

Homework

Chapter One

1. Prove what tile shapes can be used to tile a floor.
2. List the symmetry operations of a square, a rectangle, and an equilateral triangle. Prove that each list comprises a mathematical 'group'. You will have to read a little of chapter two in order to do this problem. Determine whether any of the three groups is a subgroup of another group.
3. (a) Calculate the entropy of a five-card poker hand using a normal deck of cards. (b) Suppose that someone washes your cards in bleach, rendering them black and white. Calculate the entropy of a five-card poker hand using such a deck of cards.
4. Referring to Figure Seven in chapter one, suppose you double the size of the magnet array, from (64) in the text to (128). The interaction energies remain the same. Determine whether effect, if any, this has on the critical temperature. Now double the size again, to (256), and again determine the effect on the critical temperature. What pattern do you see emerging as you make the 'system' larger?

Chapter 2

1. Go back to chapter 1, homework problem 2. Identify the group of symmetry operations for a square, rectangle, and equilateral triangle using Tinkham's notation. Tinkham's book has the character table of every point group, so you know the character table 'answer' for each group. However, prove that the character tables in Tinkham's book are correct, using the 'recipe' described in chapter 2.
2. Consider a matrix:

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 6 & -1 & -2 & 6 \\ 2 & -4 & 6 & 8 \\ 8 & 4 & -2 & -1 \end{pmatrix}. \text{ (a) Calculate the determinant of } (A), \det(A). \text{ (b) If } (A) \text{ has an}$$

inverse, find it. (c) Explain whether (A) is Hermitian or unitary. (d) Explain whether a matrix can or cannot be both Hermitian and unitary, providing an example if it can.

3. Consider a matrix:

$$B = \begin{pmatrix} 1+i & 1-i & 2 \\ 1-i & 1 & 3-i \\ 2 & 3+i & 4 \end{pmatrix} .$$
 (a) Determine whether (B) is Hermitian, unitary, both or

neither. (b) Calculate the $\det(B)$. (c) If (B) has an inverse, find it. (d) Determine the eigenfunctions and eigenvalues of (B), if (B) has such.

4. Calculate the following calculus examples.

$$(a) \int_{-3}^4 x^2 dx \quad (b) \int_0^{2\pi} \sin^2(2\theta) d\theta \quad (c) \frac{d}{dx} (e^{\sin(x)}) \quad (d) \frac{d}{dx} (\tan^2(x)) .$$

5. Consider all rational numbers between (0) and (1), including (0) and (1). Call this $\text{Rat}[0,1]$. Now consider all rational numbers between (0) and (2), including (0) and (2); call this $\text{Rat}[0,2]$. Explain whether $\text{Rat}[0,1]$ and $\text{Rat}[0,2]$ are or are not isomorphic. If they are isomorphic, prove it by writing out the isomorphism that makes one function onto the other. Can you reconcile your answer with the fact that both sets are infinite and that $\text{Rat}[0,1]$ is a proper subset of $\text{Rat}[0,2]$?

6. Consider well collimated light from a flashlight. We see 'white' light, and sending the light through a prism shows that it includes all visible colors.

(a) What is the quantization axis of the light? Is the quantization axis the same for all colors of light? Explain.

(b) What is the angular momentum of a single photon? Now, we will learn that a single photon of light, having frequency (f), has energy $E = hf$, where (h) = 6.63×10^{-34} joule second is called the *Planck constant*. Do photons of different colors (frequencies) have different angular momenta? Explain.

(c) For an ordinary vector, we only 'know' the vector if we know all components of the vector. Do we, can we, know all components of the photon angular momentum vector? Explain.

7. Imagine a type of methane molecule (CH_4) that is planar, with C-H bonds every 90° and every C-H bond equivalent to every other C-H bond.

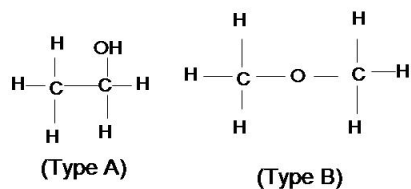
(a) What is the symmetry group of this planar methane?

(b) Calculate the character table. I know you can check the answer by looking in Tinkham, but I want you to show me you can do the calculation yourself. Besides, books have typographical errors.

(c) From the character table, what is the dimensionality of the representations? Is there any degeneracy for planar methane?

(d) If there is degeneracy, what reduction of symmetry would lift the degeneracy? I am asking both for the physical change needed and for the corresponding symmetry group.

8. Consider two forms of $\text{C}_2\text{H}_5\text{OH}$, as shown below. Both forms are planar.



- (a) Determine the symmetry group of each form.
- (b) Is either a subgroup of the other?
- (c) Calculate the character table of each form.
- (d) Determine the degeneracy of each form. Explain.
- (e) If either form has a symmetry degeneracy, propose a physical change that lifts the degeneracy, and calculate the resulting new symmetry group.

9. Consider methane (CH_4) in its realistic three-dimensional tetrahedral form; see <http://www.usetute.com.au/shapemol.html> for details.

- (a) Find all the symmetry operations and from these calculate the symmetry group.
- (b) Showing your work, calculate the character table.
- (c) Suggest the fewest number of changes to the molecular structure that will lift all degeneracies. Explain what degeneracies methane itself has, and the physical changes needed to lift all these degeneracies. Further explain whether these changes can be infinitesimal in magnitude or must be of a certain minimum size. This part of the question refers back to lecture one and the difference between a second order and first order phase transition. Finally, explain how you would actually cause the changes to occur.