CHAPTER 31  NUCLEAR PHYSICS AND RADIOACTIVITY

CONCEPTUAL QUESTIONS

1. **REASONING AND SOLUTION** Isotopes are nuclei that contain the same number of protons, but a different number of neutrons. A material is known to be an isotope of lead, although the particular isotope is not known.

   a. The atomic number $Z$ of an atom is equal to the number of protons in its nucleus. It is different for different elements, and all isotopes of the same element have the same atomic number. Therefore, if we know that a material is an isotope of lead, we can specify its atomic number ($Z = 82$).

   b. The various isotopes of the same element differ in the number of neutrons in the nucleus. Since we do not know which particular isotope of lead makes up the material, we cannot specify the neutron number.

   c. The atomic mass number is equal to the total number of protons and neutrons in the nucleus. Since we do not know which isotope of lead makes up the material, we do not know how many neutrons are in the nucleus. Therefore, we cannot specify the atomic mass number.

2. **REASONING AND SOLUTION** Two nuclei have different nucleon numbers $A_1$ and $A_2$. The two nuclei may or may not be isotopes of the same element. Two isotopes of the same element have the same number of protons in the nucleus, but differ in the number of neutrons. The nucleon number is equal to the total number of protons and neutrons in the nucleus. If the nucleon numbers differ only because the neutron numbers differ, then the two nuclei are isotopes of the same element. This would be the case for the two isotopes of actinium, $^{227}_{89}$Ac and $^{228}_{89}$Ac (see Appendix F). If the nucleon numbers differ because the nuclei contain a different number of protons, then the two nuclei are not isotopes of the same element (regardless of their respective neutron numbers). This would be the case for lanthanum and cerium, $^{139}_{57}$La and $^{140}_{58}$Ce. Therefore, two nuclei with different nucleon numbers are not necessarily isotopes of the same element.

3. **REASONING AND SOLUTION** Two nuclei that contain different numbers of protons and different numbers of neutrons can have the same radius. According to Equation 31.2, the approximate radius of the nucleus depends on the atomic mass number $A$ and is given by $r \approx (1.2 \times 10^{-15} \text{ m})A^{1/3}$. The atomic mass number $A$ is equal to the total number of protons and neutrons. Therefore, if two nuclei have different numbers of protons and different numbers of neutrons, but the sum of the protons and neutrons is the same for each nucleus, then the two nuclei have the same radius.
4. **REASONING AND SOLUTION**  Using Figure 31.6, the following nuclei have been ranked in ascending order (smallest first) according to the binding energy per nucleon: $^{232}_{90}$Th, $^{184}_{74}$W, $^{31}_{15}$P, $^{59}_{27}$Co.

5. **REASONING AND SOLUTION**  The general form for $\alpha$ decay is

$$^{A}_{Z}P \rightarrow ^{A-4}_{Z-2}D + ^{4}_{2}\text{He}$$

while the general form for the two types of beta decay are

$[\beta^{-} \text{ decay}]$

$$^{A}_{Z}P \rightarrow ^{A}_{Z+1}D + ^{0}_{-1}\text{e}$$

and

$[\beta^{+} \text{ decay}]$

$$^{A}_{Z}P \rightarrow ^{A}_{Z-1}D + ^{0}_{+1}\text{e}$$

According to Equation 31.2, the approximate radius of the nucleus depends on the atomic mass number $A$ and is given by $r \approx (1.2 \times 10^{-15} \text{ m})A^{1/3}$.

a. For $\alpha$ decay, the mass number of the daughter nucleus is four nucleons smaller than that of the parent nucleus. Therefore, according to Equation 31.2, the daughter nucleus should have a smaller radius than the parent nucleus.

b. For both types of $\beta$ decay, the mass number of the daughter nucleus is identical to that of the parent nucleus. Therefore, according to Equation 31.2, the radius of the daughter nucleus should be the same as that of the parent nucleus.

6. **REASONING AND SOLUTION**  The general forms for $\alpha$ decay, $\beta$ decay, and $\gamma$ decay are, respectively:
As discussed in Section 31.1, the identity of an element depends on the number of protons in the nucleus. Clearly, in both \( \alpha \) decay and \( \beta \) decay, the number of protons of the parent nucleus changes as the parent nucleus emits a charged particle. In contrast, in the process of \( \gamma \) decay, the parent nucleus emits only energy and the number of protons in the nucleus remains the same. Therefore, \( \alpha \) and \( \beta \) decay result in transmutations, while \( \gamma \) decay does not.

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7. **REASONING AND SOLUTION** In order for a nuclear decay to occur, it must bring the parent nucleus toward a more stable state by allowing the release of energy. The total mass of the decay products must be less than the mass of the parent nucleus. The difference in mass between the products and the parent nucleus represents an equivalent amount of energy that is given by Equation 28.5: \( E_0 = mc^2 \). This is the energy that is released during the decay.

As discussed in Example 4, uranium \( ^{238}\text{U} \) decays into thorium \( ^{234}\text{Th} \) by means of \( \alpha \) decay. The \( ^{238}\text{U} \) nucleus never decays by just emitting a single proton, instead of an alpha particle. The hypothetical decay scheme for the unobserved decay, is given below along with the pertinent atomic masses:

\[
^{238}\text{U} \rightarrow ^{237}\text{Pa} + ^{1}\text{H}
\]

The total mass of the decay products is \( (237.051\text{ u}) + (1.007\text{ u}) = 238.058\text{ u} \). This is greater, not smaller, than the mass of the parent nucleus \( ^{238}\text{U} \); therefore, the daughter nucleus is less stable than the parent nucleus. Consequently, the hypothetical decay never occurs.

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8. **REASONING AND SOLUTION** According to Equation 31.5, if there are \( N_0 \) radioactive nuclei present at time \( t = 0 \), then the number \( N \) of radioactive nuclei present at time \( t \) is \( N = N_0 e^{-\lambda t} \), where the decay constant \( \lambda \) is given by Equation 31.6: \( \lambda = (\ln 2) / T_{1/2} \), where \( T_{1/2} \) is the half-life of the nuclei. According to Equation 31.6, unstable nuclei with short half-lives have large decay constants.
When $\lambda$ is large, the term $e^{-\lambda t}$ approaches zero even for very short times. Therefore, unstable nuclei with short half-lives typically have only a small or zero abundance after a short period of time.

9. **REASONING AND SOLUTION** The half-life of a radioactive isotope is the time required for one-half of the nuclei present to disintegrate. The disintegrations do not occur continuously at regular time intervals. Individual disintegrations occur randomly; we cannot predict *a priori* which particular nucleus will decay. Any given nucleus may decay at any time. The time may be much shorter than the half-life or it may be much longer than the half-life. Therefore, even though the half-life of indium $^{115}_{49}\text{In}$ is $4.41 \times 10^{14}$ yr, it is possible for any single nucleus in the sample to decay after only one second has passed.

10. **REASONING AND SOLUTION** The activity of a radioactive sample is the number of disintegrations per second that occur. In other words, it is the magnitude of $\Delta N/\Delta t$. According to Equation 31.4, $\Delta N/\Delta t = -\lambda N$, where $\lambda$ is the decay constant that is characteristic of the isotope in question, and $N$ is the number of radioactive nuclei that are present.

   a. While each isotope has its own characteristic decay constant, the activity is proportional to the product $\lambda N$, according to Equation 31.4. Therefore, two samples with different decay constants $\lambda$, and different numbers of radioactive nuclei $N$, can have identical activities if the product $\lambda N$ is the same for each. Thus, it is possible for two different samples that contain different radioactive elements to have the same activity.

   b. Since each isotope has its own characteristic decay constant $\lambda$, two different samples of the same radioactive isotope have the same decay constant. If the two samples contain different numbers of radioactive nuclei, the product $\lambda N$ will be different for each sample. Therefore, they will have different activities.

11. **REASONING AND SOLUTION** Radiocarbon dating utilizes the $^{14}_{6}\text{C}$ isotope that undergoes $\beta^-$ decay with a half-life of 5730 yr. If the present activity of $^{14}_{6}\text{C}$ can be measured in a carbon-containing object, the age of the object can be determined. Example 11 shows how to determine the age of an object, once the present activity is known.

   Consider the following objects, each about 1000 yr old: a wooden box, a gold statue, and some plant seeds. Since 1000 yr is of the same order of magnitude as the half-life of $^{14}_{6}\text{C}$, the change in activity should be significant, and the present activity should be easily measured. However, the sample must contain $^{14}_{6}\text{C}$ atoms. Since both the wooden box and the seeds contain $^{14}_{6}\text{C}$, radiocarbon dating can be used to determine their ages. If the gold statue is made of pure gold, it will not contain any $^{14}_{6}\text{C}$ atoms. Therefore, radiocarbon dating could not be applied to the gold statue.

12. **REASONING AND SOLUTION** Two isotopes have half-lives that are short relative to the age of the earth. Today, one of these isotopes (isotope A) is found in nature and the other (isotope B) is not.
It is reasonable to conclude that the supply of isotope A has been replenished by some natural process that off-sets the radioactive decay, while the supply of isotope B has not been replenished. The replenishing of isotope A could occur, for example, as the result of the interaction of cosmic rays with the earth’s upper atmosphere or the radioactive decay of some other isotope.

13. **REASONING AND SOLUTION**  In radiocarbon dating, the age of the sample is determined from the ratio of the present activity to the original activity. Suppose that there were a greater number of $^{14}_6$C atoms in a plant living 5000 yr ago than is currently believed. According to Equation 31.4: $\Delta N / \Delta t = -\lambda N$, and the original activity would be greater than is currently believed. A longer time would be required for the activity of a larger number of $^{14}_6$C nuclei to decrease to its present value. Therefore, the age of the seeds obtained from radiocarbon dating would be too small.

14. **REASONING AND SOLUTION**  Tritium is an isotope of hydrogen and undergoes $\beta^-$ decay with a half-life of 12.33 yr. Like carbon $^{14}_6$C, tritium is produced in the atmosphere because of cosmic rays and can be used in a radioactive dating technique. The age cannot be significantly smaller or significantly larger than the half-life of the radioisotope. During a 700 yr period, $700/12.33$ or roughly 57 half-lives would occur, and the activity of a given sample would decrease to an immeasurably small fraction of its initial value (on the order of $10^{-17}$). Therefore, tritium dating could *not* be used to determine a reliable date for a 700 yr old sample.

15. **REASONING AND SOLUTION**  Because of radioactive decay, one element can be transmuted into another. Thus, a container of uranium $^{238}_{92}$U ultimately becomes a container of lead $^{206}_{82}$Pb. According to Figure 31.15, the first step is the transmutation from uranium $^{238}_{92}$U to thorium $^{234}_{90}$Th. This step has the greatest half-life, with a value of $4.5 \times 10^9$ y. Therefore, we can conclude that the complete transmutation from uranium $^{238}_{92}$U to lead $^{206}_{82}$Pb would take at least billions of years.

16. **REASONING AND SOLUTION**  According to Equation 21.2, the radius $r$ of curvature for a particle with charge $q$ and mass $m$, that moves perpendicular to a magnetic field of magnitude $B$, is given by

$$ r = \frac{mv}{qB} $$

Thus, the ratio of the radii of curvature for the motion of the $\alpha$ particle and the $\beta^-$ particle is

$$ \frac{r_\beta}{r_\alpha} = \frac{m_\beta v / q_\beta B}{m_\alpha v / q_\alpha B} = \frac{m_\beta / q_\beta}{m_\alpha / q_\alpha} = \frac{m_\beta q_\alpha}{m_\alpha q_\beta} $$

The mass of the $\alpha$ particle is 1.0026 u, while the mass of a $\beta^-$ particle (an electron) is

$$ (9.109 \ 389 \ 731 \times 10^{-31} \text{ kg}) \left( \frac{1.0073 \text{ u}}{1.6726 \times 10^{-27} \text{ kg}} \right) = 5.486 \times 10^{-4} \text{ u} $$
Since the magnitudes of the charge on the α and β⁻ particles are 2e and e, respectively, the ratio of the radii of curvature is

\[ \frac{r_β}{r_α} = \left( \frac{5.486 \times 10^{-4} \text{ u}}{4.0026 \text{ u}} \right) \left( \frac{2e}{e} \right) = 2.7 \times 10^{-4} \]

Therefore, the radius of curvature of the path of the β⁻ particle is smaller than the radius of curvature of the path of the α particle. Therefore, the trajectory of the β⁻ particle has a greater curvature than that of the α particle.