- (1) Have your partner hold the vertical meter stick while you place your thumb and forefinger opposite the 50 cm mark but not grasping it. When your partner releases the stick, grab it as soon as possible. From the distance through which the 50 cm mark fell, calculate the time of free fall of the meter stick. This is your “reaction time”.
- (2) With thumb and forefinger, grasp a vertical meter stick at the 50 cm mark. Release and grab it again as quickly as possible. From the distance through which the 50 cm mark fell, calculate the time of free fall of the meter stick. This is your “time scale for coordinated movement”. Compare this time to your reaction time. Why might the two be different?

OPTIONAL QUESTIONS:

1. Estimate the effect on your value of g of the air’s buoyant force, F_a . For air density, use $\rho = 1.2 \text{ kg/m}^3$ and for brass density (bob), $\rho_{\text{bob}} = 8700 \text{ kg/m}^3$.

Hints: $F_a = \rho_a m_{\text{bob}} g / \rho_{\text{bob}}$ (why?); $a = g[1 - (\rho_a / \rho_{\text{bob}})]$ (why?)

2. According to universal gravitation, the moon accelerates the bob with a value a_m of
$$a_m = \frac{GM_m}{r^2} = \frac{6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \text{ kg}(7.34 \times 10^{22} \text{ kg})}{(3.84 \times 10^8 \text{ m})^2} = 0.000033 \text{ m/s}^2$$

where G is the constant of universal gravitation, M_m is the mass of the moon, and r is the distance between the moon and the bob. Since this a_m is 33 times the uncertainty quoted for the local g value, why doesn’t the measurement indicate the position of the moon at the time of measurement?

Hint: Remember the acceleration g in an orbiting earth satellite provides the centripetal acceleration for the circular motion but does not appear as “weight” of an object in the satellite. While to first order the moon and sun effects are negligible, there are detectable tidal effects in the earth ($\approx 10^{-7} g$) which one corrects for in the absolute measurements. See Handbuch der Physics, Vol XLVIII, p. 811; also Wollard and Rose, p. 183.

H-3 Latent heat of vaporization of liquid-N₂

OBJECTIVE:

To measure the heat of vaporization of liquid nitrogen, L_v , at its boiling point ($T_b = 77$ K at standard atmospheric pressure).

APPARATUS:

Dewar flask; liquid nitrogen (ask for help in getting it from a large storage Dewar in room 4329 Chamberlin Hall); aluminum cylinder on a long thread; double pan balance; calorimeter plus water jacket for thermal ballast (as in H-2a); timer; thermocouple-type digital thermometer; selection of slotted masses; coffee pot for hot water.

PRECAUTIONS:

Liquid N₂ is fascinating to work with. However, keep in mind the following simple safety precautions.

1. **Never stopper a flask of liquid N₂ with an unperforated stopper. This can result in a dangerous explosion with risk of major injury. Students found attempting this will be referred to the Dean of Students for disciplinary action.**
2. Have a perforated stopper on the Dewar throughout the experiment to prevent condensation of moisture from the air on the inside of the flask.
3. **Avoid prolonged contact of liquid N₂ with your skin.** The insulating vapor layer may disappear and severe frost-bite may result.

INTRODUCTION:

When one lowers an aluminum cylinder of mass m_{Al} , that is at room temperature, T_r , into liquid N₂ at its boiling temperature, T_b , the cylinder cools to T_b . The heat given off during this cooling, Q_{Al} , will vaporize a mass m_N of liquid N₂.

You might expect to find L_v , of the nitrogen by setting

$$m_N L_v = Q_{Al} = m_{Al} c_{Al} (T_r - T_b) .$$

This method fails because c_{Al} is not constant over the $\sim 220^\circ\text{C}$ temperature range between T_r and T_b . See figure 1.

We can avoid this difficulty by noting that Q_{Al} is also the heat needed to warm the same aluminum cylinder to from T_b to T_r . You can measure this heat by placing the cold aluminum cylinder (at temperature T_b) in a "calorimeter" that contains water and observing the change in temperature of the water, $-\Delta T$, -provided that the final temperature of the water, T_f , is room temperature, T_r . It is hard to arrange for T_f to end up exactly at room temperature, but if T_f is *close* to T_r , one can accurately correct the calorimeter data for the small additional heat term, namely $m_{Al} c_{Al} (T_r - T_f)$, since over the small $T_r - T_f$ interval, $c_{Al} = 0.212$ cal/g/K is constant.

SUGGESTIONS ON PROCEDURE:

1. To maximize sensitivity (i.e. to get a large temperature change in the water) use only enough water (~ 125 - 150 grams) in the calorimeter to cover the metal cylinder.

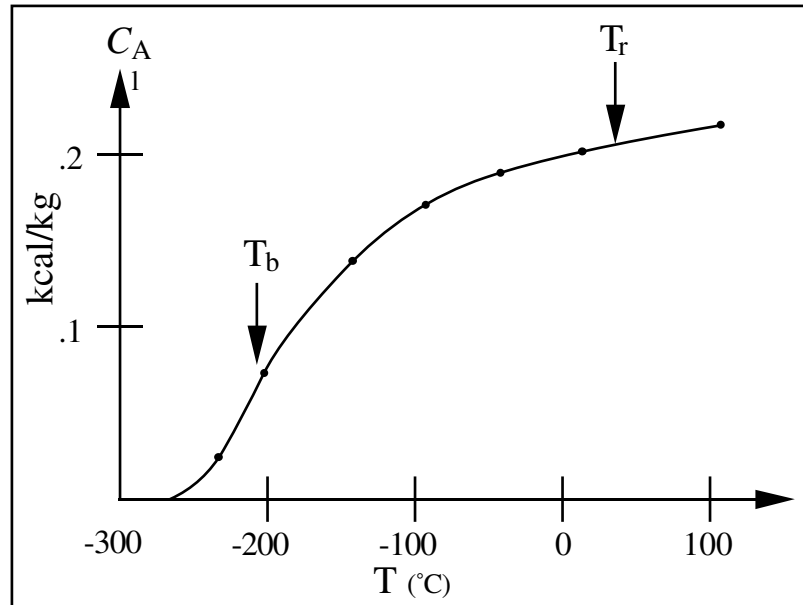
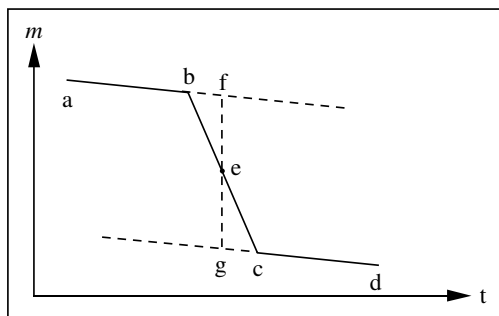


Figure 1: Specific heat capacity of Al vs. temperature.

2. The calorimeter is designed to thermally isolate the water from the surroundings. The inner vessel of the calorimeter is mounted within, but thermally isolated from, a surrounding water jacket, which is close to room temperature. The isolation isn't perfect, so there will be some small amount of heat flow between the calorimeter and the jacket. To minimize the *net* heat exchange with the water jacket, you will want to make the initial water temperature as far *above* the jacket temperature as you expect it to end up *below* the jacket temperature after the water in the inner vessel has been cooled by your Al cylinder. (The jacket temperature will remain fairly constant during the experiment.) Estimate roughly the proper initial water and calorimeter temperature. [Use the measured mass of the aluminum cylinder, m_{Al} , its specific heat (0.212 kcal/kg°C) and the b.p. of liquid N₂, $T_b = 77$ K. For this rough calculation, assume that the specific heat of Al *is* constant with temperature.] Determine the water mass, m_w , and appropriate starting temperature.
3. Place the flask containing liquid nitrogen (plus perforated stopper) on one pan of a balance and record the mass each minute for 10 minutes. (Why is the mass decreasing?)
4. Record the temperature of the metal cylinder, T_R , and then lower it (by a thread) gently to the bottom of the flask. Replace the stopper (perforated) on the top of the flask and continue recording the total mass each minute until it shows a slow steady decrease.
5. Record the initial temperature, T_i , of the calorimeter, which you have chosen so cleverly in section 2. Transfer the cold metal cylinder into the calorimeter, and note the calorimeter temperature every two minutes (while gently stirring). Record the mass of the flask of liquid N₂ on alternate minutes. When the calorimeter temperature has reached a slow steady rate of change and the mass of the flask of liquid N₂ is falling at a slow steady rate, discontinue the readings.

6. Plot the mass of the flask plus nitrogen as a function of time. For the minutes that the cylinder of metal was in the flask, subtract the mass of the cylinder. See the figure for a typical plot.



How much of the N₂ mass change was caused by the heat from the cylinder? If a-b and c-d were parallel, it would be the vertical distance between these lines. But c-d ordinarily has a smaller slope than a-b, possibly because the evaporation of liquid nitrogen between b and c has cooled the upper part of the flask.

Hence we use the average of the two rates of fall by drawing a vertical line through e, (the midpoint of line b-c). Then f-g estimates the mass, m_N , evaporated by the heat from the cylinder.

7. The final temperature, T_f , of the Al cylinder in the calorimeter usually will not be quite the same as the initial temperature ($= T_r$) of the cylinder before it was lowered into the liquid nitrogen.

Hence Q_{Al} will be the heat to warm the Al cylinder in the calorimeter to T_f plus the mass of the cylinder \times (specific heat of aluminum) \times ($T_r - T_f$).

Specifically if:

$$\begin{aligned}
 L_v &= \text{latent heat of vaporization of nitrogen} \\
 m_N &= \text{mass of nitrogen evaporated by heat from the cylinder} \\
 m_{Al}, c_{Al} &= \text{mass and specific heat of Al cylinder} \\
 T_i &= \text{initial temperature of water and calorimeter} \\
 m_w, c_w &= \text{mass and specific heat of water} \\
 m_c, c_c &= \text{mass and specific heat of calorimeter and stirrer} \\
 h_t &= \text{heat capacity of immersed part of thermometer,}
 \end{aligned}$$

then, if we neglect h_t :

$$Q_{Al} = m_N L_v = (m_w c_w + m_c c_c)(T_i - T_f) + m_{Al} c_{Al}(T_r - T_f) .$$

Calculate L_v from the above relation. The accepted value is 47.8 kcal/kg.

OPTIONAL:

1. Calculate the **apparent** specific heat of the Al block by use of

$$c_{Al} = \frac{Q}{m_{Al}(T_r - T_b)}$$

and your data. How does your result compare with the accepted value of $c_{Al} = 0.212$ kcal/kg? Explain. (See Introduction).

2. Observe (but do not touch) the following items after immersion in liquid N₂: rubber (get a piece from the instructor), pencil eraser.
3. Pour a little liquid N₂ onto the floor. Explain the behavior of the small droplets of liquid N₂.

M-11b Air Track Collisions

OBJECTIVE: To study conservation of energy and momentum in collisions.

APPARATUS:

Air track; assorted slotted masses; air supply; hose; gliders; photogates & support stands; PASCO interface and computer.

PRECAUTIONS: *The soft aluminum gliders and track surfaces damage easily: Don't drop!*

With the air pressure on, use a glider to check that the track is level, and free of high friction areas from scratches or plugged air holes.

All collisions must be free from any glider contact with the rail. In general speeds which are too slow are overly influenced by residual friction and air track leveling errors. On the other hand, speeds which too high invariably cause pitch and yaw motions of the gliders which increase the likelihood of physical contact with the track. You should perform a number of preliminary trials to discern which speeds work best.

Good alignment of the needle assembly is a necessity.

SUGGESTIONS:

1. Use the large gliders whenever possible. Small ones often tilt upon impact and hence give excessive friction.
2. Turn on the air supply and experiment with the gliders. Adjust the leveling screw so that the nominal cart acceleration is minimized. Adjust the air flow so the gliders move freely without rocking side to side.
3. Make sure that the photogates are plugged into the first two phone jack inputs in the PASCO interface module. Also set the two photogates approximately 40 to 50 cm apart and so that they track the 10-cm-long stripe in the picket fence.
4. Start the PASCO software by double-clicking "Experiment M11" in the PASCO/Physics 201 folder on the desktop. The monitor should now look as shown in Figure 1 below. Be sure to check the "Flag Length" constant for BOTH photogates in the setup window. It should be set to 10 cm.
5. To start the data acquisition, click Record. To stop it, click Stop. Each time a glider passes the photogate an entry will appear in the appropriate table column. NOTE: The photogates measure the speed but do NOT sense the direction of motion. You are responsible for the latter.
6. The results of this experiment are very technique-sensitive. Take a single glider and practice sending it through both photogates until you are able to get reasonably equivalent speed readings from both photogates. Check the behavior by launching the glider from both ends and record, in your lab book, the speed measured in four or five satisfactory trials. Do not delete the data, because it will be useful in the "Measurement of Friction" section later. Estimate the precision associated with a pair of velocity measurements, and show how this will affect your momentum and kinetic energy measurements.

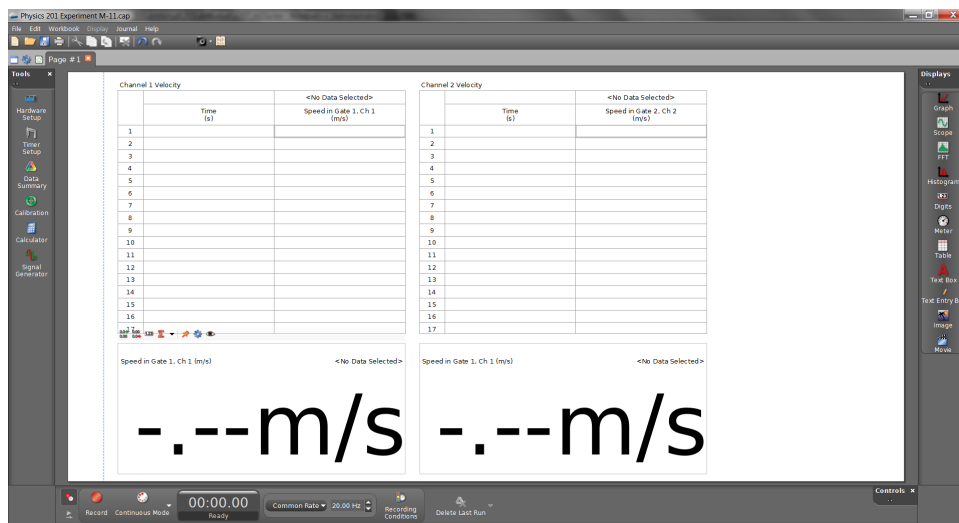


Figure 1: The Pasco Capstone display for M11.

EXPERIMENT I: ELASTIC COLLISIONS, ONE CART INITIALLY AT REST

1. Choose gliders of equal mass (or make them approximately the same by fastening weights on one). Set them on the track as in the figure below. Read step 2, and predict what will happen before attempting.
2. With glider #1 at the end of track and glider #2 at rest near the center, give #1 a push toward glider #2. After the collision, stop glider #2 before it bounces back.

Perform multiple trials until you achieve consistent results and record a few of them. Check conservation of momentum and energy in the impact. In equation,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2,$$

call velocities to the right positive, those to the left negative.

3. Comment on how well these two quantities are conserved and, if your results seem poorer than expected, suggest possible sources of error.
4. Devise a method for determining how elastic is a rubber band collision and record your results.

Suggested tabulations:

Glider:	#1	#2	Velocity Readings:	#1	#2
mass	Before impact (u)				
length	After impact (v)				

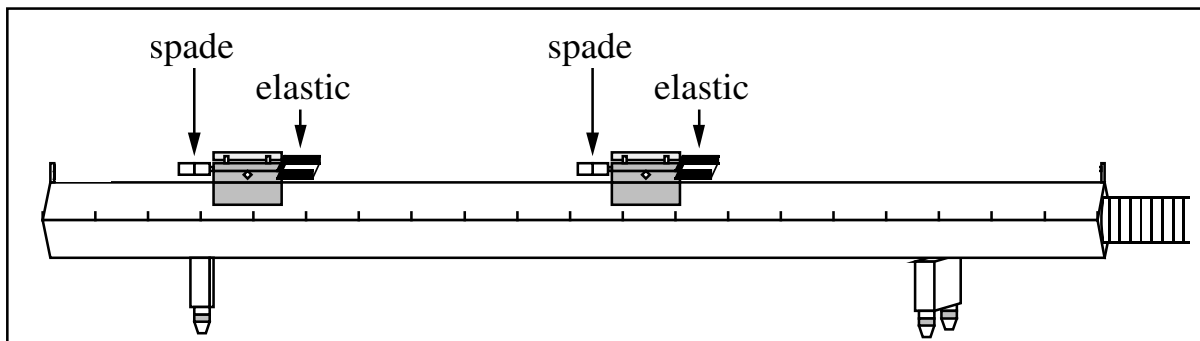


Figure 2: Sketch of the glider configuration for elastic collisions.

	<u>Before Impact</u>			<u>After Impact</u>			
$u_1 =$	_____	$u_2 =$	_____	$v_1 =$	_____	$v_2 =$	_____
$m_1 u_1 =$	_____	$m_2 u_2 =$	_____	$m_1 v_1 =$	_____	$m_2 v_2 =$	_____
$\frac{1}{2} m_1 u_1^2 =$	_____	$\frac{1}{2} m_2 u_2^2 =$	_____	$\frac{1}{2} m_1 v_1^2 =$	_____	$\frac{1}{2} m_2 v_2^2 =$	_____

change in momentum = _____; % change in momentum = _____.

change in energy = _____; % change in energy = _____.

EXPERIMENT II: ELASTIC COLLISIONS, BOTH CARTS INITIALLY MOVING

Perform the same procedure as Exp. I (steps 1, 2 and 3), except start both gliders from opposite ends of the air track and with considerably different (but not too large or too small) velocities.

EXPERIMENT III: INELASTIC COLLISIONS

Repeat Exp. II but for inelastic collisions by attaching cylinders with needle and wax inserts. Note that the needle must line up exactly with the insert or there will be significant sideways motion when the two gliders strike.

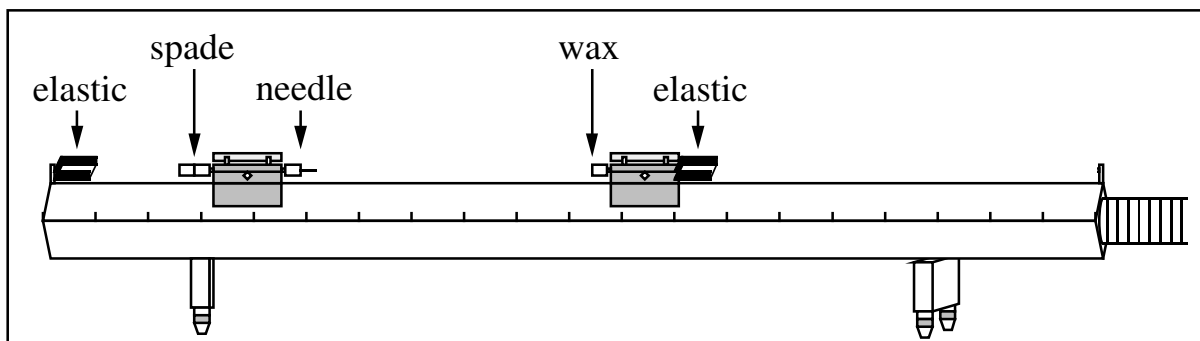


Figure 3: Sketch of the glider configuration for inelastic collisions.

EXPERIMENT IV:

Increase m_1 by adding masses and repeat Exp. I.

MEASUREMENT OF FRICTION:

Estimate the frictional force between the glider and track by using the velocity data recorded before Exp. I. From any net decrease in velocity you should be able to obtain the frictional force. Does this information help you understand the experimental data?

M-7 Simple Pendulum

OBJECTIVES:

1. To measure how the period of a simple pendulum depends on amplitude.
2. To measure how the pendulum period depends on length if the amplitude is small enough that the variation with amplitude is negligible.
3. To measure the acceleration of gravity.

APPARATUS:

Basic equipment: Pendulum ball, with bifilar support so ball swings in a plane parallel to wall; protractor; infrared photogate and mount; single pan balance.

Computer equipment: PASCO interface module; photogate sensor (plugged into DIGITAL input #1).

NOTE: The period is the time for a complete swing of the pendulum. For the most sensitivity the start and stop of the photogate timer should occur at the bottom of the swing where the velocity is maximum.

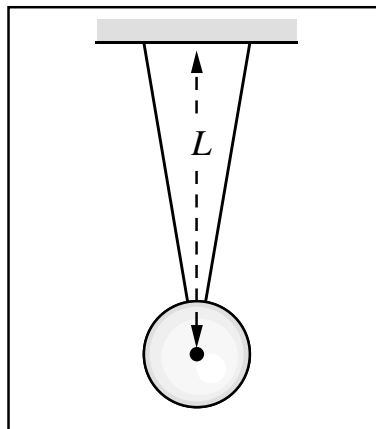




Fig. 1: Side view of pendulum ball and bifilar support.

SUGGESTED EXPERIMENTAL TECHNIQUE:

1. Adjust the infrared photogate height so that the bob interrupts the beam at the bottom of the swing. (Make sure the PASCO interface is on and that the phone plug is connected to Digital Input 1.) The red LED on the photogate lights up when the bob interrupts the beam. Rotating the photogate may help you to intercept the bob at the bottom of the swing (but perfect alignment is not essential).
2. To initiate the PASCO interface software double click the “Experiment M-7” file in the PASCO/Physics201 folder on the desktop. There will be just a single table for recording the measured period.
3. Start the pendulum swinging and then start the data acquisition by clicking the Record icon . Let the bob swing for about 17 periods. Click on the Statistics icon , and view the statistics the computer calculates. Does it calculate the standard deviation, or the standard deviation of the mean? See the “Characterization of Uncertainties” section on page 5 for the distinction between the two. Record the mean, the standard deviation, and the standard deviation of the mean.

REQUIRED INVESTIGATIONS:

1. Period *vs* Length: The period of the pendulum for small oscillations (for θ in radians, $\theta \ll 1$) is $T = 2\pi\sqrt{L/g}$, where L is the length from the support to the center of the bob.

For small oscillations, measure how T changes with L . Try five lengths from 0.10 m to 0.50 m. Change lengths by loosening the two spring-loaded clamps above protractor. Make sure to note θ for each length.

Plot the square of the measured period, (T^2), versus length (L), and extend the curve to $L = 0$. Add error bars. Plot the theoretical prediction that $T^2 = (2\pi)^2 L/g$ on the same graph. Is the discrepancy between measurement and prediction less than the uncertainty of the measurement?

2. Measurement of g : With pendulum length at its maximum, measure L and T for small-amplitude oscillations. Determine g , the acceleration of gravity.

Calculate the uncertainty in your determination of g . Note that $\Delta g = g \sqrt{(\Delta L/L)^2 + (2\Delta T/T)^2}$ (see the “Errors and Uncertainties” section of this manual).

Is the accepted value of $g = 9.803636 \pm 0.000001 \text{ m/s}^2$ within the uncertainty of your measurement?

See the “optional calculations” section below for possible real-life effects that one must take into account if one wants to be as accurate as possible

3. Period *vs* Amplitude: For a pendulum of length $L = 0.5 \text{ m}$, determine the dependence of period on angular amplitude. Use amplitudes between 5 and 50 degrees. Be careful when measuring the angle: to avoid parallax (see Appendix B on page 55) effects, position your eye so the two strings are aligned with each other when reading the protractor.

The amplitude of the swing will decrease slowly because of friction, so keep the number of swings that you time small enough that the amplitude changes by well under 5° during the timing. This effect is especially apparent at large amplitudes.

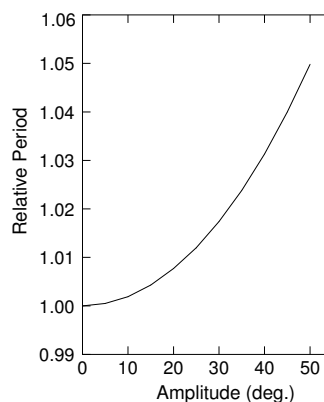
For each group of swings timed, record the average angular amplitude and the standard deviation in a table in your lab book.

The theoretical prediction for the period of an ideal pendulum as a function of amplitude θ is:

$$T = T_0 \left(1 + \frac{1}{2^2} \sin^2 \frac{\theta}{2} + \frac{1(3^2)}{2^2(4^2)} \sin^4 \frac{\theta}{2} + \dots \right)$$

where $T_0 = 2\pi\sqrt{(L/g)}$ and θ is the angular amplitude. This gives:

$\theta(\text{deg})$	T/T_0	$(T - T_0)/T_0$
5	1.0005	0.05
10	1.0019	0.19
15	1.0043	0.43
20	1.0077	0.77
25	1.0120	1.20
30	1.0174	1.74
35	1.0238	2.38
40	1.0313	3.13
45	1.0400	4.00
50	1.0498	4.98



Plot your measurements of period as a function of angular amplitude. Include error bars. Plot the theoretical prediction on the same graph. Is the discrepancy between measurement and prediction less than the uncertainty of the measurement?

- Compound Pendulum: There is a bar that you can swing into place which will give half the swing at length $L_1 + L_2$ and the other half of the swing at length L_2 . The bar should be positioned so the right face just touches the string when the pendulum is at rest and hangs freely. In your lab book first estimate the period of the motion (explain your logic) and then conduct the experiment (stating the steps in your experiment). Is your measured value close to what you expected?

OPTIONAL CALCULATIONS (these pertain to item 3 above):

- Show that the buoyant force of air increases the period to

$$T = T_0 \left(1 + \frac{\rho_{\text{air}}}{2\rho_{\text{bob}}} \right)$$

where T_0 is the period in vacuum and ρ is the density. Test by swinging simultaneously two pendula of equal length but with bobs of quite different densities: aluminum, lead, and wooden pendulum balls are available. (Air resistance will also increase T a comparable amount. See Birkhoff, "Hydrodynamics," p. 155.)

- The finite mass of the string, m , decreases the period to

$$T = T_0 \left(1 - \frac{m}{12M} \right)$$

where M is the mass of the bob (S.T. Epstein and M.G. Olsson, *American Journal of Physics* 45, 671, 1977). Correct your value of g for the mass of the string.

- The finite size of a spherical bob with radius r increases the period slightly. When L is the length from support to center of the sphere, then the period becomes (see Tipler, "Physics" 4nd ed. p. 436, problem 67):

$$T = T_0 \sqrt{1 + \frac{2r^2}{5L^2}} \simeq T_0 \left(1 + \frac{r^2}{5L^2} \right).$$

What is the resulting percent error in your determination of " g "?

NOTE: For a comprehensive discussion of pendulum corrections needed for an accurate measurement of g to four significant figures, see R. A. Nelson and M. G. Olsson, *American Journal of Physics* 54, 112, (1986).

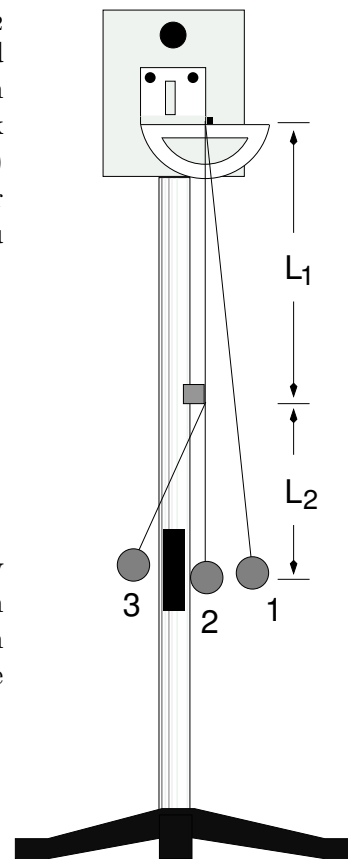


Fig. 2: A compound pendulum.

M-5 Projectile Motion

OBJECTIVE: To find the initial velocity and predict the range of a projectile.

APPARATUS:

Ballistic pendulum with spring gun and plumb bob, projectile, single pan balance, elevation stand.

PART I. BALLISTIC PENDULUM INTRODUCTION:

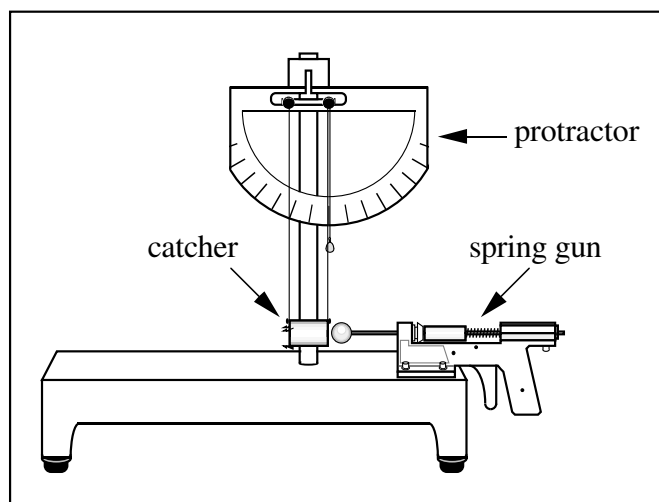


Figure 1: The spring gun.

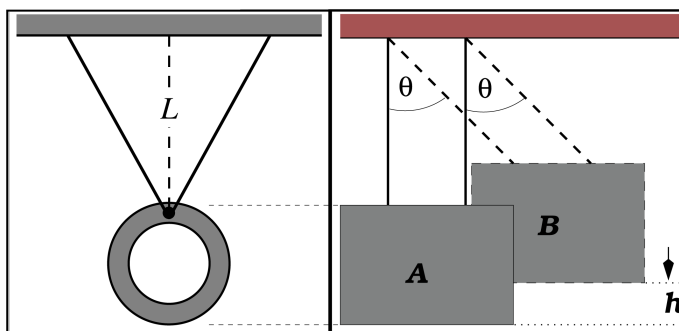


Figure 2: A side view of the catcher

A ballistic pendulum is a device commonly used to determine the initial velocity of a projectile. A spring gun shoots a ball of mass m into a pendulum catcher of mass M (See Figures 1 and 2). The catcher traps the ball; thereafter the two move together. Linear momentum is conserved, so the momentum of ball before impact equals the momentum of ball plus catcher after impact:

$$mu = (m + M)V \quad (1)$$

where u = ball's velocity before impact and V = initial velocity of combined catcher plus ball.

To find V , note that motion **after** impact conserves mechanical energy. Hence the kinetic energy of the ball plus catcher at A in Fig. 2, just after impact, equals the potential energy of the two at the top of the swing (at B). Thus

$$\frac{1}{2}(M + m)V^2 = (M + m)gh . \quad \text{Hence,} \quad V = \sqrt{2gh} . \quad (2)$$

ALIGNMENT:

If properly aligned, the suspension for the pendulum (see Figs. 1 and 2) prevents rotation of the catcher. The motion is pure translation. To ensure proper alignment, adjust the three knurled screws on the base so that

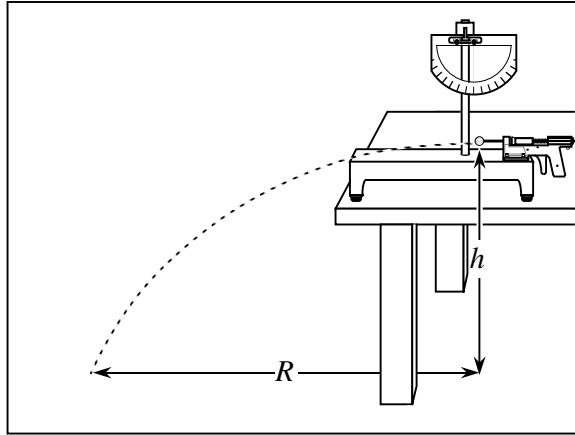
- a. the plumb bob hangs parallel to the vertical axis of the protractor;
- b. the uncocked gun axis points along the axis of the cylindrical bob. You may need to adjust the lengths of the supporting strings.

PROCEDURE:

1. Measure m , M and the length L of the pendulum (see Figure 2).
2. Cock the gun and fire the ball into the catcher. Record the maximum angular deflection of the pendulum. Repeat your measurements until you are confident in your result to within one degree.
3. Find the maximum height $h = L - L \cos \theta$ of the pendulum.
4. Calculate the initial velocity of the ball, u , as it leaves the gun.
5. Estimate the uncertainty in u . Hint: Since the largest uncertainty is likely ΔV , then Δh is important. While h is a function of the measured L and θ , the uncertainty in the angle measurement, $\Delta \theta$, will probably dominate. Estimate the uncertainty in u by calculating u for $\theta + \Delta \theta$ and for $\theta - \Delta \theta$. Remember to use absolute, not relative errors when propagating errors through addition (see "Propagation of Uncertainties" on page 7). Also make sure to use θ in radians.

PART II. RANGE MEASUREMENTS HORIZONTAL SHOT:

1. After finding u (the velocity of the ball leaving the gun), predict the impact point on the floor for the ball when shot horizontally from a position on the table.
2. To check your prediction experimentally:
 - (i) Use the plumb bob to check that the initial velocity is horizontal.
 - (ii) Measure all distances from where the ball starts free fall (not from the cocked position). All measurements refer to the bottom of the ball, so $x = 0$ corresponds to one ball radius beyond the end of the gun rod. Check that the gun's recoil does not change x .
 - (iii) Tape a piece of paper to the floor at the calculated point of impact, and just beyond the paper place a box to catch the ball on the first bounce.
 - (iv) Record results of several shots. (The ball's impact on the paper leaves a visible mark.) Estimate the uncertainty in the observed range.
 - (v) Is the observed range (including uncertainty) within that predicted?



3. Work backwards from the observed range to calculate the initial velocity u . Compare this u to the u calculated in Part I.
4. Do the two u 's agree to within the uncertainty of the u calculated in Part I?

ELEVATED SHOT:

1. Use the stand provided to elevate the gun at an angle above the horizontal. This angle can be read with the plumb bob and protractor as $90^\circ - \text{protractor reading}$. For the elevated gun, be sure to include the additional initial height above the floor of the **uncocked** ball.
2. Before actually trying a shot at an angle, again predict the range but use the value of u which you found from item 3 of the horizontal shot procedure.
3. Make several shots, record the results and compare with predictions.

QUESTION:

From the measured values of u and V in Part I of this experiment, calculate the kinetic energy of the ball before impact, $\frac{1}{2}mu^2$, and of the ball and pendulum together after impact, $\frac{1}{2}(m + M)V^2$. What became of the difference?

OPTIONAL:

1. Derive the result that for momentum to be conserved,
$$\frac{KE_{\text{before impact}}}{KE_{\text{after impact}}} = \frac{m + M}{m}$$

Is this supported by your data?

2. Find the spring constant k of the gun from $\frac{1}{2}mu^2 = \frac{1}{2}kx^2$.

M-12b Simple Harmonic Motion and Resonance (Air Track)

This lab introduces the *harmonic oscillator*, and several phenomena pertaining thereto.

OBJECTIVES:

1. To study the period of the simple harmonic oscillator as a function of oscillation amplitude.
2. To investigate whether Hooke's Law holds for real springs.
3. To observe the transfer of energy from potential energy to kinetic energy, and back again, as the phase of oscillation changes.
4. To observe the exponential decay of amplitude of the damped harmonic oscillator.
5. To investigate the phenomenon of resonance in the damped, driven harmonic oscillator.

THEORY:

The restoring force F on a mass m attached to a "simple" one-dimensional spring is proportional to the displacement from equilibrium: $F = -k(x - x_0)$, where k is the spring constant (or "stiffness") in N/m, x_0 is the position of equilibrium (no net force) and x is the actual position of the object. This is Hooke's Law.

The equation of motion of the simple spring-mass system, $F = ma = -k(x - x_0)$, is a 2nd order differential equation:

$$a = \frac{d^2x'}{dt^2} = -\frac{k}{m}(x') \quad ,$$

where $x' \equiv x - x_0$. The most general solution for this equation may be given as

$$x'(t) = C_1 \cos(\omega_0 t) + C_2 \sin(\omega_0 t) \quad ,$$

where $\omega_0 \equiv \sqrt{k/m}$. In mathematics the adjective "harmonic" has the meaning "capable of being represented by sine or cosine function", so the spring-mass system is often referred to as a *harmonic oscillator*. ω_0 is the *natural frequency* of the harmonic oscillator. C_1 and C_2 are arbitrary initial displacement parameters.

Alternatively the general solution may be written

$$x'(t) = A \sin(\omega_0 t + \phi_0) \quad ,$$

where ϕ_0 is the starting *phase* and A is the *amplitude*.

The general solutions written above are time dependent and periodic. When the phase varies by 2π (when one period $T = 1/f = 2\pi/\omega_0$ has elapsed), both the position, $x'(t+T) = x'(t)$, and velocity, $v'(t+T) = v'(t)$ return to their previous values.

Total energy (TE) is conserved in harmonic oscillation. As time passes kinetic energy (KE) is transformed to potential energy (PE) and back again. Thus at any time t :

$$TE = \text{constant} = KE(t) + PE(t) = \frac{1}{2}m[v'(t)]^2 + \frac{1}{2}k[x'(t)]^2 \quad .$$

APPARATUS:

Basic equipment: Air track; assorted slotted masses; air supply; hose; adjustable stop; glider; springs; timer; photogate & support stand; Hooke's law apparatus.

Computer equipment: PASCO interface; photogate; motion sensor; speaker with driver stem; power amplifier module.

PRECAUTIONS:

The soft aluminum gliders and track surfaces damage easily: Don't drop!

With the air pressure on, check that the glider moves freely, and that the track is level, and free of high-friction areas from scratches or plugged air holes.

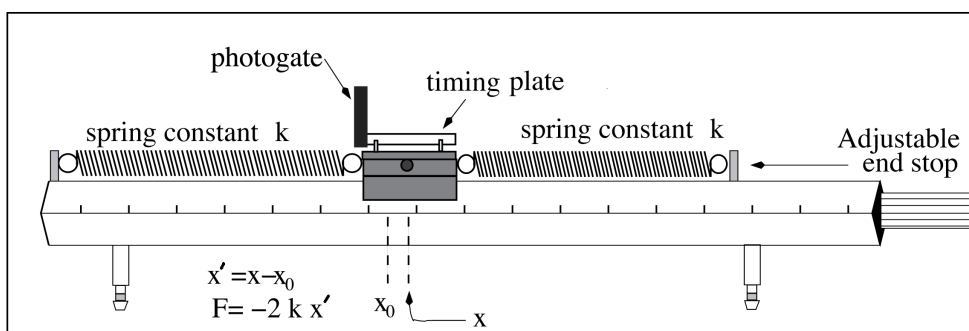


Figure 1: Sketch of the air track configuration for Expt. 0.

EXPERIMENT 0: EQUIPMENT SETUP AND CHECKOUT:

1. Choose two springs of similar lengths.
2. Attach the springs to the air track as in the diagram above, setting the adjustable end stop so that the springs are neither stretched beyond their elastic limits nor capable of sagging onto the track as the glider oscillates (oscillation amplitudes ≤ 20 cm will be requested below).
3. Choose the horizontal position of the photogate so the timing plate just cuts off the beam when glider is in the equilibrium position ($x - x_0 = 0$). The photogate phone jack should be in the PASCO input 1.
4. Turn on the air supply and increase the blower speed until the the force of friction appears to be small.
5. Displace the glider from equilibrium and then release it. Use your natural time-keeping ability to estimate the period of oscillation.
6. Initiate the Pasco software by double-clicking the "Experiment M-12 Part I" file in the PASCO/Physics201 folder on the desktop.
7. Displace the glider from equilibrium, release it, and click Record. Let the glider oscillate for about 10 periods. Click Stop.
8. Compare the mean period calculated by the computer to your estimate from above. Do you trust the computer's calculation?

9. Click on the “Glider Period (Small Amplitude) (s)” button near the top of the data table. Note that there are two choices for “Custom Timer”: “Glider Period (Small Amplitude)” and “Glider Period (Large Amplitude)”.
10. Inspect the “Timer Setup” window, in particular the “Timing Sequence” section, which gives the algorithm for how the computer calculates the glider period. Compare the timing sequences of the two custom timer measurements noted in the last step.
11. What is the minimum amplitude of oscillation that requires use of the Glider Period (Large Amplitude) custom timer, and what is special about this amplitude?

EXPERIMENT I: DEPENDENCE OF PERIOD ON AMPLITUDE:

1. Use the computer to measure the period for oscillation with initial amplitudes of 4 cm, 8 cm, 12 cm, 16, and 20 cm: acquire data for 10 oscillations for each initial amplitude. Use the appropriate custom timer for each measurement. If any period measurements seem untrustworthy, reject them and take another dataset.
2. Make a table with the following columns: “initial amplitude of oscillations A (cm)”; “mean period T (s)”; “standard deviation of the mean σ_T (s)”. Recall the distinction between “standard deviation” and “standard deviation of the mean” (see “Characterization of Uncertainties” on page 5). Which of the two does the Pasco software tell you?
3. The theory presented in the “Theory” section predicts that period is independent of amplitude. Does your data support this? Frame your answer using the formal language of error analysis, e.g. “The discrepancies between the five measurements of mean amplitude are of the same order as the uncertainties in each measurement (as characterized by the standard deviation of the mean), so therefore yes, the experimental measurements are consistent with the theoretical prediction that period is independent of amplitude.”

EXPERIMENT II: DOES HOOKE’S LAW HOLD FOR REAL SPRINGS?:

1. Make two sketches in your lab book, of the glider at equilibrium and after a displacement from equilibrium.
2. Show that the effective force constant for two identical springs of force constant k on either side of an oscillating mass is $2k$.
3. Let us try to verify whether real springs satisfy Hooke’s Law, $F = -k(x - x_0)$. Choose one of your two springs, hang it vertically from the Hooke’s Law apparatus, and set the vertical scale so that the “zero” mark aligns with the bottom end of the spring.
4. Make a table with columns “mass m (g)”, “weight F (N)”, “elongation $x - x_0$ (cm)”. Your first row of data should have zeroes in all three columns.
5. Starting with 2 g, add 5 g masses to the bottom of the spring, and measure the elongation $x - x_0$ of the spring for successively larger masses, until the elongation exceeds the 14.5 cm range of the vertical scale.

6. Plot the data using Excel in such a way that the slope of a straight line fit to the data is k .
7. Inspect the plot: do all data points fall neatly on a straight line?
 If “yes”: fit a straight line to the points, and write down “ k ” as given by the fit.
 If “no”: is it possible that if a data point or range of data points were excluded, the remainder could be well fit by a straight line? If it is, fit a straight line to the remainder points, and write down k as given by the fit, as well as the range of values of F which went into this determination of k .
8. Measure the mass m of the glider.
9. Using these measurements of k and m , calculate the period of the glider in the setup for Expt. 1. Compare to what was actually observed: of the three quantities involved in the comparison (T as measured in Expt. 1, m and k found here) identify the one that has the greatest uncertainty, estimate that uncertainty, and then compare it to the discrepancy between T and $2\pi\sqrt{m/k}$. Which is larger, the uncertainty or the discrepancy?

EXPERIMENT III: MEASURING $x(t)$:

As mentioned in the “Theory” section, the general solution of the equation of motion of the harmonic oscillator may be written

$$x'(t) = A \sin(\omega_0 t + \phi_0),$$

which implies

$$v'(t) = A\omega_0 \cos(\omega_0 t + \phi_0) = A\omega_0 \sin(\omega_0 t + \phi_0 + \frac{\pi}{2})$$

where A is the amplitude and ω_0 the natural angular frequency, as before, and ϕ_0 is the starting phase. Notice that the velocity has a *phase shift* of $\frac{\pi}{2}$ relative to the displacement.




This means that kinetic energy of motion ($1/2 mv'^2$) is transferred into potential energy ($1/2 kx'^2$) and back again.

To capture this cyclic process we will again use the PASCO interface, replacing the photogate with the motion sensor.

1. Remove the timing plate and attach an aluminum vane to the glider. Measure the mass of this glider and predict the new natural frequency.
2. See setup picture above. Place the motion sensor approximately 60 cm from the vane, collinear with the direction of oscillatory motion. Point the motion sensor at the vane and tilt it about 10° upwards, and orient the vane so that its normal points toward the motion sensor. Make sure the yellow phone jack is in Digital Input 1 and the black phone jack is in Digital Input 2.



Figure 2: Photo of setup for Expt. III.

3. Double click on the “Experiment M-12 Part III” file in the PASCO/Physics201 folder on the desktop.
4. Displace the glider approximately 20 cm from equilibrium to initiate the oscillatory motion. Click on the the Record icon to start your data acquisition. The graph will simultaneously display both absolute position and velocity versus time.
5. Inspect the position measurement for “outlier” data points which indicate faulty measurement. If any appear, stop the data acquisition, reorient the motion sensor and/or the aluminum vane, and try again.
6. Practice a few times to make sure you can reliably obtain outlier-free, smoothly-varying sinusoidal curves. Then run the data acquisition for just over five cycles.
7. Determine the initial phase (i.e., at $t = 0$), and the equilibrium position (x_0).
8. Use the Data Highlight  feature of the graph toolbar to better view a single full cycle by highlighting the desired data and then clicking the Zoom-to-Fit button .
9. Using the cross-hair  option to read out the relevant time, position and velocity, make a table as below and fill in the missing entries. Identify units.

time	phase($\omega_0 t + \phi_0$)	$x'(t)$	$v'(t)$	KE	PE	TE
0						
	$\pi/4$ (45 deg)					
	$\pi/2$ (90 deg)					
	$3\pi/4$ (135 deg)					
	π (180 deg)					

10. Is the total energy a constant of the motion?
11. What fraction of the initial energy is lost after five full cycles?

EXPERIMENT IV: DAMPING OF THE MOTION:

We expect that by this point in the lab you have seen the the amplitude of the motion diminishes over time. This suggests there might be friction. To our model

from the “Theory” section, add a velocity-dependent friction force $F_{\text{friction}} = mRv'$, where R is the drag coefficient, so that the equation of motion becomes

$$F = ma = -kx' - mRv' .$$

This is still a linear, second-order differential equation, so the solution can still be written with two free parameters, as in the “Theory” section, and turns out to be

$$x'(t) = Ae^{-Rt/2} \sin(\omega t + \phi) ,$$

where $\omega = \sqrt{k/m - R^2/4}$. The graph below shows an example of this “exponential damping”.

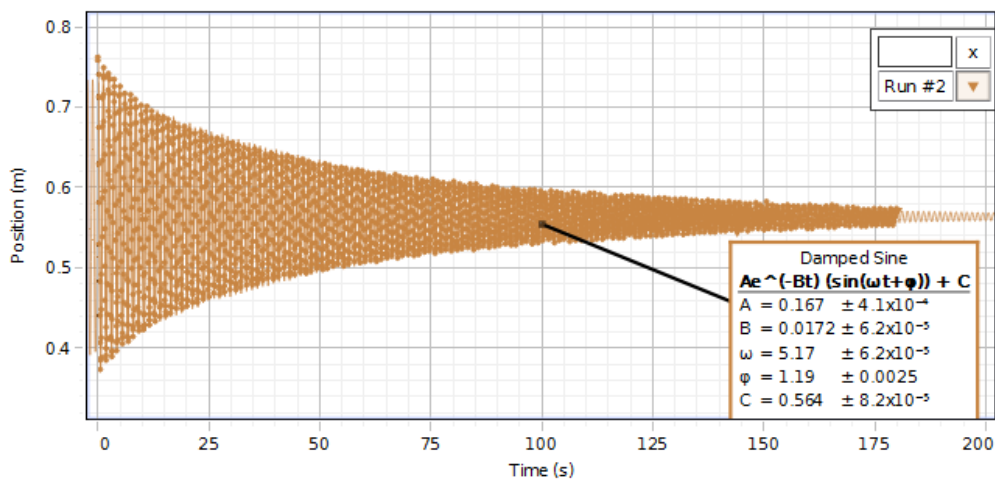




Figure 3: Exponential damping: $R=0.344 \text{ s}^{-1}$, $\omega = 5.17 \text{ s}^{-1}$.

1. Displace the cart by approximately 20 cm from equilibrium, let it go, and click Record. Make sure the position measurement is reliably free from outliers.
2. Once the position measurement is reliably free of outliers, start over: displace the cart by 20 cm from equilibrium, let it go, and click Record. Keep recording data until the amplitude of the motion has reduced to 10 % of what it was initially. This should take a few minutes.
3. Use the Data Highlight  button to select as much data as possible, then click the  button and choose the Damped Sine fit to estimate the drag coefficient, R . If there are outliers, the fit may be visibly poor.
4. State the mathematical condition that the approximation $\omega \sim \sqrt{k/m}$ is valid. Is this condition satisfied by your values of R and $\sqrt{k/m}$?

EXPERIMENT V: RESONANCE:

In the real world an oscillator is often set oscillating by an external driving force which continues to drive the oscillator as the oscillator moves. Suppose the driving force is $F_d \cos \omega_d t$, where F_d is a constant, and there is damping as in the previous section. The new equation of motion is then:

$$F = ma = -kx' - mRv' + F_d \cos \omega_d t \quad \text{or} \quad \frac{F_d}{m} \cos \omega_d t = \frac{d^2 x'}{dt^2} + R \frac{dx'}{dt} + \frac{k}{m} x' .$$

As we know from the previous section, in the absence of the driving force the system exhibits exponentially damped oscillations. If the driving force has a frequency ω_d sufficiently close to the natural oscillation frequency $\sqrt{k/m}$, energy may be fed into the oscillator faster than the damping can take it away, and the amplitude of oscillation may become very large. This is called “resonance”.

The exact solution of the equation of motion of the damped, driven oscillator is tedious. Since we are interested only in resonance, we assume oscillatory solutions (the “underdamped” case) and can quickly find solutions of the form

$$x' = A_{transient} e^{-(R/2)t} \sin(\sqrt{k/m - R^2/4}t + \phi_1) + \frac{F_d}{m [(k/m - \omega_d^2)^2 + \omega_d^2 R^2]^{1/2}} \cos(\omega_d t - \phi_2),$$

where the first term is the *transient* solution, identical to the damped harmonic oscillator described in the last section, and the second term is the *steady-state* solution. At “large” times $t \gg 2/R$, the transient solution has decayed away, and only the steady-state solution remains, so that

$$x'(t \gg 2/R) \sim \frac{F_d}{m [(k/m - \omega_d^2)^2 + \omega_d^2 R^2]^{1/2}} \cos(\omega_d t - \phi_2) .$$

The decay of the transient is illustrated in the figure below. As long as the amplitude of the transient is similar to the amplitude of the steady-state term, one observes a “beat” between the two frequencies.

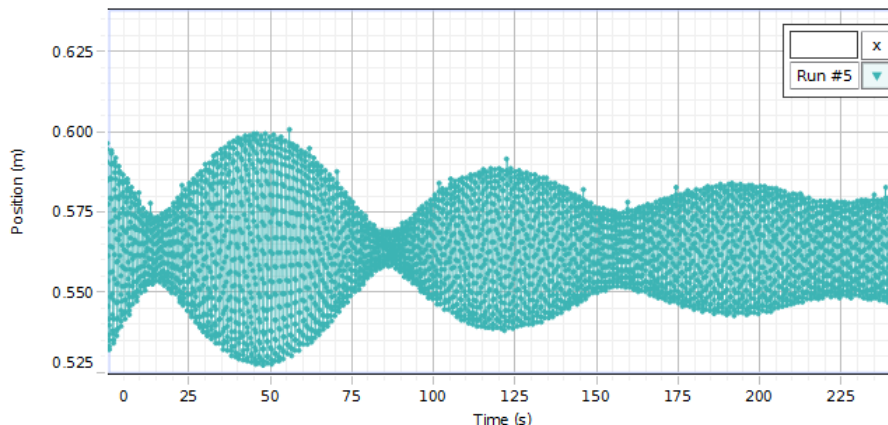


Figure 4: Transient decay: $\sqrt{k/m} = 0.824$ Hz, $\omega_d = 0.810$ Hz

Suppose that ω_d can be varied while k , m , and R remain fixed. Observe that if $\omega_d = \sqrt{k/m}$, the steady state amplitude $A \equiv \frac{F_d}{m [(k/m - \omega_d^2)^2 + \omega_d^2 R^2]^{1/2}}$ reaches its maximum value, A_{max} , which can be written $A_{max} = (F_d/k)(\sqrt{k/m}/R)$. The first element in this

product is the extension $x - x_0$ the force F_d would produce if it were statically applied. The second element is a dimensionless number which could be much greater than one if the damping is small (as it is in the air track system). This is why resonance is important!

In this lab we will only investigate the amplitude of the steady-state solution as a function of driver frequency. If $\sqrt{k/m} \gg 2/R$, we expect this to be sharply peaked about $\omega_d = \sqrt{k/m}$, as sketched in the figure below.

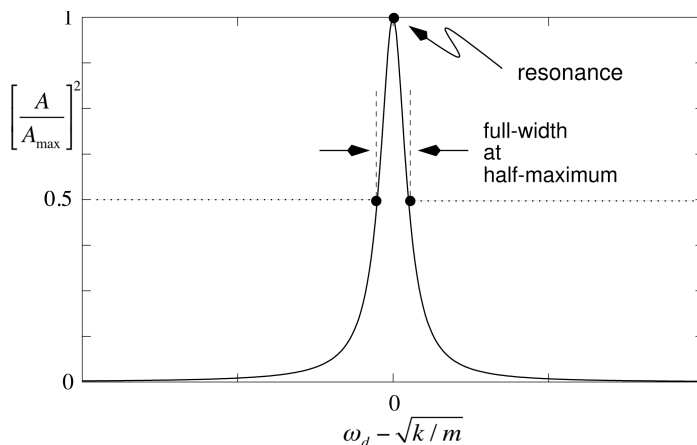



Figure 5: Sketch of the relative steady-state amplitude squared vs driver frequency.

1. Detach the fixed spring end stop and place the speaker as shown in the picture below. Loop the spring through the small slot in the speaker driver stem using the same considerations for the spring extension as in the previous experiments.



Figure 6: Photo of air track setup with motion sensor and speaker.

2. Make sure the speaker power leads are plugged into the red and black outputs of the Pasco 850.
3. Double click on the “M-12 Experiment IV” file in the PASCO/Physics201 folder on the desktop.
4. On the 850 Output 1 section of the “Signal Generator” window (see figure below), set “frequency” to your best estimate of $\sqrt{k/m}$, and set “amplitude” to 1 V. Make sure “Waveform” is set to “Sine”. Click the “On” button when you are ready to take data. NOTE: The nominal step sizes for adjusting the amplifier frequency and voltage are larger than you will want. To alter the step size use the  buttons.
5. With maximum blower airflow, displace the cart approximately 20 cm from equilibrium to initiate the oscillatory motion. Click the Record icon to start data acquisition. Verify the data is free of outliers. If not, reorient motion sensor and/or

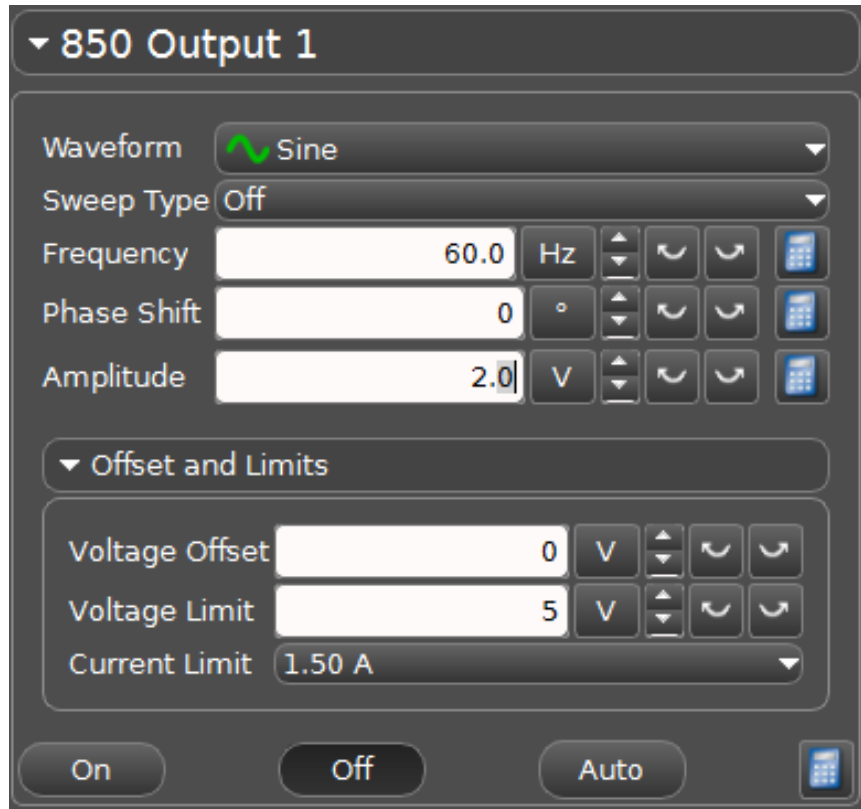



Figure 7: Pasco software Signal Generator window.

- aluminum vane. You may need to give the glider a push to keep the amplitude near 20 cm. Once outliers are gone, stop taking data.
6. Make a table with three columns: “Driver Frequency ω_d (Hz)”, “Steady-state Amplitude A (m)”, “Steady-state Frequency (Hz)”.
 7. Displace the cart approximately 20 cm from equilibrium to initiate the oscillatory motion. Click Record, and wait long enough (likely several minutes) for the transient to decay. While you are waiting, look for “beating” between the driver frequency and the natural frequency. Once the transient has decayed to nothing, record data for another 30 seconds. Highlight these last 30 seconds of data, click the  button, and choose the “Sine” option to fit a sinusoid to them. The software lets you choose data points to exclude; if there are outliers, use the software to exclude them. Verify the steady-state amplitude and frequency returned by the fit seem reasonable, and record them in the table.
 8. Set the amplifier frequency 0.01 Hz steps above and below resonance and record data until you achieve steady-state behavior. Repeat for 0.10 Hz and (if time permits) 0.40 Hz steps.
 9. Does the “Steady-state frequency” returned by the fits match the driver frequency?
 10. Plot the relative amplitude squared $(A/A_{max})^2$ vs frequency offset $(\omega_d - \sqrt{k/m})$. Estimate the full width at half maximum for this curve: it should equal R from Expt. IV.
 11. Write down one example of resonance that you have experienced in ordinary life.

S-1 Transverse Standing Waves on a String

OBJECTIVE: To study propagation of transverse waves in a stretched string.

INTRODUCTION:

A standing wave in a string stretched between two points is equivalent to superposing two traveling waves on the string of equal frequency and amplitude, but opposite directions. The distance between nodes (points of minimum motion) is one half wavelength, $(\lambda/2)$. Since the ends of the string are fixed, the only allowed values of λ are $\lambda_k = 2L/k$, $k = 1, 2, 3 \dots$

The wave velocity, v , for a stretched string is $v = \sqrt{F/\mu}$ where $\mathbf{F} = \mathbf{tension\ in\ the\ string}$ and $\mu = \mathbf{mass\ per\ unit\ length}$. But $v = f\lambda$ and hence only certain frequencies are allowed:

$$f_k = \frac{\sqrt{F/\mu}}{\lambda_k} . \quad (1)$$

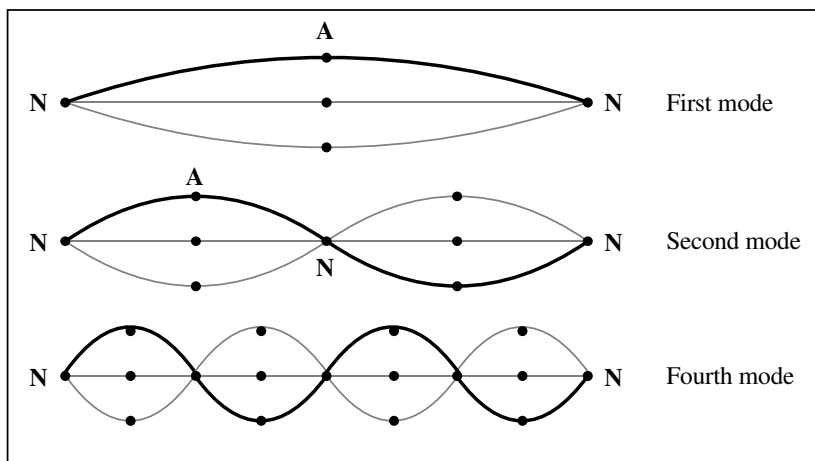


Figure 1: The Modes of a String

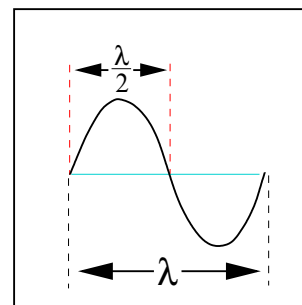


Figure 2: A close-up

PART A: Waves from a mechanical driver (i.e. a speaker)

APPARATUS:

Basic equipment: Pasco interface; electrically driven speaker; pulley & table clamp assembly; weight holder & selection of slotted masses; black Dacron string; electronic balance; stroboscope.

The set-up consists of an electrically driven speaker which sets up a standing wave in a string stretched between the speaker driver stem and a pulley. Hanging weights on the end of the string past the pulley provides the tension.

The computer is configured to generate a digitally synthesized sine wave (in volts versus time) with adjustable frequency and amplitude (max: ~ 10 V).

PASCO interface: This transforms the digital signal into a smooth analog signal.

Precautions: Decrease the amplitude of the signal if the speaker makes a rattling sound. The generator is set to produce sine waves; **do not change the waveform.**

Note: Although the speaker is intended to excite string vibrations only in a plane, the resultant motion often includes a rotation of this plane. This arises from non-linear effects since the string tension cannot remain constant under the finite amplitude of displacement. [See Elliot, *Am. J Phys.* 50, 1148, (1982)]. Other oscillatory effects arise from coupling to resonant vibrations of the string between pulley and the weight holder; hence keep this length short.

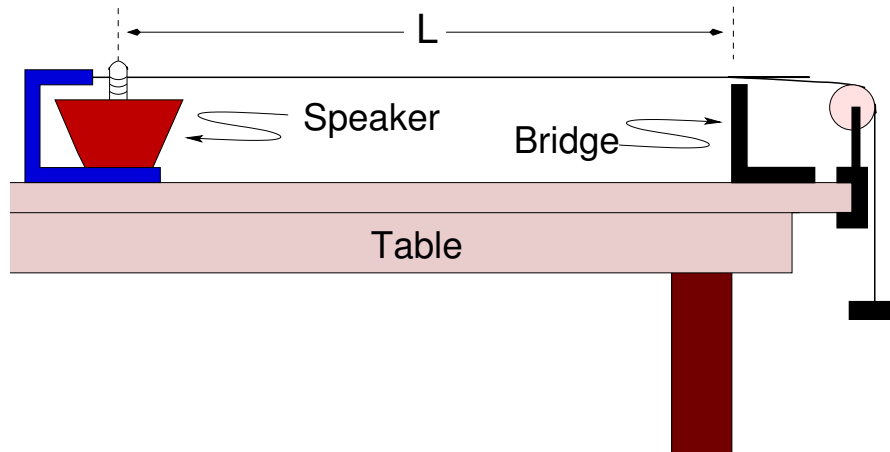




Figure 3: The apparatus

PROCEDURE: CHECKING EQUATION (1)

1. Procure a length of string about 1.5 meters long.
2. Carefully weigh the string, and calculate μ . Note this in your lab notebook.
3. Set up the speaker, bridge, pulley and string as in the above figure. Make the distance, L , between the bridge and the pin of the speaker be about 1.2 m, and measure it accurately using the two meter ruler; record this in your lab notebook.
4. Place the sheet of paper provided on the table; this will make it easier to see the vibration of the string.
5. Double click on the “Experiment S-1” file in the PASCO/Physics202 folder on the desktop. The display will appear as shown in Fig. 4.
6. You will see that the computer is set to produce a 60 Hz sine wave with an amplitude of 2 V . To start the string vibrating click the “On” button.
7. Click on the up/down arrow in order to change the amplitude or the frequency of the signal.

NOTE: The nominal step sizes for adjusting the amplifier frequency and voltage may be much too large. To alter the step size use the  buttons. To alter the current or voltage use the  buttons. You can also change the value directly by clicking the mouse cursor in the numeric window and entering a new value with keyboard number entry.

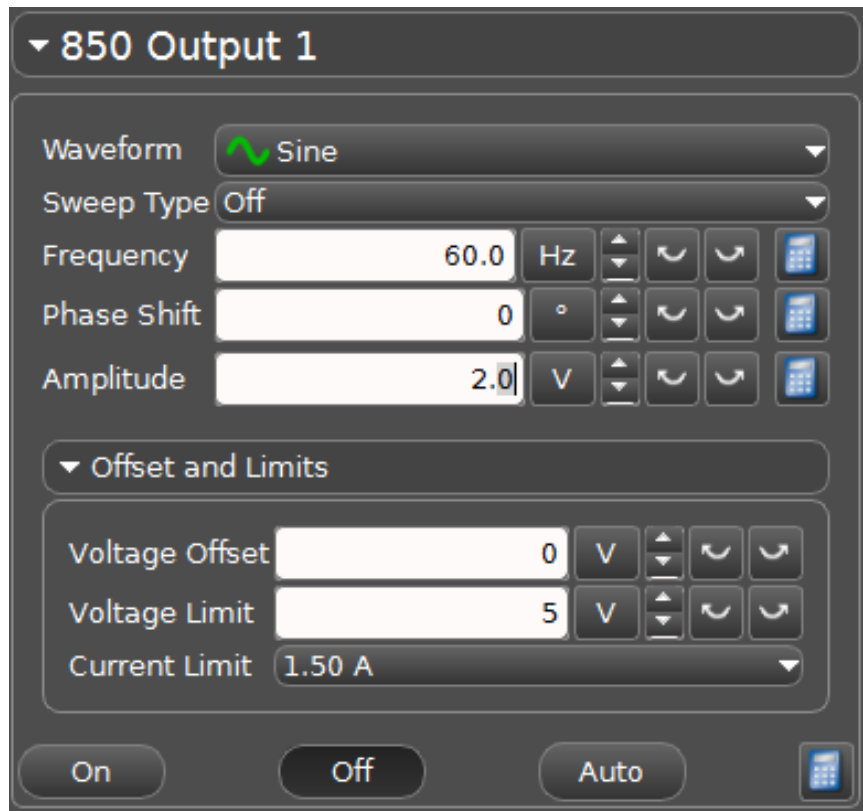


Figure 4: The Pasco Capstone display.

8. Stop the signal generator, put a 200 *g* mass on the mass hanger and restart the signal generator. Record the total mass and tension in your lab notebook. In the next few steps you'll keep the tension in the string constant and look for the frequencies of different modes.
9. Measure the frequency f_2 of the $k = 2$ mode using the following method. Once you have observed the $k = 2$ mode, decrease the voltage to the smallest value at which you can clearly see it.
10. Next, decrease the frequency until the mode is too small to see. Then, increase the frequency in 1 *Hz* steps, noting the frequency at which the mode amplitude is maximum. Continue increasing the frequency until the mode amplitude is again too small to see. Then, decrease the frequency by 1 *Hz* steps, and note when the mode amplitude is maximum. Once you have done this, discuss with your lab partners what is the best choice for f_2 .
11. (Optional) Use the stroboscope to confirm f_2 (note the stroboscope is calibrated in cycles per minute, not *Hz*). The value should be close to 70 *Hz*.
12. Find and record the frequencies f_3, f_4, f_5, \dots of as many higher-order modes as you can.
13. Find and record f_1 . Be especially careful to use the lowest possible voltage, because the mode amplitude can become large.

14. Divide the various frequencies f_k by k and enter the values in a table. Calculate the average value of f_k/k .

QUESTION:

Q1: Use this average value of f_k/k to calculate the mass per unit length of the string. How does it compare with your measurement?

15. Choose six masses between 100 g and 1 kg and enter the values in a data table.
16. Determine the resonant frequency of the second mode of the string under these different tensions and record your results. (Hint: increasing the mass by a factor of two increases f_2 by nominally a factor of $\sqrt{2}$.)
17. Plot a graph of frequency versus mass, m , and include the zero value.
18. Plot a graph of frequency versus \sqrt{m} and again include the zero value.

QUESTIONS:

Q2: Which of the two graphs can be fitted with a straight line? A parabola? Why?

Q3: From the slope of the graph having the linear relationship obtain the mass per unit length of the string.

Q4: Which of the three methods of estimating the mass per unit length of the string seems most accurate?

FOR FURTHER INVESTIGATION:

Vibrations on a circular membrane are made visible here:

<http://www.acs.psu.edu/drussell/Demos/MembraneCircle/Circle.html>

S-2 Velocity of Sound in Air

OBJECTIVE:

To calculate the velocity of sound from measurement of the wavelength in air for sound of a certain frequency.

APPARATUS:

Resonance tube with arrangement for varying water level (use only distilled water); rubber tipped hammer; tuning fork; Hg thermometer.

INTRODUCTION:

For a closed tube, resonance occurs at tube lengths of an odd multiple of one-fourth wavelength, i.e. at $\lambda/4$, $3\lambda/4$, $5\lambda/4$ etc.

SUGGESTIONS:

1. Find the positions of the water level in the tube for the first three of these resonances. Use these readings to calculate the speed of sound, $v = |\bar{v}|$. Initially have enough water that you can raise the level above the first resonance position. The tuning fork frequency is on the fork.

Since the effective end of the resonance tube is not at the tube's end, **do not use the position of the tube's top** in your calculations, but rather **take differences between the other readings**.

2. Sound waves in gases have a speed $v = \sqrt{\gamma RT/M}$. (Recall the formula for the speed of sound on a string, $v = \sqrt{T/\mu}$ (e.g., Lab S-1)). Correct your value of v to 0°C ($T = 273.16\text{ K}$) and compare with that accepted for **dry** air at 0°C : $331.29 \pm .07\text{ m/s}$, [Wong, *J. Acoust. Soc. Am.*, **79**, 1559, (1986)]. For humid air see 3. below.
3. We quantify proportions in gas mixtures by the pressure each gas contributes to the total pressure. This is called the "partial" or "vapor" pressure. Think of the speed as resulting from an average $\langle \gamma/M \rangle$,

$$\langle \gamma/M \rangle = [(\gamma_a/M_a)P_a + (\gamma_w/M_w)P_w]/(P_a + P_w),$$

so that

$$v_{dry} \cong v_{humid} \sqrt{(\gamma_a/M_a) / \langle \gamma/M \rangle},$$

where $\gamma_{air} = 1.40$, $\gamma_w = 1.33$, P_a is the partial pressure of air, P_w is the vapor pressure of water, $M_a \sim 29\text{ kg}$ and $M_w = 18\text{ kg}$.

How should the vapor pressure of water, P_w , in the tube affect the speed?

OPTIONAL: Humidity changes will affect tuning of what musical instruments?

4. What effect does atmospheric pressure have on the velocity of sound in dry air? (Assume air at these pressures is an ideal gas.)
5. Viscosity and heat conduction in the tube may reduce \mathbf{v} by $\sim 0.1\%$. See N. Feather, "The Physics of Vibrations and Waves", Edinburgh Univ. Press, (1961), p. 110-120; this reference also has a delightful historical account (including Newton's famous goof).

A Appendices

A Precision Measurement Devices

Vernier Calipers:

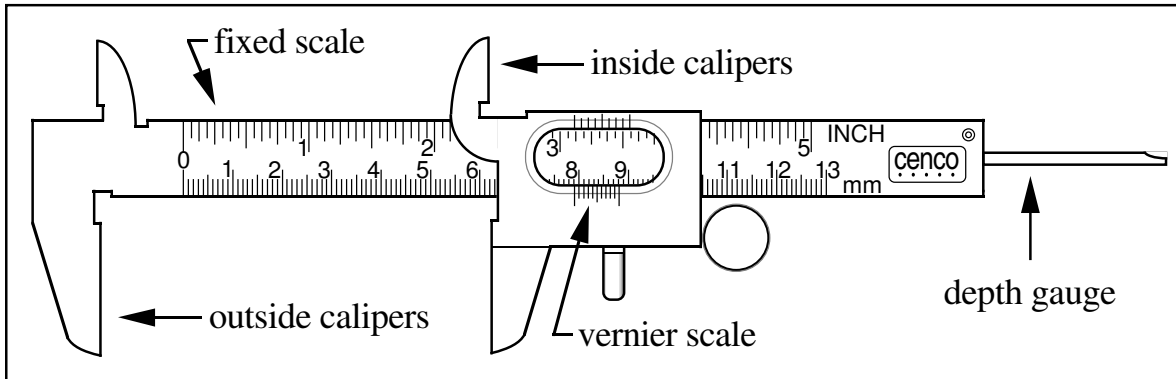


Figure 1: The vernier caliper

A vernier caliper consists of a fixed scale and a moving *vernier* scale. In a metric vernier the fixed scale is marked in centimeters and millimeters, the vernier scale is nine millimeters long, and is divided into ten parts each 0.9 millimeters long. The distances of each line from the first are therefore 0.9, 1.8, 2.7, \dots , mm or generally: $d_i = 0.9 \times i$, where d_i is the distance between the zero line and the i^{th} line of the vernier scale. If the vernier caliper is closed, so that the two jaws touch each other, the zero of the fixed scale should coincide with the zero of the vernier scale. Opening the jaws $0.03\text{ cm} = 0.3\text{ mm}$ will cause the fourth line (the *three* line which is a distance of 2.7 mm from the zero line of the of the vernier scale) to coincide with the 3 mm line of the fixed scale as shown below.

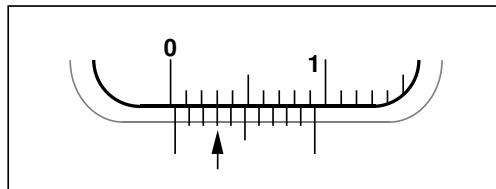


Figure 2: The vernier reads 0.03 cm

Below is another example of vernier reading; the arrow shows which mark on the vernier scale is being used.

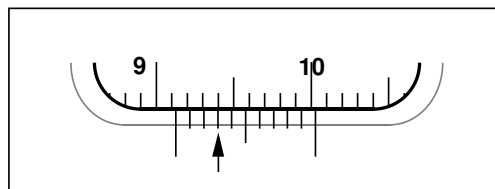


Figure 3: The vernier reads 9.13 cm

EXERCISES:

- A. Close the vernier and observe that the first vernier mark coincides with the zero of the centimeter scale.
- B. Open the jaws of the vernier very slowly and observe how the different vernier marks coincide successively with the millimeter marks on the fixed scale: the first mark coincides with the 1 *mm* mark on the fixed scale; then the second mark coincides with the 2 *mm* mark on the fixed scale; then the third mark coincides with the 3 *mm* mark on the fixed scale and so on.
- C. Estimate the dimension of an object using a meter stick and then use the vernier caliper to measure the dimension precisely.
- D. In the four examples of Fig. 4 determine the actual reading.

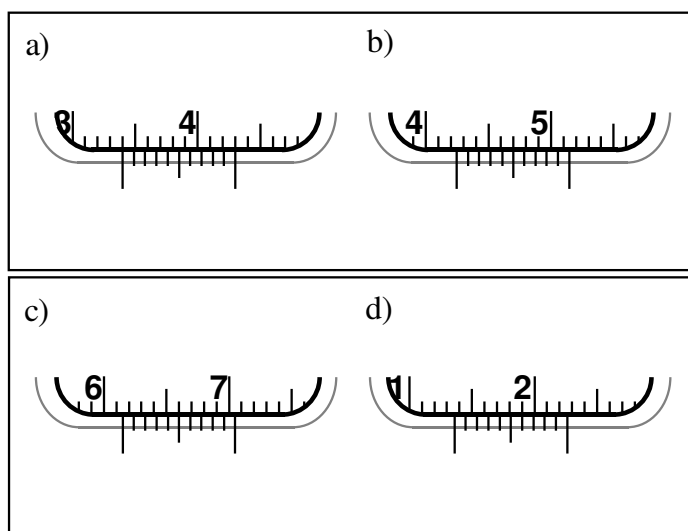


Figure 4: Test cases

Micrometer:

A micrometer can measure distances with more precision than a vernier caliper. The micrometer has a 0.5 *mm* pitch screw, this means that you read millimeters and half millimeters along the barrel. The sleeve is divided into 50 divisions corresponding to one hundredth of a millimeter (0.01 *mm*) or 10 μ each. The vernier scale on the micrometer barrel has ten divisions, marked from 2 to 10 in steps of two. The “zero” line is not marked ‘0’, but is longer than the others. The vernier allows you to read to the nearest thousandth of a millimeter, *i.e.*, to the nearest micron (0.001 *mm* = 1 μ).

Precaution:

Great care must be taken in using the micrometer caliper; a ratchet knob is provided for closing the caliper on the object being measured without exerting too much force. Treat the micrometer with care, **ALWAYS** close the calipers using the ratchet knob, this prevents tightening the screw too strongly. Closing the calipers too hard damages the precision screw.

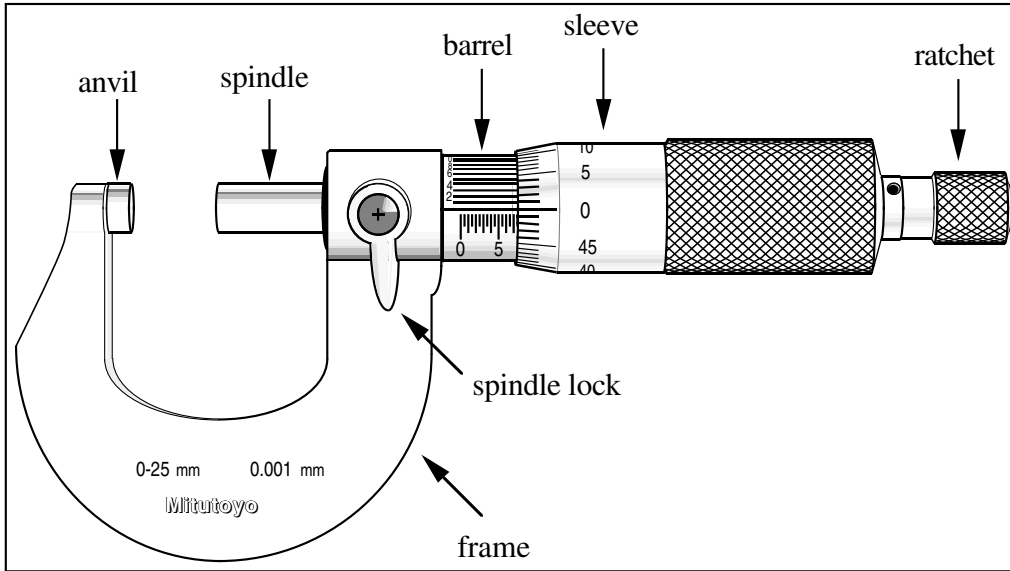


Figure 5: The micrometer calipers

Below are two examples of micrometer reading. The arrow shows which mark on the vernier scale is being used.

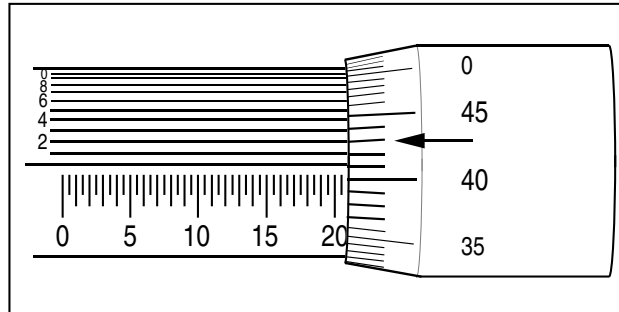


Figure 6: The micrometer reads 20.912 mm

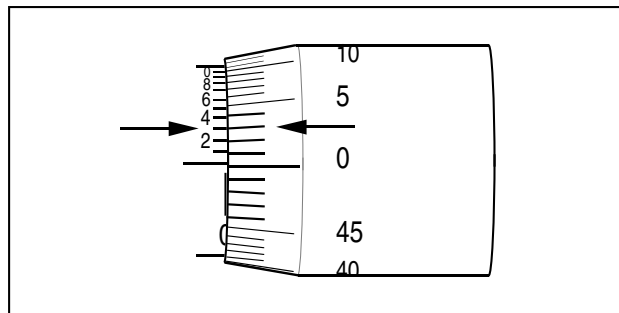


Figure 7: The micrometer reads 3 μ

Fig. 7 introduces “zero error”, which occurs when the micrometer doesn’t read zero even though anvil and spindle are touching. The zero line on the barrel is barely visible, and the vernier reads 0.003 mm = 3 μ ; the zero error is $\epsilon_0 = 3\mu$.

A negative zero error, as shown below, requires a moment of thought.

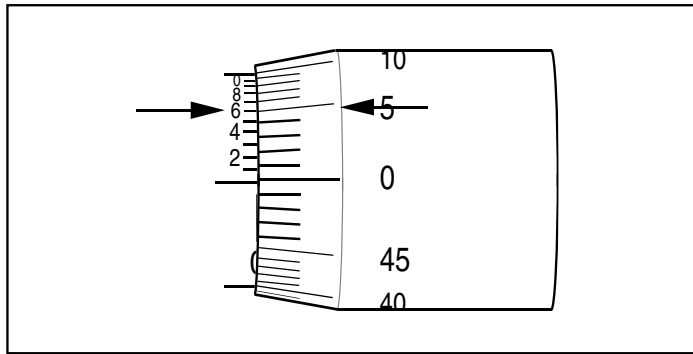


Figure 8: The micrometer reads -4μ

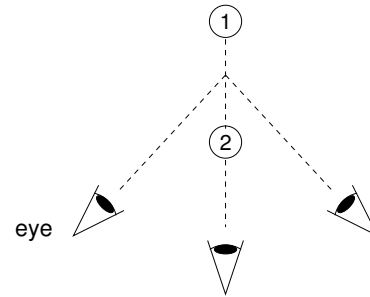
In Fig. 8 the zero line on the barrel of the micrometer is obscured by the sleeve, (the “zero” line on the sleeve is above the “zero” line on the barrel) this corresponds to a reading of -0.5 mm ; the vernier reads 0.496 mm the zero error is then $\epsilon_0 = -0.5 + 0.496 = -0.004 \text{ mm} = -4 \mu$.

B PARALLAX and Notes on using a Telescope

1. PARALLAX:

To do quantitative work in optics one must understand *parallax* and how it may be eliminated. PARALLAX is defined as *apparent motion of an object caused by actual motion of the observer*.

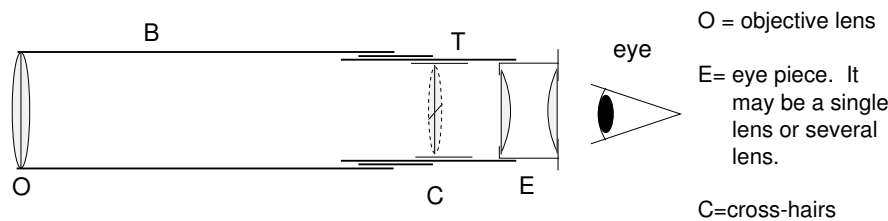
As the observer moves left and right, object 1 appears to move to the left and right of object 2. The amount by which object 1 appears to move is proportional to the distance between object 1 and object 2. If object 1 comes in front of object 2, the direction of its apparent motion reverses.



Try this with two fingers.

Note that if object 1 is an image and object 2 a cross hair, the absence of parallax shows that the cross hairs are in the plane of the image.

2. Focusing a Telescope for Parallel Rays:



The eyepiece E slides back and forth in the tube T and one should first adjust the eyepiece to give a clear image of the cross hairs. Then move the tube T back and forth in the barrel B until the image of a distant object, formed by the objective O, falls on the plane of the cross hairs. The test for this is the absence of parallax between the cross hairs and image.

The rays from a distant object are nearly parallel. For viewing a distant object, use an open window if the window glass is not accurately plane. Otherwise poor image formation may result. You can check by trying it both ways. At night use a distant object in the hallway.

The telescope, now focused for parallel rays, will stay so as long as the distance between O and C is unchanged. One may still adjust the eyepiece position to suit the observer.

3. Finding an Image in a Telescope:

If you have trouble finding an image in a telescope, locate the image first with your unaided eye, and then pull the telescope in front of your eye, aligning it with your line of sight. With high magnification it is difficult to find the image in the telescope because the alignment must be nearly perfect before the image appears in the field of view. The eye has a rather large field of view so that the image will be visible over a range of positions.