Physics 207 Lab Manual
Mechanics, Heat, Sound/Waves

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Contents

Introduction

Physics is an experimental science. In this laboratory, you will fight against experimental errors, find ways to minimize them, and (we hope) come to appreciate the need for clear and accurate record-keeping.

Physics is also a social activity. Discuss with your lab partners the purpose of each piece of apparatus; cooperate to choose who does what; critique the data as it is taken (is it as expected?); calculate independently, then compare results to check each others' work; debate the reliability of the results, and what may fairly be concluded from them.

Keep clear and accurate records, and record fair conclusions, in a laboratory notebook:

1. *Heading of the Experiment:*
   Copy from the manual the number and name of the experiment.
   Include both the current date and the name(s) of your partner(s).

2. *Original data:*
   "Original data" are the actual readings you have taken. Record them directly into your notebook as they are gathered. All partners should record all data: in case of doubt, the partners' lab notebooks can be compared to each other.
   Arrange data in tabular form when appropriate. Introduce each table with a phrase or sentence (so that you, or someone else, can make sense of it later).

3. *Housekeeping deletions:*
   Every hour or so, draw a box around any erroneous or unnecessary material and hatch three or four parallel diagonal lines across this box. Write a note explaining to yourself what was wrong *(sometimes, it turns out it wasn't wrong...).*

4. *Sketches and remarks:*
   Make simple, diagrammatic (rather than pictorial) sketches of apparatus.
   When a useful result occurs at any stage, describe it with at least a word or phrase.

5. *Graphs:*
   Affix a hand-drawn graph on graph paper with transparent tape; affix a computer-generated graph with transparent tape; or mark out and plot a simple graph directly in your notebook.
   Show points as dots, circles, or crosses, i.e., ·, ∘, or ×.
   Show trends by drawing a smooth curve (that may actually miss most of the points) rather than connecting the points with lines.

6. *Units, coordinate labels:*
   Whenever you write down a physical quantity, *always* write down its units.
   Whenever you make a graph, always label the *abscissa* (horizontal coordinate) and *ordinate* (vertical coordinate), so that the reader (who might be yourself at a later time) knows what has been graphed.

7. *Final data, results and conclusions:*
   Write a neat summary of data and results at the end of each experiment. Any conclusions must be faithful to the data. Use phrases such as "the discrepancy between our measurements and the theoretical prediction was larger than the uncertainty in our measurements."

Errors and Uncertainties

All measurements are to some degree uncertain. It is good scientific practice to **include an estimate of the uncertainty of every measurement you make**. The uncertainty of a single measurement–such as, for example, the diameter of a cylinder–comes from multiple sources, which fall into two categories: *random uncertainties* and *systematic errors*:

- There may be **actual variations** of the quantity being measured, e.g. the diameter of a cylinder may actually be different in different places. This random uncertainty can be reduced by making measurements at different places and taking the average. The standard deviation of the measurements (see below) is a measure of the uncertainty.

- The **measuring device** itself may be in error. The systematic error thus introduced will not lie equally on both sides of the true value, so averaging a large number of readings is no help. To reduce such errors, we calibrate the measuring device. In the case of the micrometer caliper, for example, we measure the zero error (the reading when the jaws are closed) and the readings on selected precision gauges that are about as large as the cylinder to be measured.

- The **measuring technique** may be a source of systematic error. When used to measure the diameter of a cylinder, the micrometer will always measure the largest diameter between its jaws; hence if there are small bumps or depressions on the cylinder, the average of a large number of measurements will not give the true average diameter but a quantity somewhat larger. This error can be reduced by making the jaws of the caliper smaller in cross section.

- The measuring technique may also be another source of random uncertainty. For example, the reading of the micrometer caliper may vary because one can't close it with the same force every time, and one's estimate of the fraction of the smallest division varies from trial to trial. Making multiple measurements also reduces the uncertainty from this source

Characterization of Uncertainties

The experimenter cannot immediately determine the cause of the variation of a set of measurements. A simpler task is to characterize this variation.

Consider the data below. Column 1 represents 10 readings of the diameter of a cylinder taken at one place, so that variations in the cylinder do not come into consideration.

Measurements	Deviation from Mean	
9.943 mm	0.0003	mm
9.942	- 0.0007	
9.944	0.0013	
9.941	- 0.0017	
9.943	0.0003	
9.943	0.0003	
9.945	0.0023	
9.943	0.0003	
9.941	- 0.0017	
9.942	- 0.0007	

How should one state the results of these measurements in a scientific fashion?

Expressed algebraically, the **mean** $\bar{x}$ of a set of n measurements $\{x_i\}$ is $\bar{x} = \sum x_i / n$. The mean of the 10 readings above turns out to be 9.9427 mm.

Column 2 gives each reading's deviation from the mean. The **standard deviation** σ (also known as the "root mean square deviation") is defined as

$$\sigma = \text{standard deviation} = \sqrt{\frac{1}{(n-1)} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

The standard deviation of the above 10 measurements is 1.225×10^{-3}.

Note that if you make one measurement of some quantity x, then $n = 1$ and $\bar{x} = x_1$ in the above equation: the standard deviation of your measurement is $\sqrt{0/0}$, which is undefined. You don't know how reliable your measurement is.

Assuming you make more than one measurement, σ tells you the typical deviation from the mean you will find for an individual measurement. It depends on your experimental method. If you make more measurements, σ will not change much.

However, the more measurements you make, the more accurately you can expect to know the value of the mean. The uncertainty of the value of the mean is called **the standard deviation of the mean** σ_μ, and in general it is less than σ:

$$\sigma_\mu = \text{standard deviation of the mean} = \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{n}} \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

The standard deviation of the mean of the above 10 measurements is 3.96×10^{-4}.

When you take more measurements in order to minimize σ_μ, you are "gathering statistics".

One states one's results as **mean $\pm$ standard deviation of the mean**. From the data above, the diameter of the cylinder was measured to be 9.9427 ± 0.0004 mm.

Error Distribution and Error Bars

For many types of measurement one may assume that the error distribution is "normal": the probability of a given error ϵ is proportional to $e^{-\epsilon^2}$. If the error distribution is normal and the number of measurements is large, 68% of the measurements will lie closer than σ to the true value.

The purpose of the error bars shown on a graph in a technical report is to guide the reader's expectation of reproducing the results in the graph using the procedure described in the report.

If the error distribution is normal, and the errors bars extend to $1\ \sigma$ above and below the data point, the reader should expect a 68% chance that a repetition of the experiment would produce a measurement that falls between the error bars.

Propagation of Uncertainties

Let $R = f(x, y, z)$ depend on measurements of three different quantities x, y, and z. The uncertainty ΔR in R which results from an uncertainty Δx in the measurement of x is

$$\Delta R = \frac{\partial f}{\partial x} \Delta x \; ,$$

and the fractional uncertainty in R is

$$\frac{\Delta R}{R} = \frac{\frac{\partial f}{\partial x}}{f} \Delta x \; .$$

In many experimental situations, the errors are uncorrelated and normally distributed. In this case the uncertainties add in quadrature (the square root of the sum of the squares):

$$\frac{\Delta R}{R} = \sqrt{\left(\frac{\frac{\partial f}{\partial x}}{f} \Delta x\right)^2 + \left(\frac{\frac{\partial f}{\partial y}}{f} \Delta y\right)^2 + \left(\frac{\frac{\partial f}{\partial z}}{f} \Delta z\right)^2} \; .$$

Δ denotes the uncertainty in a quantity, and σ refers to the standard deviation.
Some examples:

A) $R = x + y$. If the uncertainties in x and y each have a normal distribution and are independent, they combine in quadrature:

$$\Delta R = \sqrt{\Delta x^2 + \Delta y^2} \; .$$

Note that if $R = x - y$, then $\Delta R / R$ can become very large if x is nearly equal to y. Hence avoid, if possible, designing an experiment where one measures two large quantities and takes their difference to obtain the desired quantity.

B) $R = xy$. The fractional uncertainties combine in quadrature:

$$\frac{\Delta R}{R} = \sqrt{(\frac{\Delta x}{x})^2 + (\frac{\Delta y}{y})^2} \; .$$

Note the same result occurs for $R = x/y$.

C) Consider the density ρ of a solid cylinder (Exp. M-201b):

$$\rho = \frac{m}{\pi r^2 L}$$

where m = mass, r = radius, L = length, are the three measured quantities. Then

$$\frac{\partial \rho}{\partial m} = \frac{1}{\pi r^2 L} \qquad \frac{\partial \rho}{\partial r} = \frac{-2m}{\pi r^3 L} \qquad \frac{\partial \rho}{\partial L} = \frac{-m}{\pi r^2 L^2} \; .$$

If the errors have normal distribution and are uncorrelated, then

$$\frac{\Delta \rho}{\rho} = \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(2\frac{\Delta r}{r}\right)^2 + \left(\frac{\Delta L}{L}\right)^2}$$

D) Area of a circle: $A = \pi d^2 / 4$: $\Delta A / A = 2 \Delta d / d$.

Significant Figures

Suppose you measure the diameter of a circular disk to be 24.326 ± 0.003 mm and want to compute its area $A = \pi d^2/4$. Evidently $A =464.7627...$ mm^2, but how many digits are significant?

The fractional uncertainty in d is $\Delta d/d = \frac{0.003}{24.326} = 1.2 \times 10^{-4}$. From example D) on the last page, this gives $\Delta A/A = 2.4 \times 10^{-4}$, so the uncertainty in A is $\Delta A = .11$ mm^2.

This tells us that only the first five digits are significant, so A should be reported as

$$A = \pi d^2/4 = 464.77 \pm 0.11 \text{ mm}^2$$

A good rule is to use one more digit in constants than is available in your measurements, and to save one more digit in computations than the number of significant figures in the data. When you use a calculator you usually get many more digits than you need. Therefore **at the end**, be sure to **round off** the final answer to display the correct number of **significant figures.**

SAMPLE QUESTIONS

1. You measure $d = 1.0 \pm 0.2$ cm and $s = 1.0 \pm 0.2$ cm. What is the uncertainty in $r = d + s$?

   (a) 0.2 cm
   (b) 0.3 cm
   (c) 0.4 cm

2. You measure r with a percentage uncertainty of 1%. What is the percentage uncertainty in $A = \pi r^2$?

   (a) 1%
   (b) 1.4%
   (c) 2%

3. What happens to σ, the standard deviation, as you make more and more measurements? What happens to σ_μ, the standard deviation of the mean?

   (a) They both remain same
   (b) They both decrease
   (c) σ increases and σ_μ decreases
   (d) σ approaches a constant and σ_μ decreases

4. You measure $m_1 = 115.34 \pm 0.01$ g and $m_2 = 1.0 \pm 0.1$ kg, in order to find $m_3 = m_1 + m_2$. How should you report m_3?

   (a) 1.11534 ± 0.00001 kg
   (b) 1.11534 ± 0.1 kg
   (c) 1.1 ± 0.1 kg

M-200 Errors, Uncertainties, and Excel

OBJECTIVES:

1. To reassure you that nature is inherently uncertain. All measurements include some degree of uncertainty.

2. To practice the fundamentals of characterization of uncertainty and propagation of uncertainty, using Excel for calculations and plotting.

3. To develop a vocabulary for rigorous and objective reporting of measurements and their relation to predictions.

APPARATUS:

Funnel; 100 round chads from a three-hole punch, graph paper; Excel. There is a sample Excel spreadsheet called "M-200.xlsx" available on the lab computer at Lab Software/207/M-200.xlsx.

 PRELAB:

1. Read through the "Errors and Uncertainties" section, beginning on page 3.

2. Find documentation on the web for the Excel functions AVERAGE, STDEV, and LINEST that pertains to your version of Excel.

3. Familiarize yourself with how to make a scatter chart (without connecting lines) of data in Excel, how to add error bars to the data points, and how to add a trendine.

PROCEDURE

Let us use h to denote the distance between the bottom of the funnel and the paper. Your TA will choose which h you will use for this lab: $h = 10$ cm, 15 cm, 20 cm, 25 cm, or 30 cm.

1. Set the bottom of the funnel a distance h above the graph paper. Set the center of the graph paper directly below the center of the funnel, and mark the origin of a set of x-y axes on the graph paper directly below the center of the funnel. Indicate x and y directions.

2. One at a time, drop 100 chads through the funnel. Make a mark on the graph paper where the center of each chad has come to rest. Transfer data to Excel, indicating x and y coordinates of each of the chad centers. (Suggestion: work in teams, with one person dropping the chad, one person marking and reading the graph paper, and one person entering the data in Excel.)

    When you are done dropping chads, please collect all of them (check the lab table and floor for chads that have gone astray) and return them to the chad bin.

MEAN AND STANDARD DEVIATION

Below is an Excel scatter chart of the h=25 cm data. It is evident that the "center" of the distribution is somewhere near (0,0), and the "scatter" of the points is something like $\pm$ 5 cm in x and $\pm$ 5 cm in y. The concept of "mean" is (one way) to precisely quantify "center", and likewise "standard deviation" is (one way) to precisely quantify "scatter".
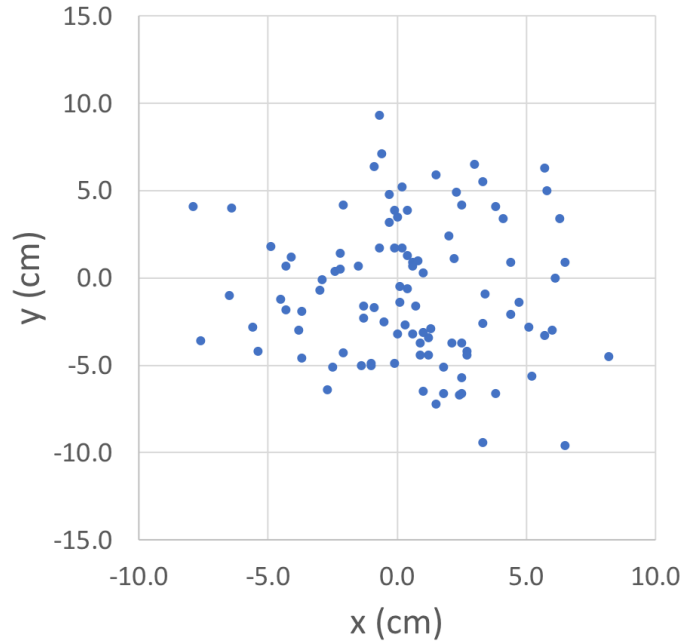
1. Make an Excel scatter chart of your data.

2. Calculate the mean $\bar{x}$ (using AVERAGE) and standard deviation σ_x (using STDEV) of the x coordinates of the data.

3. Calculate $\bar{y}$ and σ_y from the y coordinates of the data.

4. Print the scatter chart you just made (one copy for each lab partner).

5. Draw vertical lines on the chart at $x = \bar{x}$, $x = \bar{x} + 0.6745\sigma_x$, and $x = \bar{x} - 0.6745\sigma_x$.

6. Draw horizontal lines on the chart at $y = \bar{y}$, $y = \bar{y} + 0.6745\sigma_y$, and $y = \bar{y} - 0.6745\sigma_y$.

7. If the distributions of the $x-$ and $y-$ positions of the dots are "normal distributions", they should obey the following rules:

   • Half the dots are on one side of the mean, and half are on the other;
   • Half the dots are closer to the mean than 0.6745 standard deviations.

8. How many dots are to the left of $x = \bar{x}$? How many dots are below $y = \bar{y}$?

9. How many dots are within $0.6745\sigma_x$ of $\bar{x}$? How many dots are within $0.6745\sigma_y$ of $\bar{y}$?

10. Make a sketch of a distribution of 30 dots with means $\bar{x} = 0$ cm, $\bar{y} = 2$ cm, and standard deviations $\sigma_x = 1$ cm, $\sigma_y = 5$ cm. Use the same scale for x and y.

STANDARD DEVIATION OF THE MEAN

The *standard deviation of the mean* σ_μ is a measure of the uncertainty of the mean. The standard deviation of the mean σ_μ is different from the standard deviation σ discussed in the last section, which is a measure of the scatter of the data: $\sigma_\mu = \sigma/\sqrt{N}$. The larger the number N of data points you have, the smaller σ_μ becomes: .

1. Calculate the standard deviation of the mean $\sigma_{\mu,x}$ of your $x-$position data.

2. Calculate the standard deviation of the mean $\sigma_{\mu,y}$ of your $y-$position data.

3. Return to the printout you made in step 4 above, and draw a particularly dark dot at the position of $(\bar{x}, \bar{y})$.

4. Add vertical error bars to the dark dot that extend 0.6745 $\sigma_{\mu,y}$ above it and 0.6745 $\sigma_{\mu,y}$ below it.

5. Add horizontal error bars to the dark dot that extend 0.6745 $\sigma_{\mu,x}$ to either side of it.

Note that the error bars are much smaller than the "scatter" of the 100 dots, which is characterized by the standard deviations σ_x and σ_y.

The error bars convey that if someone else were to repeat the experiment of dropping 100 chads through an identical funnel, that person should expect a 50% chance that the center of the distribution of the chad locations would lie within the error bars.

PROPAGATION OF UNCERTAINTIES

The center of the distribution of dots is located at $(\bar{x}, \bar{y})$, and from this we can calculate $\bar{r} = \sqrt{\bar{x}^2 + \bar{y}^2}$, the distance of the center of the distribution from origin. The question then arises, "what is the uncertainty in $\bar{r}$?"

The answer can be worked out using multi-variate calculus (not expected of students in Physics 201 or 207):

$$\sigma_r = \frac{\sqrt{(\bar{x}\,\sigma_{\mu,x})^2 + (\bar{y}\,\sigma_{\mu,y})^2}}{\bar{r}}. \tag{1}$$

1. Calculate $\bar{r}$ and σ_r for your data.

2. Show your TA your results for $\bar{r}$ and σ_r. When your TA confirms them, please write them on the board.

You have just asked Excel to calculate the uncertainty in a quantity you did not directly measure ($\bar{r}$) based on uncertainties in quantities you did directly measure (x and y). This is an example of the "propagation of uncertainties".

3. How far from the origin did the average chad land? Calculate $\rho = \sqrt{x^2 + y^2}$ for each chad, and record $\bar{\rho}$. Is $\bar{\rho}$ the same as $\bar{r}$?

FITS AND UNCERTAINTIES IN FIT PARAMETERS

By now there should be a table on the board showing $\bar{r}$ and σ_r as functions of h. We might suppose that the larger is h, the less well we know $\bar{r}$. We will use Excel to check how well the measurements are fit by this supposition.

1. In Excel, make a scatter chart (without connecting lines) of σ_r, which measures how accurately we know $\bar{r}$, as a function of h.

2. Add a linear trendline to the chart. Note that what Excel calls a "trendline" is what in physics is called a "least-squares fit". Select "Display equation on chart".

3. Excel also calculates the uncertainties in the fit parameters. They can be accessed through the Excel function LINEST. In M-200.xlsx, see cells V6:W10 (raw data) and cells V13:W14 (trendline fit parameters and uncertainties) for an example.

4. Use LINEST to display the uncertainties in the trendline found in step 2. Two of the values LINEST returns should match the values shown on the chart. The others are the uncertainties.

5. Based on the LINEST output, what is the uncertainty in units of cm of the y-intercept of the trendline (i.e. the value of the trendline when $h = 0$)?

Excel can do quite a bit more analysis of uncertainties than needed in Physics 201 or 207.

VOCABULARY: "DISCREPANCY"

discrepancy (noun) an unexplained difference

If we lower the funnel until it sits on the paper ($h = 0$), every chad should land exactly at the origin ($r = 0$), so we expect $\sigma_r(h = 0) = 0$. Yet perhaps the linear trendline found in step 2 above does not go through the point $h = 0$, $\sigma_r = 0$. There is some kind of discrepancy (unexplained difference) between this new data point and the linear trendline.

1. What is the discrepancy (in cm) between $r = 0$ and the y-intercept of the trendline? (In physics 207 always report discrepancies as being greater than zero.)

2. What is the ratio R of (the discrepancy)/(the uncertainty in the trendline value at $h = 0$) ? (R should be a dimensionless number greater than zero.)

3. Summarize this result by writing the sentence: "The discrepancy between the observed σ_r at $h = 0$ and the value of the linear trendline at $h = 0$ is R times as big as the uncertainty in the trendline value". This elaborate phrasing is objective–it does not express an opinion on whether the observation or the trendline (or both) are in error. Also it is rigorous: it proceeds from the raw data by a set of automatic mathematical steps.

SUMMARY

1. Summarize what you did in this lab, and tell us what part of the material was new (this will help us teach you better).

2. What version of Excel are you using?

M-201 Errors & Motion

M-201a Measurement and Error

OBJECTIVES:

To develop an understanding of random and systematic errors in a measurement; to introduce PASCO hardware and software.

THEORY:

accuracy of a measurement: how close the measurement is to reality.

precision of a measurement: how consistent repeated measurements are.

systematic errors: errors which affect the *accuracy* of a measurement. Typically they are reproducible so that they always affect the data in the same way. For instance, if a clock runs slowly, its measurement of some duration will be less than the true duration. Taking more measurements does not reduce systematic error.

random errors: errors which affect the *precision* of a measurement. A process itself may have a random component (as in radioactive decay) or the measurement technique may introduce noise that causes the readings to fluctuate. Averaging over many measurements reduces random errors.

APPARATUS:

PASCO photogate and stand; PASCO 850 Universal Interface; PASCO Capstone software.

PROCEDURE:

How long is one second? The formal definition–the second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the Cesium-133 atom–is unhelpful for everyday affairs. In this lab you will test your ability to internalize the one second time interval by flicking your finger back and forth through an infrared beam sensor once per second. The duration of each full cycle (back and forth, approximately 2 seconds) will be simultaneously recorded, plotted and tabulated by the PASCO interface software.

Configure the experiment as in Fig. 1 below. Insert the phone-jack cable from the photogate into the DIGITAL INPUT #1 socket. If you pass your finger through the photogate, an infrared beam is interrupted and the red LED should light up. Adjust the photogate so that you can easily and repetitively flick your finger through the gap.

To launch the Pasco interface software, Capstone, click on the "Experiment M-201a" file in the Lab Software/201 folder on the desktop. A display should appear as in Fig. 2, showing a typical data run. In the table you can view all of the data points and the statistical analysis, including mean and standard deviation. The plot and histogram show the same data as the table.
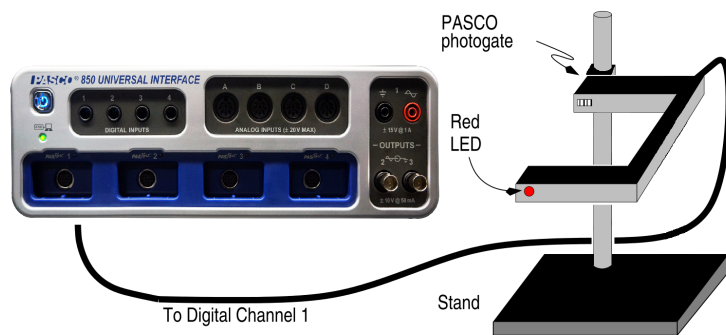
Figure 1: A schematic of the M-201a components and layout.

SUGGESTED PROCEDURE:

1. Start recording data by clicking on the "Record" icon and practice flicking a finger back and forth so that a two second interval appears in the window. The Record icon has now changed to the "Stop" icon ; click the Stop icon when done. Since the program started with one set of data, this will produce a second set.

2. Each run gets its own data set in the "Data Summary" tab under the **Tools** Palette. (If there are any data sets in existence you will *not* be able to reconfigure the interface parameters or sensor inputs.) A data set can be deleted by selecting the data set and clicking the red **X** to the right of the name. You can select which data set is displayed on a particular graph by clicking on the Data Summary icon that appears on that graph.
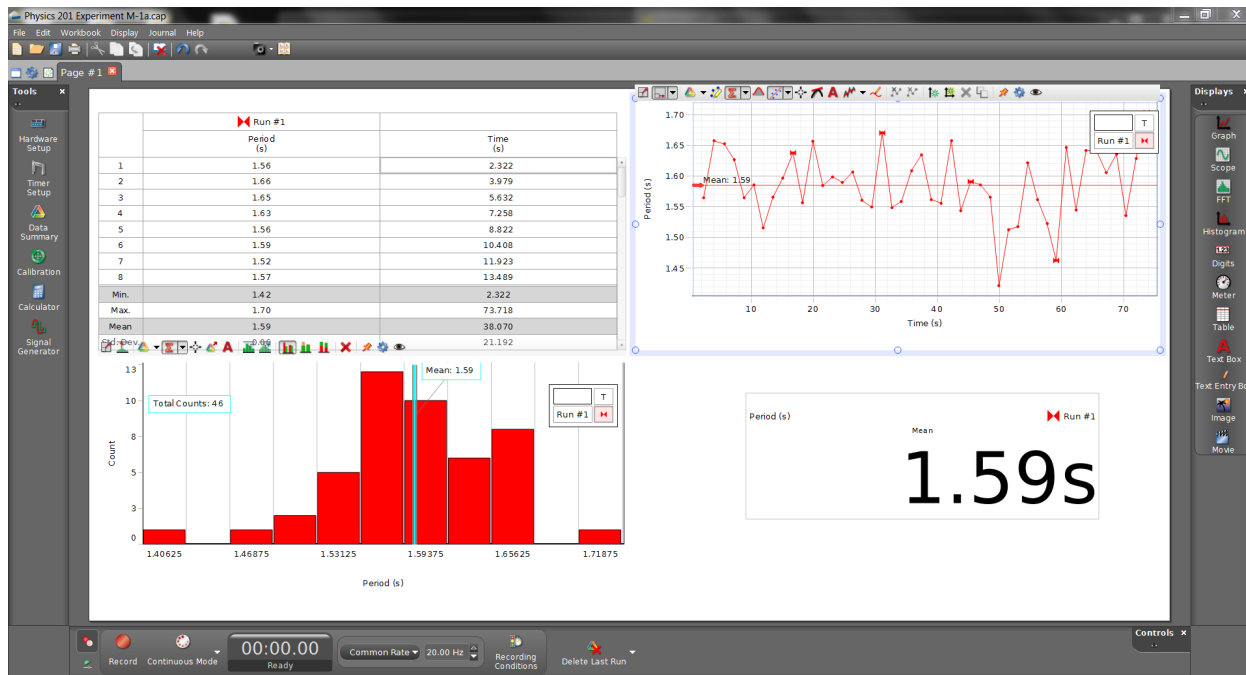
Figure 2: The PASCO Capstone display window

3. Once you are comfortable with the procedure then click on the Record icon, cycle a finger back and forth over fifty times, then click on the Stop icon. DO NOT watch the time display while you do this, since you want to find out how accurately and precisely you can reproduce a time interval of 2 s using only your mind.

4. **Table:** What is the mean time per cycle? What is the standard deviation? As described in the "Errors and Uncertainties" section on page 3, the mean $\bar{t}$ and standard deviation σ are given by:

$$\bar{t} = \sum_{i=1}^{N} t_i/N \quad \text{and} \quad \sigma = \sqrt{\sum_{i=1}^{N}(t_i - \bar{t})^2/[N-1]} \ .$$

(If these are not visible at the bottom of the table, click on the Statistics icon $\boldsymbol{\Sigma}$).

5. **Graph:** For a normal distribution approximately 68% of the data points fall within $\pm 1\sigma$ of the mean, and 95% fall within $\pm 2\sigma$. The graph display shows as a shaded region one standard deviation above and below the mean. How many data points are there, how many don't fall within one σ of the mean, and what percent of the points is this?

6. **Histogram:** A normal distribution looks like a "bell-shaped curve". What feature of the curve corresponds to the mean? What feature corresponds to the standard deviation?

7. Is the discrepancy between the mean and "reality" (in this case, two seconds) smaller than a standard deviation?

In general σ is a measure of how much a *single* measurement fluctuates from the mean, and thus describes the size of the random error. It doesn't change as more measurements are taken. The *standard deviation of the mean* $\sigma_\mu = \sigma/\sqrt{N}$ characterizes how well you know the mean. It gets smaller the more measurements you take.

8. What is σ_μ for your data?

9. The difference between the mean and reality is a test for the presence of a systematic error. If the difference between the mean and reality is much more than σ_μ, there is clear evidence of systematic effect. If the difference is much smaller than σ_μ, there is clear evidence against a systematic effect. Is there clear evidence of a systematic effect in your data set?

10. Which is larger in your dataset: the random error or the systematic error?

M-201b Errors and the Density of a Solid

OBJECTIVES:

> To learn to use vernier calipers; to learn about systematic errors and random uncertainties; to propagate uncertainties.

> NOTE: This experiment illustrates the earlier section on Errors and Uncertainties (see page 3).

APPARATUS:

> Hardened steel metal cylinder; vernier calipers; precision gauge blocks; precision balance.

PRECAUTIONS:

> Avoid dropping the metal cylinders, or deforming them in any other way.

> Disengage both thumbscrew locks on the caliper before attempting to use it.

> Improper weighing procedures may damage the precision balance. Consult your instructor if in doubt.

> In handling the gauge blocks avoid touching the polished surfaces since body acids are corrosive.

INTRODUCTION:

> First read the material on Errors and Uncertainties (page 3). You will measure the density (mass per unit volume) of a metal cylinder by first measuring the cylinder's mass m (on a balance), and then calculating the volume ($h\pi r^2 = h\pi d^2/4$) from measurements of the cylinder's diameter d and height h. Then you will estimate the uncertainty in your determination of density by propagating the uncertainties in your determinations of m, d, and h.

> Calculations may conveniently done in Excel: use Excel "average" function to calculate mean, and Excel "stdev" function to calculate standard deviation.

THE VERNIER CALIPER:

> The vernier scale was invented in 1631 by Pierre Vernier. Work through Appendix A for practice reading a vernier scale. A helpful video is here: https://mediaspace.wisc.edu/media/M-201b_Vernier_Calibration_Rev/1_vm3rtq18 Experiment with one of the large verniers in the lab until you are confident you can use it correctly.

Precautions on use of the calipers:

1. Unclamp **both** top thumbscrews to permit moving caliper jaws.

2. Open caliper to exceed the dimension being measured by about 1 mm.

3. Close right thumbscrew to lock position of lower horizontal knurled cylinder. Rotate the knurled cylinder to make the measurement, then rotate it the other direction to release the object being measured. **Never over-tighten!**

CALIBRATION:

1. Zero Error: wipe the caliper jaws with cleaning paper. Close the jaws and make a reading. Record the reading, separate the jaws, close them again, and make another reading. Continue in this way until you have recorded five readings. The mean of these readings is the *zero error*. Record the zero error, and standard deviation of your measurements.

2. Measure all four calibration gauge blocks (6, 12, 18 and 24 mm): Set the gauge blocks on end, well-in from the edge of the table, thus freeing both hands to handle the caliper. Record the actual (uncorrected) reading. A single measurement of each block will suffice.

3. Using data from the two steps above, plot a correction curve for your measuring device: plot errors as ordinates (vertical axis) and nominal measurements (0, 6, 12, 18, and 24 mm) as abscissae (horizontal axis). Fit a straight line to your measurements. This line represents your estimate of the systematic error of your use of this particular vernier caliper.

DENSITY DETERMINATION:

1. Make five measurements (should be in millimeters) of the height and ten of the diameter. Since our object is to determine the volume of the cylinder, distribute your measurements so as to get an appropriate average length and average diameter. Avoid any small projections which would result in a misleading measurement. If not possible to avoid, estimate their importance to the result. Record actual readings and indicate, in your lab book, how you distributed them.

2. Calculate and record the mean height, mean diameter, and standard deviations thereof.

3. Use your correction curve to correct for systematic error. If you were to use the uncorrected values, how much fractional error would this introduce?

4. From the corrected mean dimensions, calculate the volume. From the fractional uncertainties in the dimensions, calculate the fractional uncertainty in the volume, and then the uncertainty in the volume. Refer to the "Propagation of Uncertainties" section (page 5) for guidance. Record your results.

5. Weigh the cylinder twice on the electronic balance; estimate to 1 mg; record the mean and standard deviation of the two measurements.

6. Calculate the density (in units of g/cm^3), the fractional uncertainty in the density, and thus the uncertainty in the density. Record your results.

7. Some familiar metals (of which the cylinder might be made) are: Aluminum, bronze, Iron, Lead, Nickel, steel, Tin. Which of these are ruled out by your measurements?

Link to the world's premier institution devoted to improving the accuracy of measurements:

National Institute of Science & Technology at http://www.nist.gov/

M-201c Position, Velocity and Acceleration

OBJECTIVES:

Become familiar with the PASCO interface hardware and software; relate subjective observation of motion to mathematical description by experimenting with 1-D motion.

THEORY:

The motion of an object is described by indicating its distances x_1 and x_2 from a fixed reference point at two different times t_1 and t_2. From the change in position between these two times one calculates the average *velocity* (remember velocity is a vector, and specifies direction) for the time interval:

$$\text{average velocity} \equiv \overline{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \ m/s$$

The *acceleration* of an object may be calculated by finding its velocity v_1 and v_2 at two different times t_1 and t_2. From the change in velocity between t_1 and t_2 one calculates the average acceleration for the time interval:

$$\text{average acceleration} \equiv \overline{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \ m/s^2$$

FUNDAMENTAL CONCEPTS OF LINEAR MOTION:

1. The equation that describes the motion of an object that moves with *constant velocity* is: $x = x_0 + v \cdot t$. x_0 is the position of the object at time $t=0$, and v is the (constant) velocity.

2. The equations that describe the motion of an object that moves with *constant acceleration* are: $x = x_0 + v_0 \cdot t + \frac{1}{2}at^2$ and $v = v_0 + at$. x_0 again represents the position of the object at $t=0$, v_0 is the object's velocity at $t=0$, and a is the (constant) acceleration.

APPARATUS

PASCO motion sensor, and stand; PASCO 850 Interface; PASCO dynamic track with magnetic bumpers; cart with reflecting vane; meter stick; one or two steel blocks.

The PASCO motion sensor emits a series of short pulses of sound (each pulse sounds like a "click"), and receives a series of echoes reflected from a nearby object. The time elapsed between the emission and the reception of a sound pulse is equal to the distance to the object divided by the speed of sound (this is also how bats find flying insects and military ships locate submarines).

Thus the motion sensor directly measures the position of an object as a function of time, then calculates from this the velocity and acceleration of the object, using successive position measurements, by an algorithm like that given above in the "THEORY" section.

EXPERIMENT I: BASIC OPERATION AND MOTION SENSOR CALIBRATION:

PROCEDURE:

1. Connect the yellow plug of the motion sensor cable to digital channel 1, and the black plug to digital channel 2.

2. Double click the "Experiment M-201c" file in the Lab Software/201 folder on the desktop. Fig. 1 below shows the display that should appear. All three measured quantities–position, velocity, and acceleration–are displayed simultaneously.
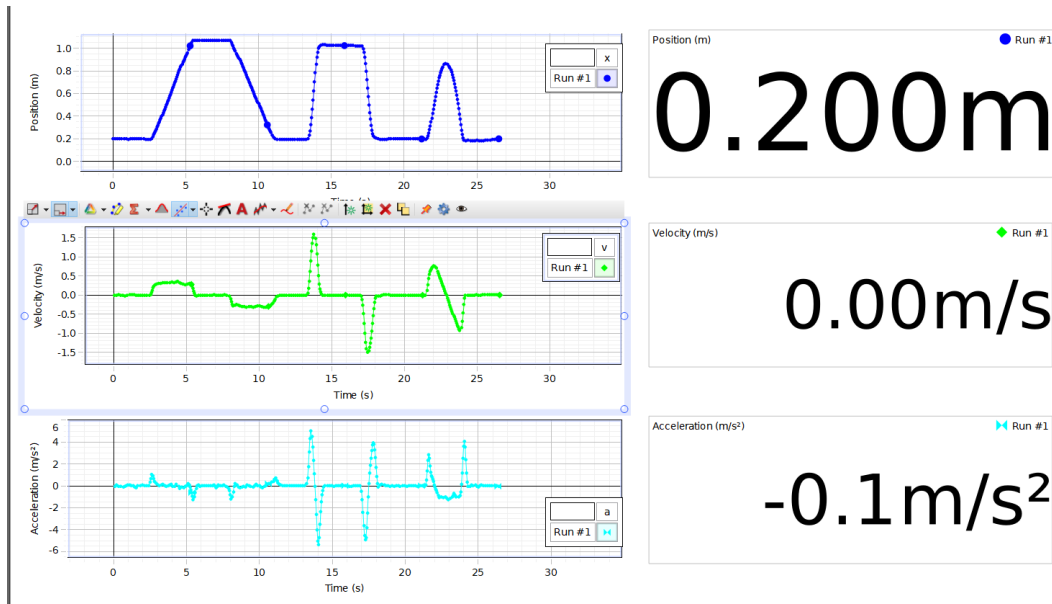

Figure 1: The PASCO Capstone display format

3. Referring to the above figure: identify a time when the object is not moving, and also not accelerating.

4. Identify a time when the object is moving, but not accelerating.

5. Identify a time when the object is accelerating, but not moving.

6. Attach the reflecting vane (which is magnetic) to the cart. Put the motion sensor on the stand, with the top of the reflective disk at the same height as the top of the vane, and facing it, as in the figure below. Tilt the motion sensor 5° upward so that the sound it emits doesn't reflect from the bottom of the cart. Note the minimum range of the motion sensor is approximately 20 cm.
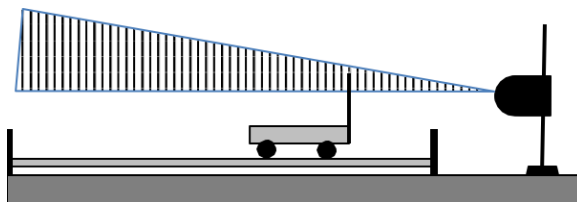

Figure 2: How to position the motion sensor

7. To start the data acquisition click on the Record icon . To stop it click on the same icon which will have become the Stop icon .

8. Each run gets its own data set in the Data Summary tab under the **Tools** Palette.

In what follows, make sure you don't drop the cart or allow it to roll off the table. This damages the bearings and drastically increases the friction.

9. Start the data acquisition, and try to reproduce Fig. 1 by moving the cart to and fro and while watching the position, velocity and acceleration displays. When you are satisfied, stop the data acquisition, print out the screen using Ctrl-P, and tape it into your lab notebook.

10. Stop the data acquisition.

11. Explore what some of the other buttons do:

   - Click on the Zoom-to-Fit icon on one of the plots.
   - Click on the Data Highlight icon and the Zoom-to-Fit icon to zoom in to particular regions of interest. Use Ctrl-Z to get back to the original plot area.
   - Click on the Crosshair icon and move about within a plot window.
   - Click on the Statistics icon to bring up statistics for a given set of data.

   Make sure that all members of the group have an opportunity to test these. It will facilitate the rest of the course if the basic operations on the software interface are understood by everyone!

12. Delete the data set by clicking on the red 'X' next to RUN #1 item in the Data Summary tab under the **Tools** Palette.

13. What is the minimum distance the vane can be to the motion sensor and still have its position read accurately? (Expect the distance to be close to 20 cm.)

14. If necessary, move the motion sensor further from the track so that when the cart touches the near magnetic bumper the motion sensor still records accurately.

15. How accurate is the motion sensor? Measure the position at two distances approximately 0.8 m apart and compare the printed centimeter scale with the motion sensor readout. Does the motion sensor appear to be at least as accurate as the scale?

EXPERIMENT II: INCLINED PLANE AND MOTION:

1. Raise the side of the track closest to the motion sensor using one of the blocks.

2. Find an appropriate release point that allows the cart to roll down the track slowly enough that it bounces off the magnetic bumper *without* striking it.

3. Click the Record button and release the cart, letting it bounce three or four times, and then click the Stop button.

4. Qualitatively describe the shape of the three curves: position, velocity and acceleration; use words like "constant", "linear", and "parabolic".

5. Use Ctrl-p to obtain a hard copy of this data.

6. Label/identify the various key features in the various curves by writing descriptive phrases like "cart bounces off bumper", "cart accelerating", "cart decelerating", or "cart motionless" directly on the hard copy, and paste it in your notebook.

7. The cart bounces less high with every bounce for two reasons: there is friction in the bearings, so the cart slows down as it rolls, and when it hits the bumper the bumper flexes a little bit, so the cart doesn't come off the bumper as fast as it went in. Write down (using complete sentences) how these two effects are visible in the hard copy.

EXPERIMENT III: ACCELERATION AT g (~ 9.8 m/s^2):

1. Have one member of the group stand *carefully* on a chair and hold the position sensor facing downward above a second member's head, while the second member holds a notebook above his or her head.

2. Start a new data set and have the student holding the notebook jump straight up, trying to remain in the air as long as possible, keeping the notebook flat to the top of the head. Be sure the jumper doesn't bump into the motion sensor, or the person holding it.

3. Stop the recording and look at the data. If there doesn't appear to a be a range of times when the position is a parabolic function of time, take another dataset.

4. When the notebook-holder is moving closer to the motion sensor (ie upwards), what is the sign of the velocity?

Try to measure the acceleration g due to gravity in three ways by making three fits to data.

5. First, use the Data Highlight icon to select the position data you think corresponds to the time when the notebook-holder was off the ground. Write down a good criterion by which to select this data.

6. Then, click on the Fit icon to get at the pull-down menu; choose "Quadratic". Record the estimate of g and uncertainty that results. Make sure to include units.

7. Next, choose the appropriate velocity data. What is a good criterion to use to identify the moment the notebook holder leaves the grouns?

8. Make a "Linear" fit to the velocity data. Record the estimate of g that results.

9. Finally, choose the appropriate acceleration data, and use the statistics button to find its mean, and record the resulting estimate of g.

10. It is very unlikely all three fits will yield the same results (that is the nature of experimental science!). Which fit do you trust the most–and why?

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M-204   Acceleration in Free Fall

OBJECTIVE: To measure g, the acceleration of gravity; to propagate uncertainties.

APPARATUS:

    Spark timer; cylindrical bob; paper tape; cushion.

INTRODUCTION:

    A spark timer is a device that generates a spark at very precise intervals of time. A heavy bob is connected to a paper tape, which is routed through the spark timer (see figure below). When the tape is released, the bob accelerates downlard. Sparks pass between points A and B every 1/60 of a second, leaving little burn marks on the paper. From the positions of these burn marks, the acceleration of the bob can be determined. In order to make as an accurate as possible a measurement of g, the acceleration due to gravity, we will attempt to account for two effects that reduce the acceleration of the bob: the drag of the paper tape as it passes through the spark timer ("tape drag"), and air drag.

PROCEDURE:

1. Find a stable horizontal surface about 1.5 m above the floor. Position spark timer chassis near the edge of the surface, and with the convex surfaces (A and B) extended beyond the surface. Put cushion on floor directly under A and B.

2. Cut a 1.5 m length of paper tape, and insert one end between the two halves of the cylindrical bob. Fasten together with thumb screw.

3. Team up with your lab partners: have one partner find a stable place to stand, and hold the tape end high enough (vertically) above A and B that the bob just touches below A and B. Have another lab partner stand in front of, and several meters away from, the spark timer, and tell the first lab partner when the tape end is centered side-to-side over the spark timer. When the tape is centered, have the third partner press the button on the back of the spark timer, and immediately say "Go!", at which time the first lab partner is to release the tape. Discard any part of tape which fell through the spark gap after bob hit the cushion.
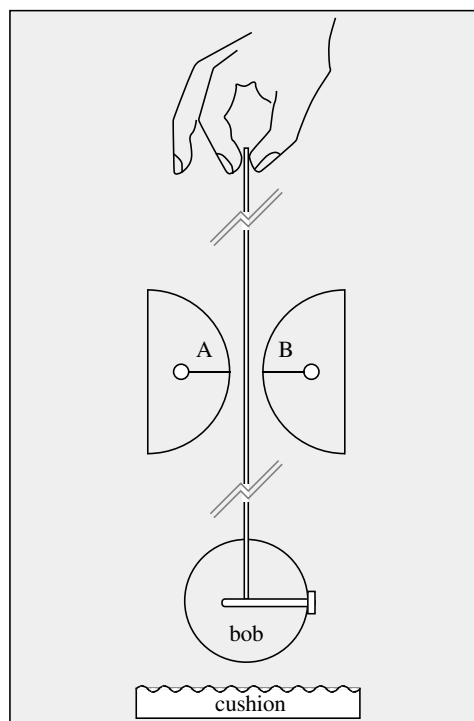
4. Place a meter stick on its side (on top of tape) so that the ends of the mm graduations are as close as possible to the dots on the tape.

5. Recording and analysis of data may conveniently be done in Excel. Start a table of data, with column headings "dot number", "position x (cm)", "uncertainty in position (cm)", "time (s)".

6. There should be a giant black splodge on the tape where many sparks passed through the tape before the tape started moving. Label this "dot 0", but ignore it in the analysis.

7. Starting with dot 1, record the position of each dot as you see it with your eye directly above the dot, in order to avoid parallax error (see the section on Parallax in Appendix B, p. 67). Make sure the meter stick and tape don't move while you are making measurements.

8. Aside: this is an excellent occasion to become acquainted with parallax error. Pick a dot, and record the position of the dot with your eye two cm to one side of the position of the dot. By how much did the position of the dot appear to change, compared to your reading from the previous step? This is parallax error. The meter stick is about 1/4-inch thick.

9. Estimate the uncertainty in your measurement of the position of each dot. If you are less certain of the position of one or more dots than of the others, make sure to record their uncertainties as larger than the others.

10. Assume spark timer timing errors are negligible, so the time between sparks is 1/60 second, and the uncertainty in the time of a position measurement is zero.

11. Make a graph of the data, including error bars indicating the uncertainty in the position measurement. Make sure to label axes appropriately.

12. Scrutinize the graph. Do you see any evidence of drag? If so, record it.

13. Now start a new table for the recording of speeds. Make the headings "nominal dot number", "$x_j - x_{j-1}$", "v (cm/s)", "uncertainty in v (cm/s)", and "nominal time (s)".

14. The first row will be for nominal dot number 1.5. Enter the distance between the first two dots, $x_2 - x_1$, in the second column. Divide this by the time interval between the dots, 1/60 s, to get the average speed of the bob during the time interval between the first two dots, and enter this in the third column. Propagate the uncertainties in the dot positions, and enter the result in the fourth column (for guidance see the section on Propagation of Error on page 5). Take the average of the times of dot 2 and dot 1 and enter it in the fifth column.

15. Complete the remaining rows in an analogous fashion.

16. Make a plot of speed vs. time, including error bars and axis labels.

17. Air drag is proportional to speed. The effect of air drag on a freely-falling object is to make the acceleration decrease as the velocity increases (eventually, the acceleration

reaches zero, at which point the object is falling with "terminal velocity"). The effect of tape drag is unpredictable–in a worst-case scenario, the tape could get tied in a knot and get stuck in the spark timer, in which case the velocity of the bob would drop to zero. Scrutinize the plot of speed vs. time, and write down any evidence you see for air drag and tape drag.

18. Following the above scrutiny, determine which data points should be excluded from further analysis because they were affected by air drag or tape drag. State which dots are excluded.

19. Make a final plot, showing the positions of just the dots that haven't been excluded as a function of time. Fit a 2nd-order polynomial trendline to this data set. Display the equation on the chart. Record the value of g given by the fit.

20. What are the uncertainties in the fit parameters given by the trendline? An Excel Trendline is apparently an application of the Excel "LINEST" function. Let us suppose the position data is in cells B4:B31 and the time data is in cells D4:D31. In order to show the coefficient of x^2 shown in the trendline, select an empty cell and type the following into it:

    = INDEX(LINEST($B$4:$B$31,$D$4:$D$31ˆ{1,2},TRUE,TRUE),1,1)
    The cell should now display the coefficient of x^2 shown on the graph.

    In an adjacent cell, type
    =INDEX(LINEST($B$4:$B$31,$D$4:$D$31ˆ{1,2},TRUE,TRUE),2,1)
    This cell should now display the uncertainty in the coefficient of x^2.

    Based on the above LINEST parameters, record your measurement of g, and its uncertainty. Make sure to include units.

21. The local value of g has been measured to be 9.803636 ± 0.000001 m/s^2.[1] Compare this to your value of g: is the discrepancy between this value of g and your value of g larger or smaller than the uncertainty in your value of g?

In measurements made before the semester, the discrepancy was found to exceed the uncertainty by a factor of about 7. This suggests that more measurements would be needed to fully account for drag (either air or tape).

Follow-up Questions

A. *Meaning of "uncorrelated" errors* In the section on Propagation of Uncertainties on page 5, certain rules were given for propagating uncertainties in the case that "errors are uncorrelated". If we were to attempt to calculate the average acceleration of the bob, as a function of time, by taking the difference in successive velocity measurements, we would encounter a situation where errors where not uncorrelated.

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[1] Wollard and Rose, "International Gravity Measurements", UW Geophysical and Polar Research Center, (1963) p. 211 and p. 236.

Suppose an error of δ was made in the measurement of the position of dot j. What effect would this have on $v_{j+1/2}$, the speed associated with nominal dot *j+1/2*, and on $v_{j-1/2}$, the speed associated with nominal dot *j-1/2*?

B. We might dutifully calculate the acceleration of the bob at that time as $a_j = (v_{j+1/2} - v_{j-1/2})/(1/60s)$. However, there would be a correlation between the errors in $v_{j+1/2}$ and $v_{j-1/2}$. Describe this correlation.

C. *Measurement of reaction time* Find a partner. Have your partner hold a meter stick by the top, letting it dangle vertically, while you place your thumb and forefinger opposite the 50 cm point, but not grasping it. Ask your partner to release the stick without giving you a warning. When your partner releases the stick, grab it as soon as possible. From the distance through which the 50 cm mark fell, calculate the time of free fall of the meter stick. This is your "reaction time". Record the distance the meter stick fell, and the reaction time you calculated.

D. *M-204 in Space* The International Space Station orbits the earth, which has a radius of about 6400 km, at an altitude of about 400 km. Newton's law of gravitation predict the force of gravity drops off as $1/r^2$. If the acceleration due to gravity at the Earth's surface is ∼9.80 m/s², one would predict that the acceleration due to gravity at the altitude of the space station would be about

$$9.80 \,\text{m/s}^2 \times \left(\frac{6400 \,\text{km}}{6400 \,\text{km} + 400 \,\text{km}} \right)^2 \sim 8.68 \,\text{m/s}^2.$$

However, if lab M-4 were performed as written by an astronaut on the space station, the astronaut would measure an acceleration of 0 m/s²–the bob would appear to be "weightless". How can this be?

For further investigation: The acceleration due to the Earth's gravity at the location of the top of the bob is slightly smaller than the acceleration due to the Earth's gravity at the bottom of the bob, which is ∼ 3 cm closer to the center of the Earth. This is an example of a "tidal" effect (effect due to the gradient of a gravitational field). Tidal effects can be as large as ≈ 10^{-7} *g*, and one must correct for them. See Handbuch der Physics, Vol XLVIII, p. 811; also Wollard and Rose, p. 183.

M-205 Projectile Motion

OBJECTIVE: To find the initial speed and predict the range of a projectile, by use of the laws of conservation of momentum and conservation of energy.

APPARATUS: Ballistic pendulum with spring gun, plumb bob, catcher, protractor, and elevation stand; balance; large plumb bob.
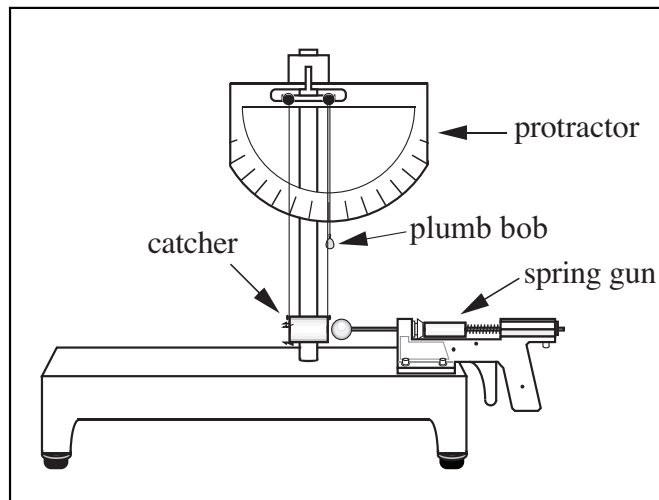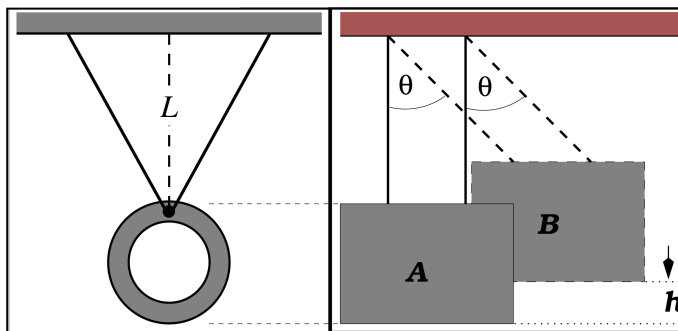

Figure 1: The ballistic pendulum.

Figure 2: Front and side views of the catcher

THEORY:

A ballistic pendulum is a device commonly used to determine the initial speed of a projectile. A spring gun shoots a ball of mass m into a pendulum catcher of mass M (see Figs. 1 and 2). The catcher traps the ball; thereafter the two move together. Linear momentum is conserved, so the momentum of ball before impact equals the momentum of ball plus catcher after impact:

$$mu = (m + M)V \tag{1}$$

where u = ball's speed before impact and V = initial speed of combined catcher plus ball.

To find V, note that motion **after** impact conserves mechanical energy. Hence the kinetic energy of the ball plus catcher at A in Fig. 2, just after impact, equals the potential energy of the two at the top of the swing (at B). Thus

$$\frac{1}{2}(M + m)V^2 = (M + m)gh . \qquad \text{Hence,} \qquad V = \sqrt{2gh} . \qquad (2)$$

We will measure h, and estimate its standard deviation σ_h from a set of measurements of h. We will use these data to calculate u and its uncertainty σ_u. This is an example of the "propagation of uncertainties". Using the ideas set forth in the section "Propagation of Uncertainties", on page 5, we can show that, if $y = Ax^n$, where A and n are constants, then

$$\frac{\sigma_y}{y} = n\frac{\sigma_x}{x}.$$

PART I: LEVELING, ALIGNMENT, AND CALIBRATION:

1. Leveling: rotate the knurled screw on the front underside of the ballistic pendulum until the plumb bob hangs at $90°$, as indicated by the protractor.

2. Alignment: adjust the lengths of the four strings suspending the catcher, by adjusting each of the four screws at the top of the ballistic pendulum, until the axis of the catcher is collinear with the axis of the spring gun.

3. Measure m, M and the length L of the pendulum (see Figure 2). Also record ΔL, the uncertainty in L. Minimize parallax error (see Appendix B) when measuring L.

4. Practice cocking the gun, firing the ball into the catcher, and recording the maximum angular deflection of the pendulum. Make sure to position your eye so that, at the moment of maximum deflection of the pendulum, the string closer to you on the back of pendulum appears to be directly in front of the string further from you on the back of the pendulum, thus avoiding parallax error. Repeat until you are confident in your result to within one degree.

5. For three trials, measure θ, the maximum deflection of the pendulum ($= 90°-$ reading on the protractor), and its uncertainty $\Delta\theta$, and from θ find the increase in height of the pendulum $h = L - L\cos\theta$. Record θ in units of radians.

6. Calculate the mean rise $\bar{h}$ and standard deviation σ_h for the three trials.

7. The purpose of this step is to compare the shot-to-shot variation in the pendulum height increase, calculated in the previous step, to the uncertainty of a particular measurement of the pendulum height increase. It can be shown (using the ideas set forth in the section "Propagation of Uncertainties", on page 5) that the uncertainty Δh associated with h is given by:

$$\frac{\Delta h}{h} = \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \frac{\sin^2\theta}{(1 - \cos\theta)^2}\Delta\theta^2} \qquad (3)$$

Calculate Δh for each of the three shots. Is the shot-to-shot variation in h much bigger than the typical Δh, much smaller, or about the same?

8. From the measured value of h, and σ_h, use the conservation of energy (after impact) and the conservation of momentum (during impact) to calculate the initial speed of the ball as it leaves the gun, u, and its uncertainty.

9. What fraction of the initial kinetic energy of the ball was lost during the collision?

PART II. RANGE MEASUREMENTS
HORIZONTAL SHOT:

1. Remove the catcher from the ballistic pendulum; move the strings out of the way.

2. Measure and record D, the height of the bottom of the ball above the floor.

3. Given u and its uncertainty from Part I, predict the range R (see diagram below) and its uncertainty for the ball when shot horizontally from a position on the table.
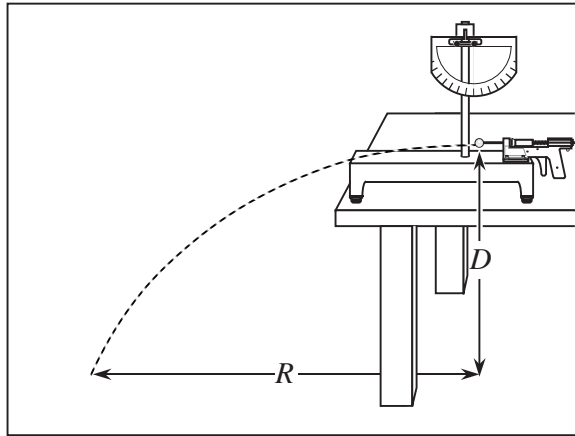
4. Check your prediction experimentally by carrying out this and the next few steps. Let $x=0$ correspond to the horizontal location of the center of the ball when it starts free fall. What is the relationship between $x=0$ and the end of the spring gun, when the spring gun is fully extended? (Hint: the radius of the ball is involved somehow; record the radius of the ball).

5. Using the large plumb bob (the one not attached to any ballistic pendulum), put a piece of tape on the floor and make a mark on it where $x=0$. Also put a piece of tape on the table just behind the rear feet of the ballistic pendulum, so that you can move it back into position if it recoils when fired.

6. Tape a piece of paper on the floor at the predicted point of impact (the ball's impact on the paper leaves a visible mark). Just beyond the paper put a box to catch the ball on the first bounce.

7. Take five shots. Record the mean and standard deviation of the observed range. Did the ballistic pendulum recoil during any of the three shots?

8. Which is larger, and by what percentage, the uncertainty in the predicted R (which is based on the uncertainty in u), or the standard deviation of the observed R?

9. What is the discrepancy between the predicted and observed R, and is it greater or less than the larger of the two quantities mentioned in the last step?

10. Suggest one thing you might do, if you had more time, to try to track down the source of the discrepancy.

11. Calculate a revised value of u, based on the observed range.

ELEVATED SHOT:

1. Use the stand provided to elevate the gun at an angle above the horizontal. Call this angle ϕ: it can be read with the plumb bob and protractor: $\phi = 90°$ - protractor reading. Record ϕ.

2. Measure the new value of D, the height of the bottom of the ball above the floor at the first instant it is no longer supported by the spring gun.

3. Remark $x = 0$ on the floor.

4. Predict the range, using the value of u which you found in the "horizontal shot" procedure above. Keep in mind that the initial vertical and horizontal velocities of the ball are $u \sin \phi$ and $u \cos \phi$.

5. Make five shots; record the mean and standard deviation of the ranges.

FINAL QUESTIONS:

1. Which measurement was more repeatable, horizontal shot range or elevated shot range? State your answer in one english sentence, supported by one mathematical inequality involving the fractional variation $\Delta R/R$ in the observed ranges.

2. When the gun is cocked, the spring is compressed by about 4.6 cm. From your data from Part I, find the spring constant k of the gun, in units of N/m, from $\frac{1}{2}mu^2 = \frac{1}{2}kx^2$.

M-206 Uniform Circular Motion

OBJECTIVE: To verify whether the centripetal force F_c is correctly calculated as

$$F_c = \frac{mv^2}{r} = m\omega^2 r$$

APPARATUS:

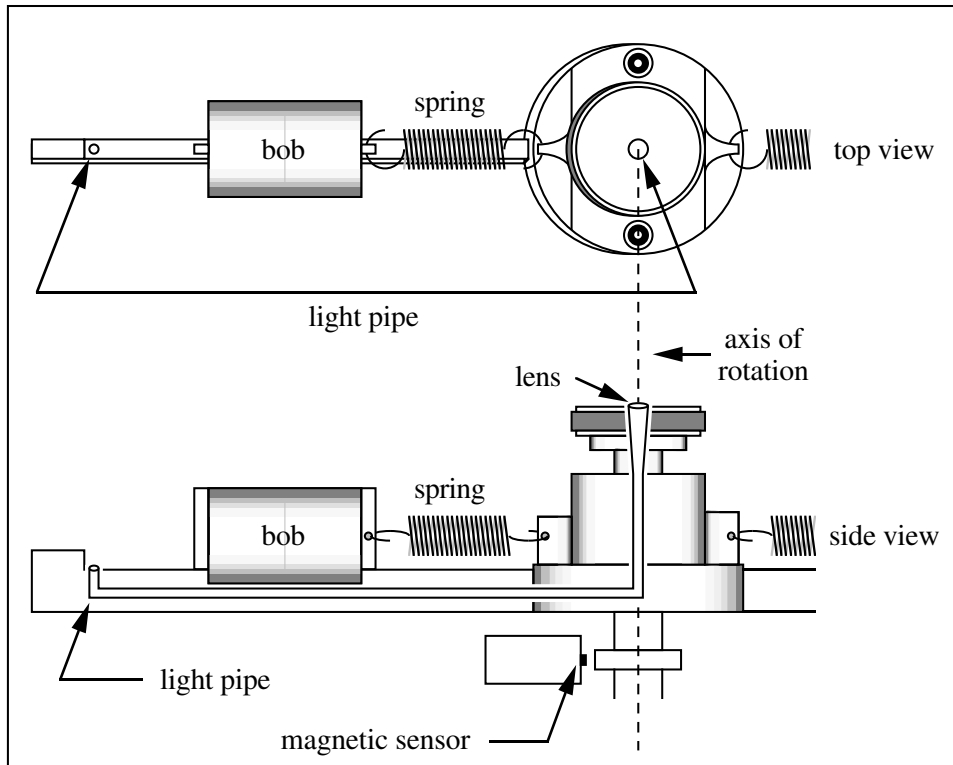Fig. 1 is a schematic of the rotating system. The bobs and springs are removable for weighing.

Figure 1: The rotating sytem (only one side shown).

A variable speed motor drives the rotating system which has two slotted bobs which slide on a low-friction bar.

A revolutions counter is on the shaft. The counter operates by sensing rotating magnetic poles. The display shows the frequency of revolution in rpm.

A spring (plus any friction) supplies the centripetal force required to keep the bob traveling in a circle.

Suppose one measures the angular frequency ω required for one bob to just cover the light pipe, and then measures the force F needed to pull the bob out to the same distance when the system is not rotating. Assuming $F = F_c$, one can check whether

$$F_c = m\omega^2 r$$

PROCEDURE:

1. Measurement of m: measure the mass of the nickel-plated brass bob, and the mass of the spring. Record the masses and the uncertainties thereof.

2. Measurement of ω: Replace the brass bob, spring, and plastic cover. Adjust the motor speed until the light pipe at the center of the rotating system goes dark.

   To correct for frictional effects of the bob on the bar, record the frequencies both as the speed is slowly *increased* to the correct value and as the speed is *decreased* from too high a value. Since the direction of the frictional force reverses for the two cases, take the average of ω^2 to eliminate the frictional effect.

   Repeat several times. Record your best value for, and uncertainty of, ω^2.

3. Measurement of F: Use the string, pulley, weight holder, and slotted weights to measure the force required to stretch the spring so that the optical light pipe is again just covered.

   Make sure the string is collinear with the spring. Devise a way to account for error caused by the friction at the pulley and of the sliding bob on the bar. Record F and the uncertainty thereof.

4. Measurement of r: while the spring is stretched to its proper length (item 3 above), measure the distance r from the axis of rotation to the center of mass of the bob. The center of mass is marked on the bob. Record r and the uncertainty thereof.
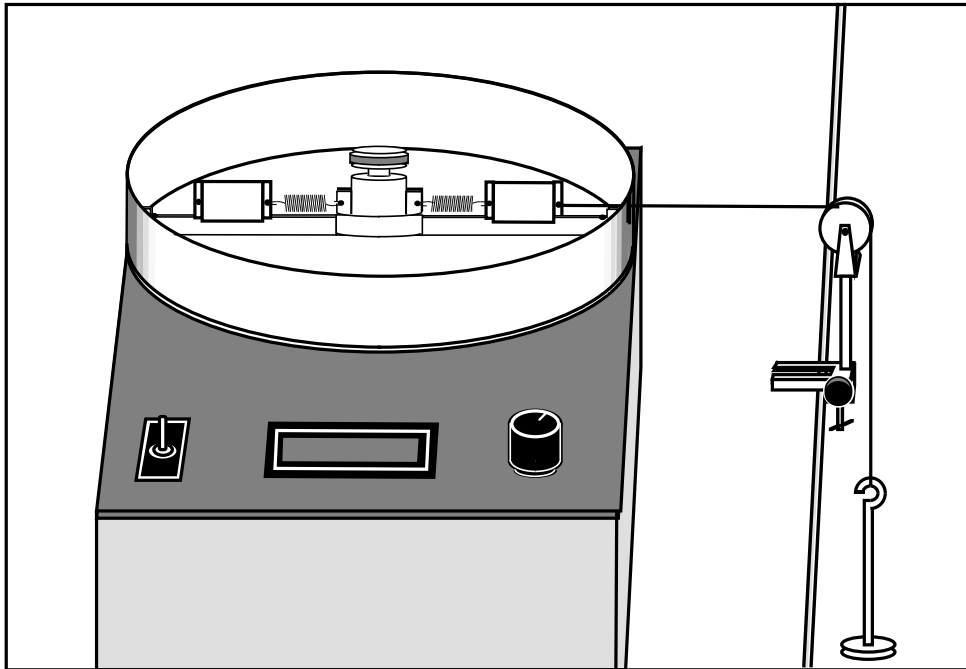

Figure 2: Static measurement of the force using hanging weights

5. Compute the putative centripetal force $F_c = m\omega^2 r$, and its uncertainty, from your measurements of m, ω, and r. Recall that if $y = Ax^n$, where A and n are constants, then

$$\frac{\sigma_y}{y} = n\frac{\sigma_x}{x},$$

and that if $y = Av^l w^m x^n$, and the uncertainties in quantities v, w, and x are uncorrelated, then the uncertainties add in quadrature:

$$\frac{\sigma_y}{y} = \sqrt{\left(\frac{l\sigma_v}{v}\right)^2 + \left(\frac{m\sigma_w}{w}\right)^2 + \left(\frac{n\sigma_x}{x}\right)^2}.$$

Compare F_c to the static measurement of the force, F. Is the discrepancy between F and F_c larger or smaller than the uncertainties in F and F_c?

6. Replace the brass bob with the aluminum bob, and repeat steps 1, 2, and 5 above.

7. In computing the centripetal force, it can be shown (Weinstock, *American Journal of Physics*, 32, p. 370, 1964) that $\approx 1/3$ of the spring mass should be added to the mass of the bob to obtain the total effective mass. Compute the magnitude of this correction for both bobs, in N.

8. Summarize the evidence given by your results to support (or otherwise) the conjecture that centripetal force is calculated by $F_c = m\omega^2 r$. If you had more time, which of your measurements would you try to improve?

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M-209 Angular Acceleration, Moment of Inertia, Angular Momentum

OBJECTIVES:

To study rotational motion resulting from constant *torque*, causing constant *angular acceleration*; to investigate the *moment of inertia* of an object about different axes of rotation; to demonstrate conservation of *angular momentum.*

THEORY

In this lab we consider a rigid object rotating about a fixed axis with **constant** angular acceleration. We use the notation:

$\theta(t)$: orientation (i.e. angle) of object at time t. Units: radians.
θ_0: orientation of object at $t = 0$. Units: radians.
$\omega(t)$: *angular speed* of rotating object at time t. Units: radians/second.
ω_0: angular speed of rotating object at $t = 0$. Units: radians/second.
α: angular acceleration of rotating object. Units: radians/second^2.

For a constant α, $\theta(t) = \theta_0 + \omega_0 \cdot t + \frac{1}{2}\alpha t^2$ and $\omega(t) = \omega_0 + \alpha t$. If $\alpha = 0$, these simplify to $\theta(t) = \theta_0 + \omega \cdot t$, and $\omega = \omega_0$.

In this lab always use radians and never degrees. One full revolution of an object corresponds to 2π radians.

A constant α is caused by a constant torque τ, via $\tau = I\alpha$, where I is the moment of inertia of the rotating object.

In the absence of torque from external forces the angular momentum $L = I\omega$ of a rotating system is a constant.

Check units on each equation that you write. For example, if you are using v to denote linear speed (units = m/s), and you find yourself writing an equation like $v = \omega$, then you must recheck your work–the units do not match.

APPARATUS:

PASCO interface; rotational assembly; photogate sensor and support; pulley; solid plastic disk; aluminum bar; black metal ring; black metal square; a length of string; weight hanger and various slotted masses; scale for mass measurements.

PRE-LAB QUESTIONS:

The figure below depicts a solid disk of mass M and radius R that is free to rotate about the axis as shown. The inner hub is massless but has radius r. Around this inner hub is wrapped a string (as shown) that goes over a frictionless pulley and this is attached to a hanging mass m. The mass m is acted upon by gravity. The normal force of the table cancels the weight of the disk. The disk starts at rest. Thereafter, the string slowly unwinds without slipping on the pulley or the hub.
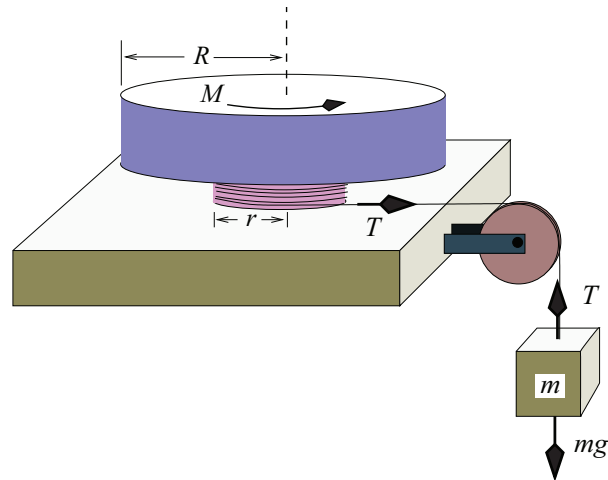
1. Draw a free body diagram of the mass m, and use it to write an equation relating a, the linear acceleration of the mass m, to the tension T in the string.

2. Draw a free body diagram of the disk, and use it to relate the angular acceleration of the disk, α, to T, the mass M of the disk, and the radii (R and r). The moment of inertia of a solid disk about its axis of symmetry is $I = MR^2/2$.

3. Relate a to α.

4. Write α in terms of M, m, R, and r.

(In Experiment 1 below you will measure, M, m, R, and r, and α, and compare the predicted and measured moments of inertia of the disk.)

EXPERIMENT 0: PRELIMINARY

1. Measure the mass, M, of the disk, and estimate its uncertainty ΔM. Also measure the radius R of the disk, and estimate its uncertainty ΔR.

2. Connect the photogate to Digital Input # 1 of the PASCO interface. Install it on the PASCO rotational motion assembly as shown in the figure below. Make sure it is neither so low it rubs against the slotted wheel, nor so high it rubs against the disk. It will measure the angular speed of the disk.
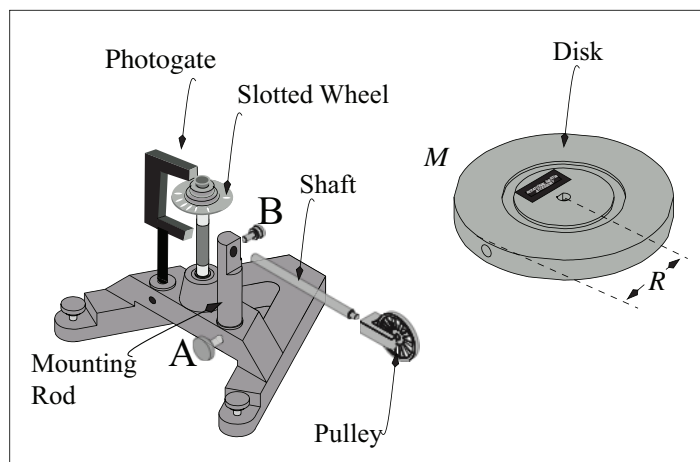

Figure 2: Mounting of photogate and pulley

3. Screw the pulley onto the shaft. Orient the pulley so its axis is parallel to the floor, and tighten thumbscrew B. This will decrease friction from string rubbing.

4. Thread the string downwards through the middle hole of the inner hub, and tie a big enough knot in the end so the knot doesn't slide through the hole. Spin the slotted wheel clockwise so that the string winds a few turns around the middle channel of the hub. Lay the string over the pulley, and pull it taut while holding the hub so it doesn't spin. Rotate the mounting rod so the string runs in the plane of the pulley, and then secure the mounting rod by tightening thumbscrew A. This will also decrease friction from string rubbing.

5. Move the assembly so the pulley sticks out over the edge of the table.

6. With the string fully unwound, affix the weight hanger to the string (perhaps by using a bowline knot) so that the weight hanger hangs an inch above the floor.

7. Rotate the hub clockwise, winding the string on the top hub, the weight of the hanger keeping the string taut. Make sure the string winds neatly.

8. Mount the solid grey disk horizontally onto the rotation shaft. Make sure it doesn't rest on or rub against the photogate.

9. Measure the effective hub radius, r, and estimate its uncertainty. One might use a vernier caliper, but this neglects string diameter and tracking. Here is a better way:

   (i) Align a meter stick vertically from the floor so that you can track the net displacement of the string.
   (ii) Identify a calibration point on the rim of the grey disk. (Masking tape should work well enough.) Align this near the edge of the meter stick and use the meter stick edge to locate a reproducible starting point.
   (iii) Use the weight hanger to identify a height reading. Then lower the string and hanger ten full turns of the disk and take a second height reading.
   (iv) The relationship between the change in height δh and change in angle $\delta\theta$ is given by $\delta h = r\,\delta\theta$, where $\delta\theta$ is in *radians*. Use the measured values of δh and $\delta\theta$ to find r, and calculate $\Delta r/r = \sqrt{(\Delta\delta h/h)^2 + (\Delta\delta\theta/\theta)^2}$.

10. Initiate the Pasco interface software by double clicking on the "Experiment M-209a" file in the Lab Software/201 folder on the desktop.

11. Start the data acquisition by clicking on the Record icon [Record] . To stop the data acquisition click on the Stop icon [Stop] .

12. Using the 50 gram hanger plus another 100 grams, start data acquisition and release the disk. Since the torque is constant, it is predicted that the angular speed should increase linearly.

13. Use the "highlight" tool to select the five data points on the graph of angular speed vs. time where the angular speed is closest to 2.0 rad/s, and use the "fit" tool to make a linear fit to this data. Record the slope of the curve (which is α) and its uncertainty. Make sure to record units.

14. Is there any evidence of departure from linearity?

EXPERIMENT I: COMPARING PREDICTED AND MEASURED MOMENTS OF IN-
ERTIA

1. Calculate the theoretical moment of inertia of the disk if it is rotated about its axis of symmetry, $I_{axis} = MR^2/2$.

2. Using the measured uncertainties in M and R, calculate the uncertainty in I_{axis} (refer to the Propagation of Uncertainties section on page 5; if you find that the fractional uncertainty in one measured quantity is much larger than the fractional uncertainty in the other, you may neglect it).

3. For nominal hanging masses of 50 g, 100 g, 150 g, 200 g, 250 g, and 300 g, measure and record the actual value of each mass, its uncertainty, and the fractional uncertainty.

4. For each mass, measure and record α, the angular acceleration of the disk, and its uncertainty, using the method of step 13 from Exp. 0. For the 150 g mass, use the measurement of α made in that step.

5. Compute the fractional uncertainty in each measurement of α.

6. Compare the fractional uncertainties $\Delta m/m$, $\Delta r/r$, and $\Delta\alpha/\alpha$: which quantity is the most accurately known, m, r, or α? Which is the least?

7. From the measured α, calculate the measured moment of inertia of the disk for each of the six measurements, starting from

$$I = mr^2 \left(\frac{g}{r\alpha} - 1 \right) .$$

Compute $g/r\alpha$ for the largest value of α you measured in step 4, and show that it is much larger than one. This allows you use the approximation

$$I \approx mrg/\alpha .$$

8. Propagate the uncertainties in α and r to give the uncertainty in each measurement of I.

9. Find the mean and standard deviation of the six measurements; call them I_{axis}^{meas} and ΔI_{axis}^{meas}.

10. Is the discrepancy between I_{axis} and I_{axis}^{meas} larger or smaller than ΔI_{axis}^{meas}?

11. Look up or derive the expected moment of inertia I_{dia} of a disk rotated about its diameter in terms of M and R, and write down the expected ratio I_{dia}/I_{axis} (you should be able to express it as the ratio of two integers).

12. Remove the disk, and remount it vertically using one of the holes drilled in its side. Experimentally measure I_{dia}^{meas}, the moment of inertia about this new axis, using the 150 g hanging mass. What is the ratio $I_{dia}^{meas}/I_{axis}^{meas}$?

13. We have neglected friction in this lab, despite some evidence it is measurable (see item 14 in Exp. 0). Does that mean our measurements of I are likely to be incorrectly high, or incorrectly low? Justify your answer.

EXPERIMENT II: CONSERVATION OF ANGULAR MOMENTUM:

1. Measure M', R_1, and R_2, the mass and inner and outer radii of the black metal ring. Use $I_{ring} = \frac{1}{2}M'(R_1^2 + R_2^2)$ to calculate the moment of inertia of the ring.

2. Untie the string from the hub. Re-install the grey disk horizontally, as it was in the first part of Exp. 1.

3. Spin the grey disk and click on the Record icon [Record] .

4. While the grey disk is spinning hold the ring just above it. Carefully drop the ring onto the disk so that it remains centered, and afterwards rotates at the same angular velocity as the disk. Your data should look something like Fig. 3 below.
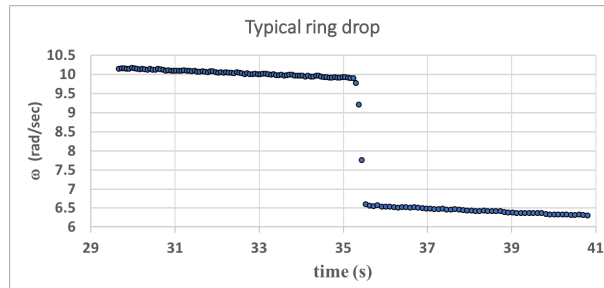

Figure 3: Ring drop, superimposed on gradual angular deceleration.

5. Repeat four more times.

6. Note that the angular speed is observed to decrease steadily both before and after the "collision", due to friction. Use the PASCO data (perhaps downloaded into a spreadsheet) to determine this constant (negative) angular acceleration.

7. Determine the decrement in angular speed due to the collision. Note that the collision has a finite duration (typically about 0.25 s), so the angular speed decreases by a non-zero amount due to friction during this time. Develop a plan to account for this non-zero decrease; state your plan and carry it out.

8. Calculate the initial and final angular momenta and kinetic energies for each of the five trials. Calculate the fraction of angular momentum and kinetic energy that were conserved in each trial. Calculate the mean and standard deviation of the five trials.

    Initial values:
    $$L_i = I_{axis}^{meas} \, \omega_i \quad \text{and} \quad E_i = \frac{1}{2}I_{axis}^{meas} \, \omega_i^2$$

    Final values:
    $$L_f = (I_{axis}^{meas} + I_{ring}) \, \omega_f \quad \text{and} \quad E_f = \frac{1}{2}(I_{axis}^{meas} + I_{ring}) \, \omega_f^2$$

9. Is angular momentum conserved to within the standard deviation of the measurement?

10. What was the average fraction of kinetic energy that was lost?

11. Friction measurement. Use your estimate of the angular acceleration of the disk (in rad/sec^2) from step 6 above, and I_{axis}^{meas} from Exp. 1, to estimate the torque (in N m) due to friction. Use the value of r from Exp. 0 to estimate the weight, and thus the mass (in g), of a hanging mass that would balance that frictional torque. State how you would use this estimate to correct your result from Exp. 1 for friction (but do not go back to do the correction).

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M-211 Elastic and Inelastic Collisions

OBJECTIVE: To study conservation of energy and momentum in collisions.

APPARATUS:

Air track; assorted slotted masses; air supply; hose; gliders; photogates & support stands; PASCO interface and computer.

PRECAUTIONS: *The gliders and track surfaces damage easily: Don't drop!*
With the air pressure on, check that a glider glides smoothly, and the track is free of high friction areas from scratches or plugged air holes.
All collisions must be free from any glider contact with the rail.
Speeds which are too slow are overly influenced by friction and air track leveling errors. Speeds which are too fast cause pitch and yaw motions of the gliders which increase the likelihood of physical contact with the track.
You should perform a number of preliminary trials to discern which speeds work best.
In Experiment III, good alignment of the needle assembly is a necessity.

EXPERIMENT 0: PRELIMINARIES

1. Turn on the air supply and experiment with the gliders.

2. Adjust the leveling screw so that a glider placed at rest remains at rest. Adjust the air flow so the gliders move freely without rocking side to side.

3. Make sure that the photogates are plugged into the first two phone jack inputs in the PASCO interface module. Set the two photogates 50 cm apart and at the right height to track the 10-cm-long timing plate.

4. Start the PASCO software by double-clicking "Experiment M211" in the Lab Software/201 folder on the desktop. The monitor should look as in Figure 1 below.
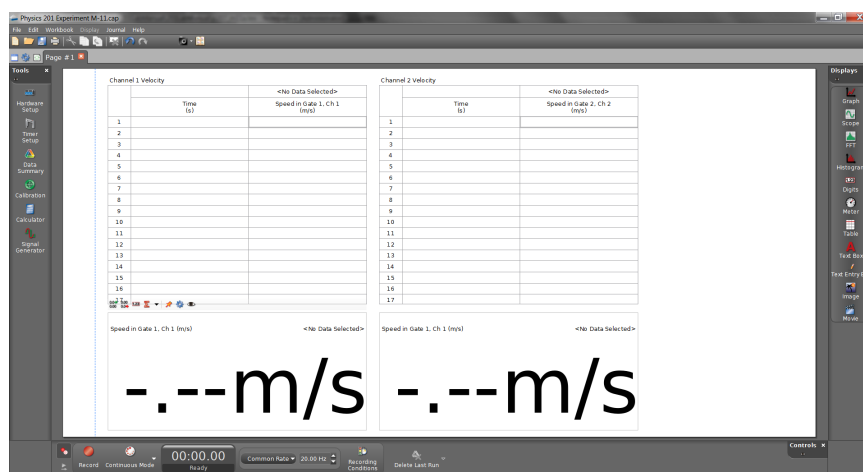

Figure 1: The Pasco Capstone display for M211.

5. To start the data acquisition, click Record. To stop it, click Stop. Each time a glider passes the photogate an entry will appear in the appropriate table column. NOTE: The photogates measure the speed but do NOT sense the direction of motion. You are responsible for the latter.

6. Take a single glider and practice sending it through both photogates. Make sure the two photogates record similar speeds. Keep speeds under about 0.7 m/s.

7. Launch the glider from the right end of the track and record, in a spreadsheet, the speeds measured by both photogates, as well as the direction of motion. Include units.

8. Repeat, launching the glider from the left end.

9. Repeat eight more times, launching the glider first from the right end, then the left (you should have 20 speed measurements at the end of this step).

10. Measure m, the mass of the glider, and estimate Δm, the uncertainty in your measurement of m. What is $\Delta m/m$?

EXPERIMENT I: ELASTIC COLLISIONS, ONE GLIDER INITIALLY AT REST
The adjective "elastic" means "energy-conserving".

1. Affix spade and elastic to both gliders (see Fig. 2); add masses to one glider (henceforth "glider #1"). Measure the mass of each glider, and the uncertainty in each measurement. Record your data in a spreadsheet. Specify units for all data.

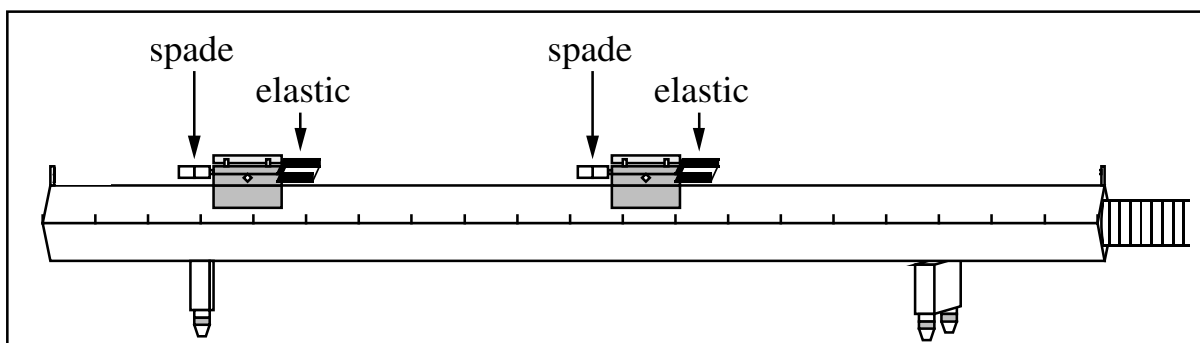| Glider:	#1	#2
mass (g)		
uncertainty in mass (g)		

Figure 2: Sketch of the glider configuration for elastic collisions.

2. Read the following step; before performing it, predict which way will glider #1 be moving after the collision: left or right?

3. Start with glider #1 at the left end of track and glider #2 at rest near the center, in between the two photogates. Give #1 a push toward #2. Make sure your hand has left #1 before #1 reaches the photogate, and make sure #1 clears the photogate before it collides with #2. Call this "collision 1". After the collision, stop glider #2 before it bounces back. Which way was glider #1 moving after the collision?

4. Tabulate the velocities for collision 1 as shown below. Use a spreadsheet for convenience. Record velocities to the right as positive, to the left as negative.

Raw data	Before impact		After impact	
	v_1 (m/s)	v_2 (m/s)	v_1' (m/s)	v_2' (m/s)
collision 1				
collision 2				
collision 3				

5. Repeat; record the velocities for this collision ("collision 2").

6. Now put #1 at rest in the center, send #2 in from the left, and record the velocities from the collision ("collision 3").

7. Recall the definitions of momentum ($p = mv$) and kinetic energy ($E = 1/2mv^2$). For a two-body collision the laws of conservation of momentum $p_{init} = p_{final}$ and conservation of kinetic energy $E_{init} = E_{final}$ may be written:

$$\text{conservation of momentum}: \ m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2',$$

$$\text{conservation of kinetic energy}: \ \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^{\,2} + \frac{1}{2}m_2v_2'^{\,2}.$$

Tabulate momenta and energy as shown below; record momenta to the right as positive, those to the left as negative.

p (kg m/s)	Before impact			After impact		
	p_1	p_2	p_{init}	p_1'	p_2'	p_{final}
collision 1						
collision 2						
collision 3						

E (kg m^2/s^2)	Before impact			After impact		
	E_1	E_2	E_{init}	E_1'	E_2'	E_{final}
collision 1						
collision 2						
collision 3						

8. Tabulate the fraction of initial total momentum conserved in each collision and the fraction of the initial kinetic energy conserved in each collision, as shown below.

	Fraction of momentum conserved	Fraction of kinetic energy conserved
collision 1		
collision 2		
collision 3		

9. Using data from the final table, calculate the mean and standard deviation of the fraction of conserved momentum and the fraction of conserved kinetic energy.

10. Are the measured fractions of conserved energy and momentum within a standard deviation of the predicted fractions (=1)?

EXPERIMENT II: ELASTIC COLLISIONS, BOTH GLIDERS INITIALLY MOVING

1. Spend a few minutes bouncing the gliders against each other, and letting them bounce off the ends. When you see a collision about to occur, try to predict what is about to happen before it happens. Here is a game: try to send the two gliders together in such a way that one of them is motionless after the collision. Let every lab partner have a turn. No need to make any measurements.

2. Start the gliders at opposite ends of the air track. Record the data for three collisions. Tabulate data, and compute fraction of energy and momentum conserved (with uncertainties), as in Exp. I.

3. Compare σ_p and σ_E, the standard deviations in the fractions of conserved p and E, for this experiment to those from Exp. 1. Why is σ_p larger than in Exp. I?

   (Hint: If the two gliders initially have equal and opposite momenta, $p_{init} = 0$. Friction during the collision may cause $p_{final} \neq 0$. One should not conclude that momentum has appeared from nowhere, but rather that one should avoid designing experiments that depend on subtracting two nearly equal quantities (in this case the initial momenta of the gliders), since small losses or uncertainties in measurement can lead to spurious conclusions.

EXPERIMENT III: INELASTIC COLLISIONS
The adjective "inelastic" means "energy non-conserving".

1. Attach needle and wax insert to the gliders as in Fig. 3, so that they now stick together when they collide (inelastic collision). Make sure the needle lines up exactly with the wax or there will be significant sideways motion during the collision. Measure masses of gliders (with uncertainties). Repeat Exp. II. Start gliders from the same side (so that one overtakes the other), in order that $p_{init} \neq 0$.

   When two gliders that are stuck together pass through a photogate, they ought to both have the same speed, but sometimes the measured speeds differ, because of friction or a non-level track. If this happens, is it better (for the purposes of accounting for p and E during the collision) to use the speed of the lead glider for both gliders, or use different speeds for the two gliders?
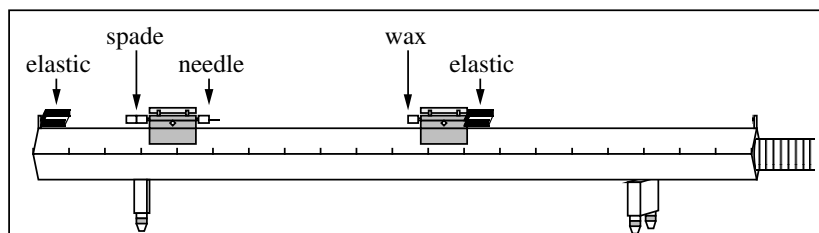

Figure 3: Sketch of the glider configuration for inelastic collisions.

2. Did you observe a large change in the fraction of energy conserved in this experiment, compared to the preceding two experiments?

EXPERIMENT IV: ERRORS AND UNCERTAINTIES

In this section, we will try to quantify three sources of uncertainty or error: i. uncertainty in measurements of speed; ii. effect of friction; iii. effect of possibly non-level track.

1. *Uncertainty in measurements of speed.* The uncertainty in the photogate measurement of speed is $\Delta v = 0.005$ m/s. Read the "Propagation of Uncertainties" section (page 5). If $(\Delta m/m)/(\Delta v/v) \ll 1$ we may neglect the uncertainty in mass when calculating the uncertainties in momentum and energy. Using your answer from the last question in Exp. 0, calculate how large v would have to be in order for $(\Delta m/m)/(\Delta v/v) = 1$. (Since all speeds in the experiment are less than this, neglect Δm in what follows.)

2. Derive formulae for $\Delta p/p$ and $\Delta E/E$ in terms of $\Delta v/v$ in the case where $\Delta m/m$ can be neglected.

3. Using a formula you derived above, propagate the uncertainty in the speed measurements through the calculation of momentum for collision 2 of Exp. II. Use the tabulation below.

p and Δp (kg m/s)	p_1	Δp_1	p_2	Δp_2	p_{init}	Δp_{init}
(before collision)						
p and Δp (kg m/s)	p'_1	$\Delta p'_1$	p'_2	$\Delta p'_2$	p_{final}	Δp_{final}
(after collision)						

4. If the collision were ideal the change in total momentum would be zero. Calculate the "anomalous" change in momentum, $p_a \equiv p_{init} - p_{final}$, and equate Δp_a, the uncertainty in p_a, with σ_{pa}, the standard deviation in p_a. By how many standard deviations does p_a differ from 0?

5. Using the other formula you derived above, propagate the uncertainty in speed through the calculation of energy for collision 2 of Exp. II. Use a similar tabulation to what you used for momentum.

6. Calculate the change in energy, $E_a = E_{init} - E_{final}$, and its uncertainty ΔE_a. Assert $\Delta E_a \sim \sigma_{Ea}$. By how many standard deviations does E_a differ from 0?

7. *Effect of friction.* Review the data from Exp. 0. For each passage of the cart, calculate w, the speed decrement between photogate passages, and then calculate the mean w for the 10 passages. What is the ratio between the mean w and the uncertainty in measuring v, $\Delta v = 0.005$ m/s?

8. *Effect of non-level track.* In the data from Exp. 0, is there any evidence of a non-level track? If so–estimate the change in speed a cart experiences between passing through the two photogates, and state which end of the track is higher, left or right.

9. *Uncertainty in measurements of speed; effect of friction; effect of non-level track:* which of these is a random error, and which is a systematic error?

10. Based on the data, which is the largest: uncertainty in measurements of speed; effect of friction; or non-levelness of the track?

11. Based on your calculations in steps 4, 6, 7, and 8, is there evidence for an additional source of uncertainty or error (i.e. loss of energy and/or momentum) that has not been accounted for? If so, state a hypothesis of what it might be.

M-212 Simple Harmonic Motion

This lab introduces the *harmonic oscillator*, and several phenomena pertaining thereto.

OBJECTIVES:

1. To study the period of the simple harmonic oscillator as a function of oscillation amplitude.
2. To investigate whether Hooke's Law holds for real springs.
3. To observe the transfer of energy from potential energy to kinetic energy, and back again, as the phase of oscillation changes.
4. To observe the exponential decay of amplitude of the damped harmonic oscillator.

THEORY:

The restoring force F on a mass m attached to a "simple" one-dimensional spring is proportional to the displacement from equilibrium: $F = -k(x - x_0)$, where k is the spring constant (or "stiffness") in N/m, x_0 is the position of equilibrium (no net force) and x is the actual position of the object. This is Hooke's Law.

The equation of motion of the simple spring-mass system, $F = ma = -k(x - x_0)$, is a 2nd order differential equation:

$$a = \frac{d^2 x'}{dt^2} = -\frac{k}{m}(x') \ ,$$

where $x' \equiv x - x_0$. The most general solution for this equation may be given as

$$x'(t) = A \sin(\omega_0 t + \phi_0) \, ,$$

where the starting *phase* ϕ_0 and the *amplitude* A may take on any values, but $\omega_0 \equiv \sqrt{k/m}$ is determined.

In mathematics the adjective "harmonic" has the meaning "capable of being represented by a sine or cosine function", so the spring-mass system is often referred to as a *harmonic oscillator*. ω_0 is the *natural frequency* of the harmonic oscillator, and $T \equiv 2\pi/\omega_0$ is called the *period*.

The general solution written above is time-dependent and periodic. When the phase varies by 2π (when one period $T = 1/f = 2\pi/\omega_0$ has elapsed), both the position, $x'(t+T) = x'(t)$, and velocity, $v'(t+T) = v'(t)$ return to their previous values.

Total energy (TE) is conserved in harmonic oscillation. As time passes kinetic energy (KE) is transformed to potential energy (PE) and back again. Thus at any time t:

$$TE = \text{constant} = KE(t) + PE(t) = \frac{1}{2}m[v'(t)]^2 + \frac{1}{2}k[x'(t)]^2 \quad .$$

APPARATUS:

Basic equipment: Air track; assorted slotted masses; air supply; hose; adjustable stop; glider; springs; timer; photogate & support stand; Hooke's law apparatus.

Computer equipment: PASCO interface; photogate; motion sensor; speaker with driver stem; power amplifier module.

PRECAUTIONS:

The soft aluminum gliders and track surfaces damage easily: Don't drop!
With the air pressure on, check that the glider moves freely, and that the track is level, and free of high-friction areas from scratches or plugged air holes.
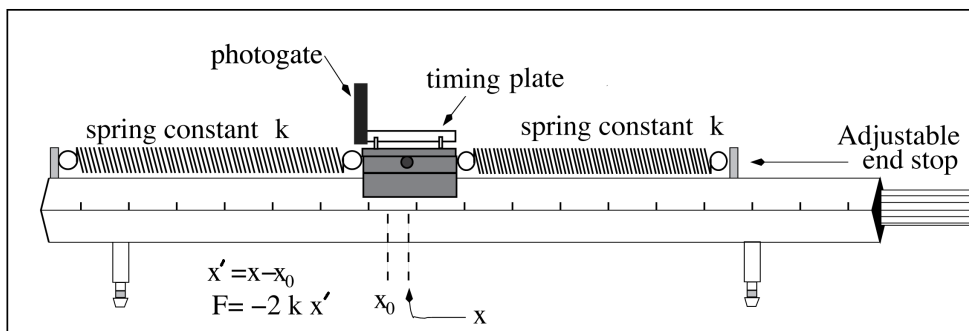
Figure 1: Sketch of the air track configuration for Expt. 0.

EXPERIMENT 0: EQUIPMENT SETUP AND CHECKOUT:

1. Choose two springs of similar lengths.

2. Attach the springs to the air track as in the diagram above, setting the adjustable end stop so that the springs are neither stretched beyond their elastic limits nor capable of sagging onto the track as the glider oscillates (oscillation amplitudes $\leq$ 20 cm will be requested below).

3. Choose the horizontal position of the photogate so the timing plate just cuts off the beam when glider is in the equilibrium position ($x - x_0 = 0$). The photogate phone jack should be in the PASCO input 1.

4. Turn on the air supply and increase the blower speed until the the force of friction appears to be small.

5. Displace the glider from equilibrium and then release it. Use your natural time-keeping ability to estimate the period of oscillation.

6. Initiate the Pasco software by double-clicking the "Experiment M-212 Part I" file in the Lab Software/201 folder on the desktop.

7. Displace the glider from equilibrium, release it, and click Record. Let the glider oscillate for about 10 periods. Click Stop. Print a screenshot of your data and include it in your report.

8. Compare the mean period calculated by the computer to your estimate from above. Do you trust the computer's calculation?

9. Click on the "Glider Period (Small Amplitude) (s)" button near the top of the data table. Note that there are two choices for "Custom Timer": "Glider Period (Small Amplitude)" and "Glider Period (Large Amplitude)".

10. Inspect the "Timer Setup" window, in particular the "Timing Sequence" section, which gives the algorithm for how the computer calculates the glider period. Compare the timing sequences of the two custom timer measurements noted in the last step.

11. What is the minimum amplitude of oscillation that requires use of the Glider Period (Large Amplitude) custom timer, and what is special about this amplitude?

EXPERIMENT I: DEPENDENCE OF PERIOD ON AMPLITUDE:

1. Use the computer to measure the period for oscillation with initial amplitudes of 4 cm, 8 cm, 12 cm, 16, and 20 cm: acquire data for 10 oscillations for each initial amplitude. Use the appropriate custom timer for each measurement. If any period measurements seem untrustworthy, reject them and take another dataset.

2. Make a table with the following columns: "initial amplitude of oscillations A (cm)"; "mean period T (s)"; "standard deviation of the mean σ_T (s)". Recall the distinction between "standard deviation" and "standard devation of the mean" (see "Characterization of Uncertainties" on page 3). Which of the two does the Pasco software tell you?

3. The theory presented in the "Theory" section predicts that period is independent of amplitude. Does your data support this? Frame your answer using the formal language of error analysis, e.g. "The discrepancies between the five measurements of mean amplitude are of the same order as the uncertainties in each measurement (as characterized by the standard deviation of the mean), so therefore yes, the experimental measurements are consistent with the theoretical prediction that period is independent of amplitude."

EXPERIMENT II: DOES HOOKE'S LAW HOLD FOR REAL SPRINGS?:

1. Make two sketches in your lab book, of the glider at equilibrium and after a displacement from equilibrium.

2. Show that the effective force constant for two identical springs of force constant k on either side of an oscillating mass is $2k$.

3. Let us try to verify whether real springs satisfy Hooke's Law, $F = -k(x - x_0)$. Choose one of your two springs, hang it vertically from the Hooke's Law apparatus, and set the vertical scale so that the "zero" mark aligns with the bottom end of the spring.

4. Make a table with columns "mass m (g)", "weight F (N)", "elongation $x - x_0$ (cm)". Your first row of data should have zeroes in all three columns.

5. Starting with 2 g, add 5 g masses to the bottom of the spring, and measure the elongation $x - x_0$ of the spring for successively larger masses, until the elongation exceeds the 14.5 cm range of the vertical scale.

6. Plot the data using Excel in such a way that the slope of a straight line fit to the data is k (and not, say, $1/k$).

7. Inspect the plot: do all data points fall neatly on a straight line?
   If "yes": fit a straight line to the points, and write down "k" as given by the fit.
   If "no": is it possible that if a data point or range of data points were excluded, the remainder could be well fit by a straight line? If it is, fit a straight line to the remainder points, and write down k as given by the fit, as well as the range of values of F which went into this determination of k.

8. Use LINEST to find the uncertainty in your determination of k (for an example of the usage of LINEST see p. 23 (M-204), or look in M-200.xlsx).

9. Measure the mass m of the glider, and estimate the uncertainty Δm of your measurement.

10. Using these measurements of k and m, predict the period of the glider in the setup for Expt. 1, using $T = 2\pi \sqrt{m/2k}$. Calculate the percentage discrepancy between prediction and measurement ($|(T_{calc} - T_{meas})/T_{meas}| \times 100\%$).

11. Write down the percentage uncertainty in the three experimentally measured quantities involved in the calculation in the previous step (T as measured in Expt. 1, m and k found here). Which quantity is the most uncertain? Are the uncertainties in the experimentally measured quantities sufficiently large to explain the discrepancy between the calculated and measured periods?

EXPERIMENT III: MEASURING $x(t)$:

As mentioned in the "Theory" section, the general solution of the equation of motion of the harmonic oscillator may be written (note $'$ does not mean derivative):

$$x'(t) = A \sin(\omega_0 t + \phi_0) ,$$

where $x'(t)$ is the displacement from equilibrium. This implies

$$v'(t) = A\omega_0 \cos(\omega_0 t + \phi_0) = A\omega_0 \sin(\omega_0 t + \phi_0 + \frac{\pi}{2})$$

where $v'(t)$ is the velocity. A is the amplitude, ω_0 the natural angular frequency, and ϕ_0 is the starting phase, as before. Notice that the velocity has a *phase shift* of $\frac{\pi}{2}$ relative to the displacement.

This means that kinetic energy of motion ($1/2\,mv'^2$) is transferred into potential energy ($1/2\,kx'^2$) and back again.

To capture this cyclic process we will again use the PASCO interface, replacing the photogate with the motion sensor.
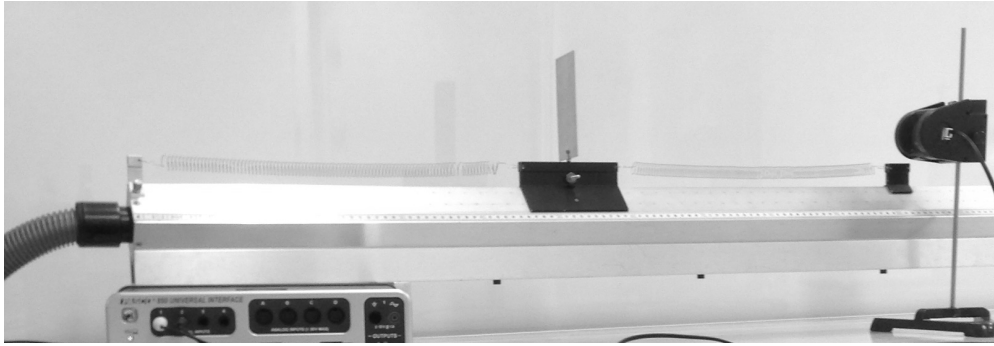
Figure 2: Photo of setup for Expt. III.

1. Remove the timing plate and attach an aluminum vane to the glider. Measure the mass m_{vane} of this glider and predict the new natural frequency $\omega = \sqrt{k/m_{vane}}$.

2. See setup picture below. Place the motion sensor approximately 60 cm from the vane, collinear with the direction of oscillatory motion. Point the motion sensor at the vane and tilt it about $10°$ upwards, and orient the vane so that its normal points toward the motion sensor. Make sure the yellow phone jack is in Digital Input 1 and the black phone jack is in Digital Input 2.

3. Double click on the "Experiment M-212 Part III" file in the Lab Software/201 folder on the desktop.

4. Displace the glider approximately 20 cm from equilibrium to initiate the oscillatory motion. Click on the the Record icon to start your data acquisition. The graph will simultaneously display both absolute position and velocity versus time.

5. Inspect the position measurement for "outlier" data points which indicate faulty measurement. If any appear, stop the data acquisition, reorient the motion sensor and/or the aluminum vane, and try again.

6. Practice a few times to make sure you can reliably obtain outlier-free, smoothly-varying sinusoidal curves. Then run the data acquisition for just over five cycles.

7. Using $x'(t) = A\sin(\omega_0 t + \phi_0)$, and $x' \equiv x - x_0$, determine the initial phase ϕ_0 and the equilibrium position x_0 (note that x_0 is not $x(t=0)$).

8. Use the Data Highlight ✎ feature of the graph toolbar to better view a single full cycle by highlighting the desired data and then clicking the Zoom-to-Fit button ⬓.

9. Use the cross-hair ✛ option to read out time, position and velocity. Convert position to displacement-from-equilibrium (x') by subtracting x_0. Make a table as below and fill in the missing entries. Identify units.

time	phase($\omega_0 t + \phi_0$)	$x'(t)$	$v'(t)$	KE	PE	TE
	0					
	$\pi/4$ (45 deg)					
	$\pi/2$ (90 deg)					
	$3\pi/4$ (135 deg)					
	π (180 deg)					

10. Plot TE vs. time. Neglecting friction, should TE be a constant of the motion?

11. Record five full cycles of motion, and print out a copy of your waveforms. What fraction of the initial energy is lost after five full cycles?

EXPERIMENT IV: DAMPING OF THE MOTION:

We expect that by this point in the lab you have seen the the amplitude of the motion diminishes over time. This suggests there might be friction. To our model from the "Theory" section, add a velocity-dependent friction force $F_{\text{friction}} = mRv'$, where R is the drag coefficient, so that the equation of motion becomes

$$F = ma = -kx' - mRv' \ .$$

This is still a linear, second-order differential equation, so the solution can still be written with two free parameters, as in the "Theory" section, and turns out to be

$$x'(t) = Ae^{-Rt/2}\sin(\omega t + \phi) \ ,$$

where $\omega = \sqrt{k/m - R^2/4}$. The graph below shows an example of this "exponential damping".
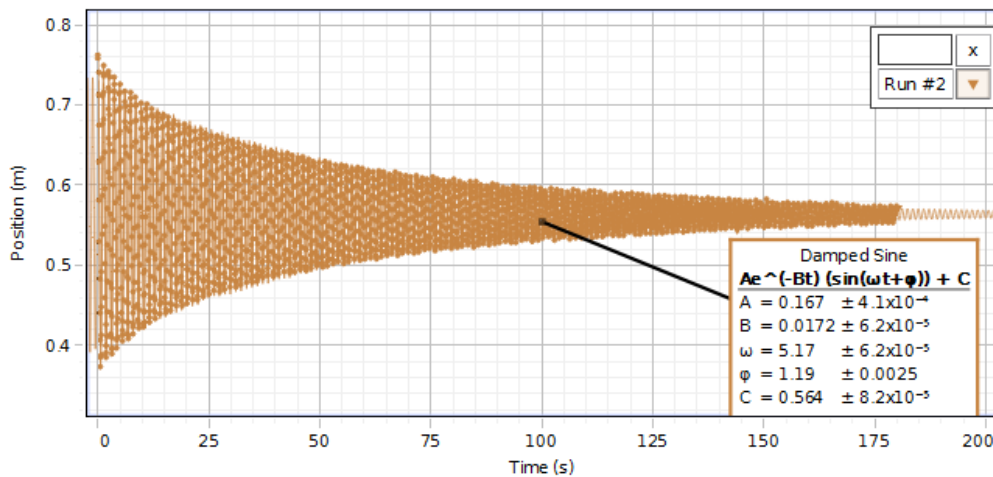
Figure 3: Exponential damping: R=0.0344 s^{-1}, $\omega = 5.17$ s^{-1}.

1. Displace the cart by approximately 20 cm from equilibrium, let it go, and click Record. Make sure the position measurement is reliably free from outliers.

2. Once the position measurement is reliably free of outliers, start over: displace the cart by 20 cm from equilibrium, let it go, and click Record. Keep recording data until the amplitude of the motion has reduced to 10 % of what it was initially. This should take a few minutes.

3. Use the Data Highlight ✐ button to select as much data as possible, then click the ▦▾ button and choose the Damped Sine fit to estimate the drag coefficient, R. If there are outliers, the fit may be visibly poor.

4. State the mathematical condition that the approximation $\omega \sim \sqrt{k/m}$ is valid. Is this condition satisfied by your values of R and $\sqrt{k/m}$?

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H-201 The Ideal Gas Law and Absolute Zero

OBJECTIVES:

To observe the behavior of an "ideal" gas (air); to determine absolute zero (in Centigrade).

PRECAUTIONS:

This lab uses liquid N_2. It is fascinating to work with. However, please keep in mind the following simple safety precautions:

(a) **Never stopper a flask of liquid N_2 with an unperforated stopper. This can result in a dangerous explosion with risk of major injury. Students found attempting this will be referred to the Dean of Students for disciplinary action.**

(b) Have a perforated stopper on the Dewar throughout the experiment to prevent condensation of moisture from the air on the inside of the flask.

(c) **Avoid contact of liquid N_2 with your skin.** The insulating vapor may disappear and severe frost-bite may result.

FUNDAMENTAL CONCEPTS:

The behavior of an ideal gas under varying conditions of pressure and temperature is described by the "Ideal Gas Law":

$$PV = nRT \tag{1}$$

Where P is the pressure (in Pascals, Pa), V is the volume (in m^3), T is the temperature in Kelvins (K= C+273.15), n is the number of moles present (1 mole $\equiv 6.023 \times 10^{23}$), and R is the gas constant (8.31 J/mol ·K).

This equation is a combination of two laws which were discovered previously:

(a) *Boyle's* Law relates volume and pressure of a fixed quantity of gas at a constant temperature: $PV = constant$

(b) *Charles's* Law relates volume and temperature of a fixed quantity of gas at a constant pressure $V/T = constant$

Gay-Lussac finally combined the two laws into the ideal gas law shown in Eqn. 1.
In Expt. I Boyle's law will be verified by varying V and P (assuming fixed T).
In Expt. II You will vary T and V to determine the value of absolute zero.

APPARATUS:

Lab stand; 60 cm^3 plastic syringe attached to a plastic "quick-connect" coupling; PASCO pressure sensor (mounted on lab stand); PASCO interface module; PASCO steam generator; water jacket container; steel Dewar flask; small stainless steel can with a volume of 98 cm^3 with a "quick release" connector; FLUKE digital thermometer and temperature probe; liquid N_2 (kept in 4329 Chamberlin).

EXPERIMENT I: BOYLE'S LAW

1. Launch the Pasco interface software by double clicking the "Experiment H-201a" file in the Lab Software/201 folder on the desktop.
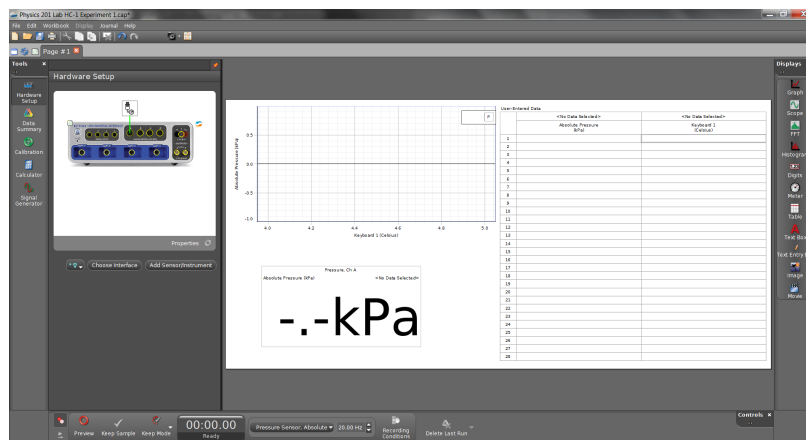
Figure 1: Image of PASCO Capstone display.

2. The computer screen should look like Fig. 1. In this window there is a graph configured to display pressure vs. volume, a table for these values, and a panel meter for the instantaneous pressure reading.

   Run the Pasco software in "Keep Mode": after clicking the "Preview" button ⬛ it will then change to the "Keep" button ⬛. Clicking on this button will generate a pop-up window for data entry. To terminate the run you must click on the red square immediately to the left of the Keep button.

   The volume is recorded in *milliliters* (1 mL = 1 cm^3) and the pressure in *kilopascals* (kPa). In this case the ideal gas law ($PV = nRT$, T in Kelvin) uses $R = 8.31 \times 10^3 (\text{kPa·cm}^3)/(\text{mol·K})$.

3. If the syringe is attached to the pressure sensor, disconnect it at the twist-lock quick connect coupling. Set the syringe plunger to the 60 cm^3 mark and then reconnect it to the pressure sensor. Make sure that the equipment looks as sketched in Fig. 2.
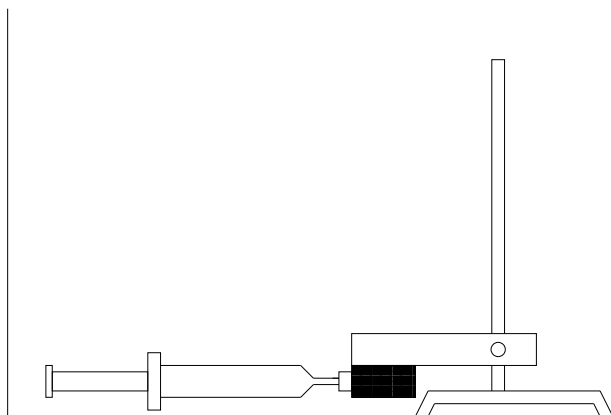
Figure 2: Sketch of pressure sensor and syringe/plunger assembly.

4. Initiate the data acquisition by clicking on the Preview button. Write in your lab-book the readout precision of the pressure sensor. Check for leaks: slowly push the plunger in to the 50 cm^3 position (about 10 seconds) and back to the original setting while watching the panel pressure meter. What would be the evidence of a leak? Is there evidence of a leak? Record the measurements you used to back up your determination that there is/is not evidence for a leak.

5. Now begin taking data. With the plunger at 60 cm^3, press the Keep button to record a data point. Confirm that a new data point appears on the plot and new values in the appropriate row of the table.

6. Reduce the volume slowly in discrete 5 cm^3 increments while recording a data point at each step (see the previous item) until you reach 20 cm^3. Then, as a check, slowly increase the volume in 5 cm^3 steps (recording a data point at each step) until you return to 60 cm^3.

7. Stop the data acquisition and transfer your data to a table in your lab write-up and comment on the reproducibility of your measurements. You should make two plots, one with respect to volume and one with respect to *inverse* volume. Do your plots have the correct functional behavior? Refrain from printing out the graphs until you have performed a linear regression using either the PASCO graphical analysis capabilities or Excel.

8. Which of the two graphs do you expect to be a straight line and why? For this "curve" what is the expected y intercept? Now fit this data to a line (using linear regression) and record the slope and intercepts with the appropriate units. Print out the respective plots and include them in your lab write-up.

9. OPTIONAL: Repeat the procedure of item 6 except move from 60 cm^3 to 20 cm^3 as rapidly as possible. Are the pressure readings similar to those of the slow-moving experiment? If not, can you suggest a reason?

QUESTIONS:

I. Does the line go through the origin as expected? By how much should you change the pressure readings so that the line goes through the origin?

II. What are the possible sources of error in this experiment? For each source, try to decide what effect it might have on the experimental results.

III. If a barometer is available: read the barometer. Ask your instructor for help if you are having trouble. Convert the pressure from cm of Hg to kPa and compare this value with the one obtained at 60 cm^3 in step 5. Is the difference between the two readings important? Does it affect your conclusions?

IV. OPTIONAL: Assuming that there are 22.4 liters per mole of an ideal gas at STP, how well does your observed slope agree with your expected slope?

EXPERIMENT II: IDEAL GAS LAW

In this part the Ideal Gas Law is verified by observing the pressure of a fixed volume of gas at four different temperatures: room temperature, 100 °C, 0 °C and -197 °C. Since $T = T_C + T_0$, where T_C is the temperature in Celsius and T_0 is absolute zero on the Celsius scale, we can rewrite the Ideal Gas Law $PV = nRT$ as

$$P(T_c) = \frac{nR}{V}T_C + \frac{nR}{V}T_0 \,,$$

and thereby determine the value of absolute zero from a straight-line fit to P as a function of T_C.

SUGGESTED PROCEDURE:

1. Double click on the "Experiment H-201b" file in the Lab Software/201 folder on the desktop.

2. Attach the small stainless steel can (volume of 98 cm^3) as shown below to the PASCO pressure sensor using the plastic "quick release" connector. Raise the assembly up if necessary. Turn on the Fluke thermometer. Unless the stainless steel can has been recently used it should now be at room temperature.
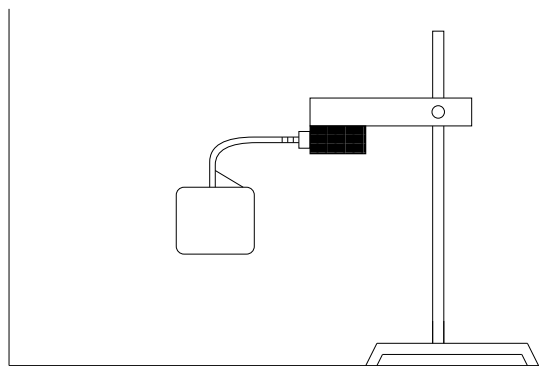
Figure 3: Sketch of pressure sensor and stainless-steel can assembly.

3. Once again initiate the experiment by clicking on the Preview icon. There should be a single plot (pressure vs. keyboard entry or, equivalently, temperature). Record the Fluke meter reading by clicking on the Keep icon and then entering the temperature in the keyboard entry window.

4. Fill the PASCO steam generator with water and turn it on. The level of the water should be ∼3 cm below the top, or about $1/2 - 3/4$ full (Fig. 4). Turn the knob to 9 until the water boils and then turn it down somewhat.

5. As soon as the water in the steamer is boiling, raise the pressure sensor stand and then lower it so as to submerge the stainless steel can in the boiling water.

6. Wait until the water is boiling again and then put the temperature probe in the boiling water. Resume the data acquisition by clicking on the Record icon. Watch the pressure readout and after it is stable (with time) record the Fluke meter reading in the data entry window. Turn the PASCO steam generator off and remove the small stainless steel can from the bath. Pause the data acquisition.
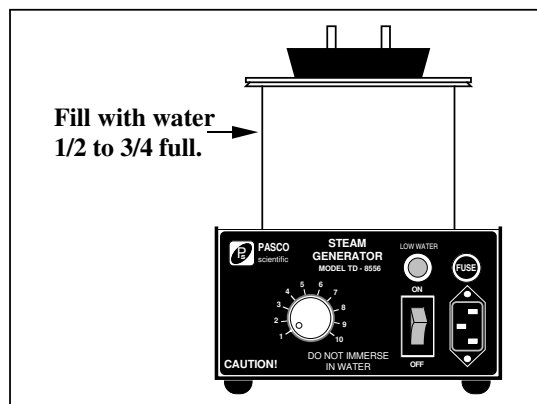
Figure 4: PASCO steam generator.

7. Fill the water jacket container with water and ice. The level of the water should be ~5 cm below the top. Note that water reaches maximum density at 4 ° C, and that the sensitive part of the temperature probe is at the tip, and place the temperature probe tip accordingly. Obtain the 0 °C pressure reading by submerging the can as you did before, and waiting for the pressure readout to stabilize as you did before. Empty and dry the stainless steel dewar.

8. Take the stainless steel dewar and ask the instructor to fill your dewar with liquid Nitrogen and repeat step 3 for the container at liquid Nitrogen temperature.

9. Stop the data acquisition and transfer your data to a table in your write-up. Plot P as a function of T_C: does it look like a straight line, as predicted by the Ideal Gas Law?

10. Fit this data to a straight line and record the slope and intercepts with the appropriate units. Print out the plot and include it in your lab write-up.

11. Calculate absolute zero using the appropriate fit parameters. State the accepted value of absolute zero, and identify the largest source of uncertainty in your measurements.

QUESTIONS

I. What is the percentage difference between the value you found and the accepted value for absolute zero?

III. OPTIONAL: Qualitatively, what error results from the gas in the small tube not being always at the can temperature?

IV. OPTIONAL: Does thermal expansion of the can affect your results? In what way?

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S-201 Transverse Standing Waves on a String

OBJECTIVE: To study propagation of transverse waves in a stretched string.

INTRODUCTION:

A standing wave in a string stretched between two points is equivalent to superposing two traveling waves on the string of equal frequency and amplitude, but opposite directions. The distance between nodes (points of minimum motion) is one half wavelength, $(\lambda/2)$. Since the ends of the string are fixed, the only allowed values of λ are are $\lambda_k = 2L/k, \ \ k = 1, 2, 3 \ldots$

The wave velocity, v, for a stretched string is $v = \sqrt{F/\mu}$ where **F = tension in the string** and μ = **mass per unit length**. But $v = f\lambda$ and hence only certain frequencies are allowed:

$$f_k = \frac{\sqrt{F/\mu}}{\lambda_k} = k\,\frac{\sqrt{F/\mu}}{2L} \quad . \tag{1}$$
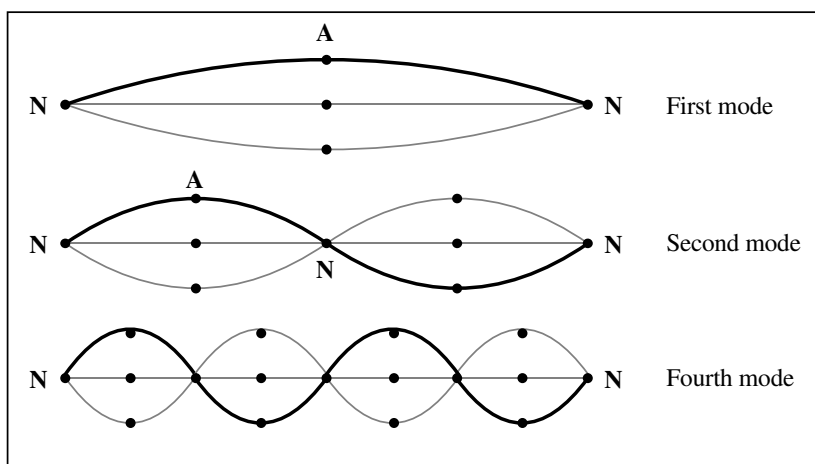
Figure 1: The Modes of a String
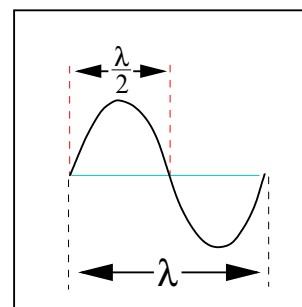
Figure 2: A close-up

PART A: Waves from a mechanical driver (i.e. a speaker)

APPARATUS:

Basic equipment: Pasco interface; electrically driven speaker; pulley & table clamp assembly; weight holder & selection of slotted masses; black Dacron string; electronic balance; stroboscope.

The set-up consists of an electrically driven speaker which sets up a standing wave in a string stretched between the speaker driver stem and a pulley. Hanging weights on the end of the string past the pulley provides the tension.

The computer is configured to generate a digitally synthesized sine wave (in volts versus time) with adjustable frequency and amplitude (max: ∼10 V).
PASCO interface: This transforms the digital signal into a smooth analog signal.

Precautions: Decrease the amplitude of the signal if the speaker makes a rattling sound. The generator is set to produce sine waves; **do not change the waveform**.

Note: Although the speaker is intended to excite string vibrations only in a plane, the resultant motion often includes a rotation of this plane. This arises from non-linear effects since the string tension cannot remain constant under the finite amplitude of displacement. [See Elliot, **Am. J Phys.** 50, 1148, (1982)]. Other oscillatory effects arise from coupling to resonant vibrations of the string between pulley and the weight holder; hence keep this length short.
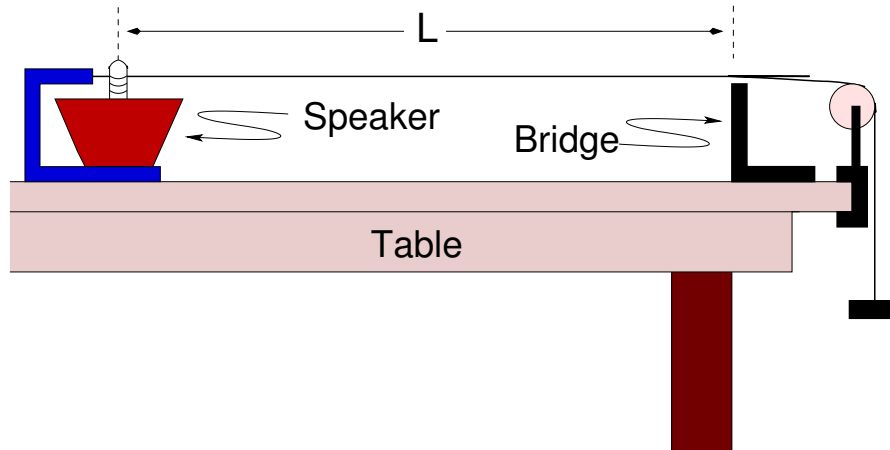
Figure 3: The apparatus

PROCEDURE: CHECKING EQUATION (1)

1. Procure a length of string about 1.5 m long.

2. Carefully weigh the string, and calculate μ. Note this in your lab notebook.

3. Set up the speaker, bridge, and pulley as in the above figure. To connect the string to the speaker: tie the string to the metal support, push (gently) the spring down, and thread the string through the notch in the pin mounted on the speaker cone. Make the distance, L, between the bridge and the pin of the speaker be about 0.75 m, and measure it accurately using the two meter ruler; record this in your lab notebook.

4. Place the sheet of paper provided on the table; this will make it easier to see the vibration of the string.

5. Double click on the "Experiment S-201" file in the Lab Software/201 folder on the desktop. The display will appear as shown in Fig. 4.

6. You will see that the computer is set to produce a 60 Hz sine wave with an amplitude of 2 V. To start the string vibrating click the "On" button.

7. Click on the up/down arrow in order to change the amplitude or the frequency of the signal.

   NOTE: The nominal step sizes for adjusting the amplifier frequency and voltage may be much too large. To alter the step size use the ⌄⌄ buttons. To alter the
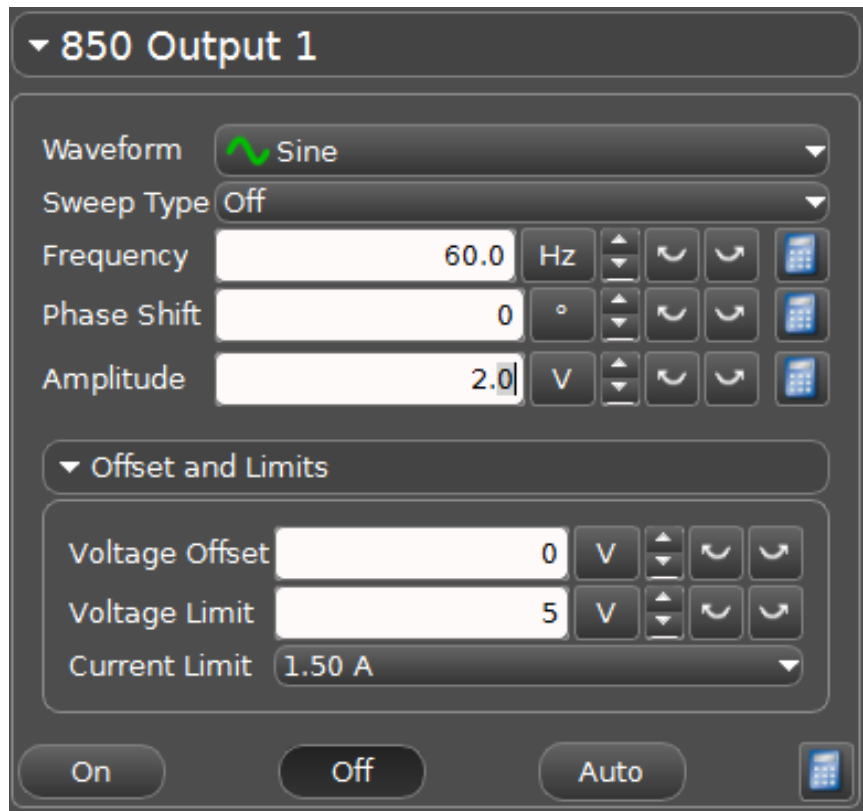
Figure 4: The Pasco Capstone display.

current or voltage use the ▲▼ buttons. You can also change the value directly by clicking the mouse cursor in the numeric window and entering a new value with keyboard number entry.

8. Stop the signal generator, put a 200 g mass on the mass hanger and restart the signal generator. Record the total mass and tension in your lab notebook. In the next few steps you'll keep the tension in the string constant and look for the frequencies of different modes.

9. Predict the frequencies f_k for $k = 1$ to $k = 6$.

10. Measure the frequency f_2 of the $k = 2$ mode using the following method. First, obtain the $k = 2$ mode, then decrease the voltage to the smallest value at which you can clearly see the mode.

11. Next, decrease the frequency until the mode is too small to see. Then, increase the frequency in 1 Hz steps, noting the frequency at which the mode amplitude is maximum. Continue increasing the frequency until the mode amplitude is again too small to see. Then, decrease the frequency by 1 Hz steps, and note when the mode amplitude is maximum. Once you have done this, discuss with your lab partners what is the best choice for f_2.

12. (Optional) Use the stroboscope to confirm f_2 (note the stroboscope is calibrated in cycles per minute, not Hz). The value should be close to 70 Hz.

13. Find and record the frequencies f_3, f_4, f_5,...of as many higher-order modes as you can.

14. Find and record f_1. Be especially careful to use the lowest possible amplitude, because the mode amplitude can become large.

15. Divide the various frequencies f_k by k and enter the values in a table. Calculate the average value of f_k/k.

QUESTION:

Q1: Use this average value of f_k/k to calculate the mass per unit length of the string. How does it compare with your measurement?

16. Choose six masses between 100 g and 500 g and enter the values in a data table.

17. Determine the resonant frequency of the second mode of the string under these different tensions and record your results. (Hint: increasing the mass by a factor of two increases f_2 by nominally a factor of $\sqrt{2}$.)

18. Plot a graph of frequency versus mass, m, and include the zero value.

19. Plot a graph of frequency versus $\sqrt{m}$ and again include the zero value.

QUESTIONS:

Q2: Which of the two graphs can be fit with a straight line? A parabola? Why? State your reasoning.

Q3: From the slope of the graph having the linear relationship obtain the mass per unit length of the string.

Q4: Which of the three methods of estimating the mass per unit length of the string seems most accurate?

FOR FURTHER INVESTIGATION:

Vibrations on a circular membrane are made visible here:

http://www.acs.psu.edu/drussell/Demos/MembraneCircle/Circle.html

Appendices

A   Precision Measurement Devices
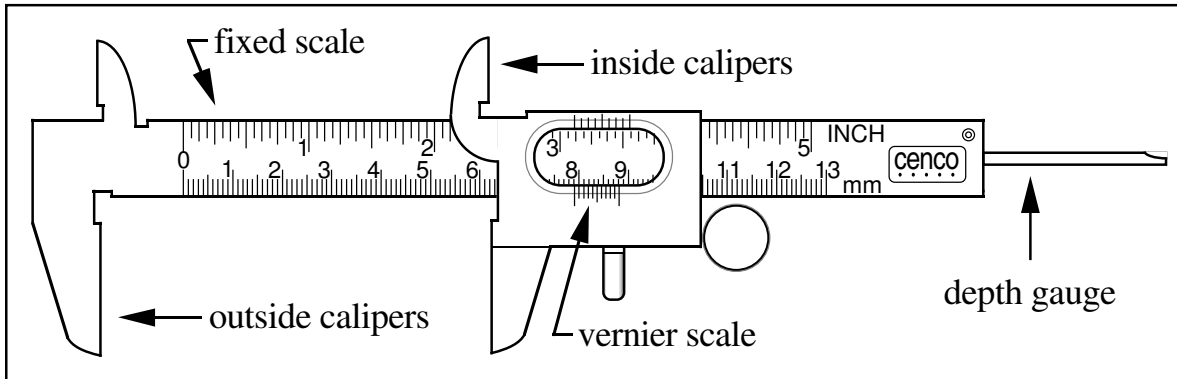
Vernier Calipers:

Figure 1: The vernier caliper

A Vernier consists of a fixed scale and a moving *vernier* scale. In a metric vernier the fixed scale is marked in centimeters and millimeters, the vernier scale is nine millimeters long, and is divided into ten parts each 0.9 millimeters long. The distances of each line from the first are therefore 0.9, 1.8, 2.7 ... mm or generally: $d_i = 0.9 \times i$, where d_i is the distance between the zero line and the ith line of the vernier scale. If the vernier caliper is closed, so that the two jaws touch each other, the zero of the fixed scale should coincide with the zero of the vernier scale. Opening the jaws 0.03 cm = 0.3 mm will cause the fourth line (the *three* line which is a distance of 2.7 mm from the zero line of the of the vernier scale) to coincide with the 3 mm line of the fixed scale as shown below.
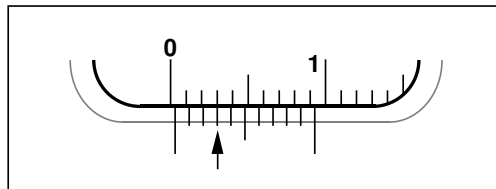
Figure 2: The vernier reads 0.03 cm

Below is another example of vernier reading; the arrow shows which mark on the vernier scale is being used.
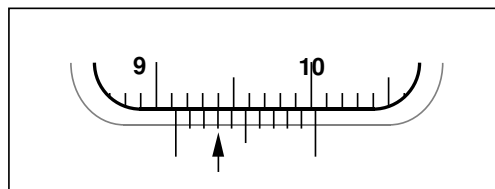

Figure 3: The vernier reads 9.13 cm

EXERCISES:

A. Close the vernier and observe that the first vernier mark coincides with the zero of the centimeter scale.

B. Open the jaws of the vernier very slowly and observe how the different vernier marks coincide successively with the millimeter marks on the fixed scale: the first mark coincides with the 1 mm mark on the fixed scale; then the second mark coincides with the 2 mm mark on the fixed scale; then the third mark coincides with the 3 mm mark on the fixed scale and so on.

C. Estimate the dimension of an object using a meter stick and then Use the vernier caliper to measure the dimension precisely.

D. In the four examples of Fig. 4 determine the actual reading.
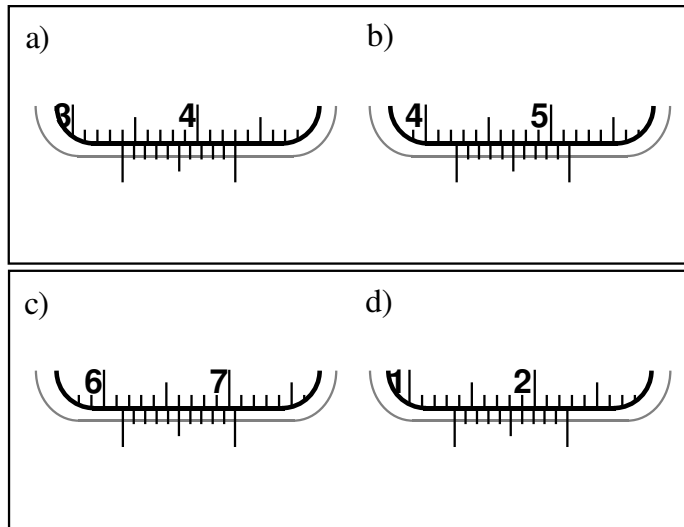
Figure 4: Test cases

Micrometer:

A micrometer can measure distances with more precision than a vernier caliper. The micrometer has a 0.5 mm pitch screw, this means that you read millimeters and half millimeters along the barrel. The sleeve is divided into 50 divisions corresponding to one hundredth of a millimeter (0.01 mm) or 10 μm each. The vernier scale on the micrometer barrel has ten divisions, marked from 2 to 10 in steps of two. The "zero" line is not marked '0', but is longer than the others. The vernier allows you to read to the nearest thousandth of a millimeter, *i.e.*, to the nearest micron (0.001 mm = 1 μm).

Precaution: Great care must be taken in using the micrometer caliper; A ratchet knob is provided for closing the caliper on the object being measured without exerting too much force. Treat the micrometer with care, **ALWAYS** close the calipers using the ratchet knob, this prevents tightening the screw too strongly. Closing the calipers too hard damages the precision screw.
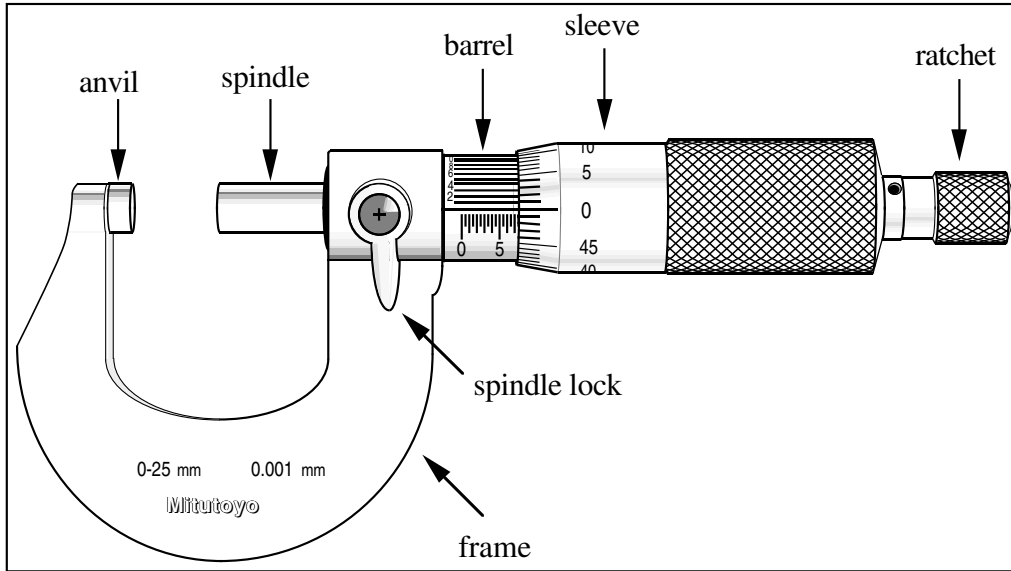
Figure 5: The micrometer calipers

Below are two examples of micrometer reading; the arrow shows which mark on the vernier scale is being used.
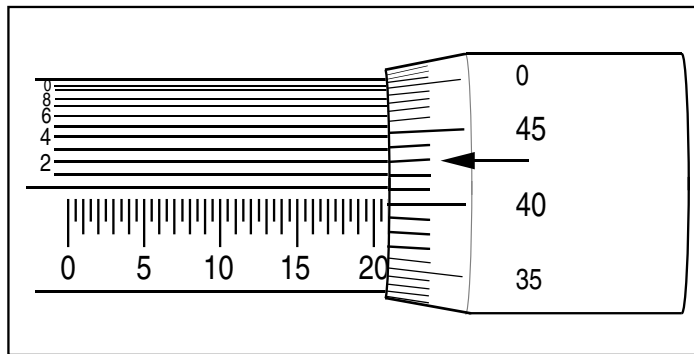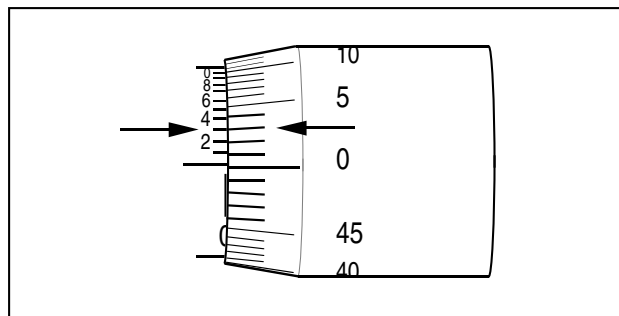

Figure 6: The micrometer reads 20.912 mm.

Figure 7: The micrometer reads 3 μm.

An uncalibrated micrometer may have a *zero error*–may read something other than zero when the anvil and spindle are touching. Before attempting precise work with

a micrometer, the zero error should be measured, so that it may be subtracted from subsequent measurements.

In Fig. 7 the zero line on the barrel is barely visible, and the vernier reads 0.003 mm = 3 μm; the zero error is $\epsilon_0 = 3\mu$m.

A negative zero error, as shown below, requires a moment of thought:

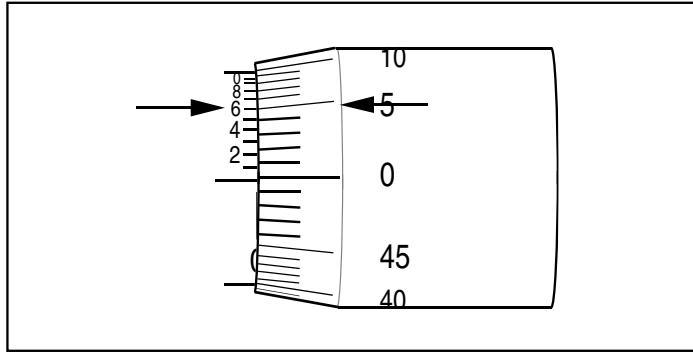
Figure 8: The micrometer reads -4 μm

In Fig. 8 the zero line on the barrel of the micrometer is obscured by the sleeve, (the "zero" line on the sleeve is above the "zero" line on the barrel); this corresponds to a reading of -0.5 mm. The vernier reads 0.496 mm. The zero error is then $\epsilon_0 = -0.5$ mm + 0.496 mm = -0.004 mm = -4 μm.
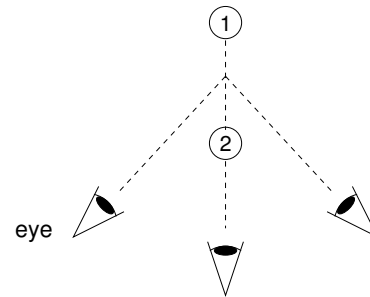
B   Parallax and Parallax Error

1. **Parallax:**

   To do quantitative work in optics one must understand *parallax* and how it may be eliminated. PARALLAX is defined as *apparent motion of an object caused by actual motion of the observer.*

   As the observer moves left and right, object 1 appears to move to the left and right of object 2. The amount by which object 1 appears to move is proportional to the distance between object 1 and object 2. If object 1 comes in front of object 2, the direction of its apparent motion reverses.

   Try this with two fingers.

2. **Parallax Error:**

   If an observer wants to measure the location of the object 2 using a measuring scale in the location of object 1 (that is, "behind" object 2), the apparent position of object 2 will depend on the location of the observer. The observer has no way of knowing which location is the "correct" location from which to make the measurement. Such a measurement is said to include a "parallax error".

   One solution is to make sure that the object and the measuring scale are at the same distance from the observer (that is, the object is right next to the measuring scale).

   Another solution is to have a mirror run alongside the measuring scale. The observer then knows that the correct location from which to make a measurement is the location at which the object appears to be directly in front of its reflection in the mirror. This solution is used in high-quality analog measuring devices.