Defining Terms

Linear network: A network in which the parameters of resistance, inductance, and capacitance are constant with respect to voltage or current or the rate of change of voltage or current and in which the voltage or current of sources is either independent of or proportional to other voltages or currents, or their derivatives.

Maximum power transfer theorem: In any electrical network which carries direct or alternating current, the maximum possible power transferred from one section to another occurs when the impedance of the section acting as the load is the complex conjugate of the impedance of the section that acts as the source. Here, both impedances are measured across the pair of terminals in which the power is transferred with the other part of the network disconnected.

Norton theorem: The voltage across an element that is connected to two terminals of a linear, bilateral network is equal to the short-circuit current between these terminals in the absence of the element, divided by the admittance of the network looking back from the terminals into the network, with all generators replaced by their internal admittances.

Principle of superposition: In a linear electrical network, the voltage or current in any element resulting from several sources acting together is the sum of the voltages or currents from each source acting alone.

Reciprocity theorem: In a network consisting of linear, passive impedances, the ratio of the voltage introduced into any branch to the current in any other branch is equal in magnitude and phase to the ratio that results if the positions of the voltage and current are interchanged.

Thévenin theorem: The current flowing in any impedance connected to two terminals of a linear, bilateral network containing generators is equal to the current flowing in the same impedance when it is connected to a voltage generator whose voltage is the voltage at the open-circuited terminals in question and whose series impedance is the impedance of the network looking back from the terminals into the network, with all generators replaced by their internal impedances.
FIGURE 6.12 Block diagram of cathode ray oscilloscope.

FIGURE 6.13 Schematic diagram of cathode ray tube.
The Types of Oscilloscopes

Electronic equipment can be classified into two categories: analog and digital. Analog equipment works with continuously variable voltages, while digital equipment works with discrete binary numbers that represent voltage samples. A conventional phonograph is an analog device, while a compact disc player is a digital device.

Oscilloscopes can be classified similarly – as analog and digital types. For many applications, either an analog or digital oscilloscope will do. However, each type has unique characteristics that may make it more or less suitable for specific applications. Digital oscilloscopes can be further classified into digital storage oscilloscopes (DSOs), digital phosphor oscilloscopes (DPOs) and sampling oscilloscopes.

Analog Oscilloscopes

Fundamentally, an analog oscilloscope works by applying the measured signal voltage directly to the vertical axis of an electron beam that moves from left to right across the oscilloscope screen – usually a cathode-ray tube (CRT). The back side of the screen is treated with luminous phosphor that glows wherever the electron beam hits it. The signal voltage deflects the beam up and down proportionally as it moves horizontally across the display, tracing the waveform on the screen. The more frequently the beam hits a particular screen location, the more brightly it glows.

The CRT limits the range of frequencies that can be displayed by an analog oscilloscope. At very low frequencies, the signal appears as a bright, slow-moving dot that is difficult to distinguish as a waveform. At high frequencies, the CRT’s writing speed defines the limit. When the signal frequency exceeds the CRT’s writing speed, the display becomes too dim to see. The fastest analog oscilloscopes can display frequencies up to about 1 GHz.

When you connect an oscilloscope probe to a circuit, the voltage signal travels through the probe to the vertical system of the oscilloscope. Figure 13 illustrates how an analog oscilloscope displays a measured signal. Depending on how you set the vertical scale (volts/div control), an attenuator reduces the signal voltage and an amplifier increases the signal voltage.

Next, the signal travels directly to the vertical deflection plates of the CRT. Voltage applied to these deflection plates causes a glowing dot to move across the screen. The glowing dot is created by an electron beam that hits the luminous phosphor inside the CRT. A positive voltage causes the dot to move up while a negative voltage causes the dot to move down.
The signal also travels to the trigger system to start, or trigger, a horizontal sweep. Horizontal sweep refers to the action of the horizontal system that causes the glowing dot to move across the screen. Triggering the horizontal system causes the horizontal time base to move the glowing dot across the screen from left to right within a specific time interval. Many sweeps in rapid sequence cause the movement of the glowing dot to blend into a solid line. At higher speeds, the dot may sweep across the screen up to 500,000 times per second.

Together, the horizontal sweeping action and the vertical deflection action trace a graph of the signal on the screen. The trigger is necessary to stabilize a repeating signal – it ensures that the sweep begins at the same point of a repeating signal, resulting in a clear picture as shown in Figure 14.

In addition, analog oscilloscopes have focus and intensity controls that can be adjusted to create a sharp, legible display.

People often prefer analog oscilloscopes when it is important to display rapidly varying signals in “real time” – or, as they occur. The analog oscilloscope’s chemical phosphor-based display has a characteristic known as intensity grading that makes the trace brighter wherever the signal features occur most often. This intensity grading makes it easy to distinguish signal details just by looking at the trace’s intensity levels.

Digital Oscilloscopes

In contrast to an analog oscilloscope, a digital oscilloscope uses an analog-to-digital converter (ADC) to convert the measured voltage into digital information. It acquires the waveform as a series of samples, and stores these samples until it accumulates enough samples to describe a waveform. The digital oscilloscope then re-assembles the waveform for display on the screen. (see Figure 15)

Digital oscilloscopes can be classified into digital storage oscilloscopes (DSOs), digital phosphor oscilloscopes (DPOs), and sampling oscilloscopes.

The digital approach means that the oscilloscope can display any frequency within its range with stability, brightness, and clarity. For repetitive signals, the bandwidth of the digital oscilloscope is a function of the analog bandwidth of the front-end components of the oscilloscope, commonly referred to as the ~3dB point. For single-shot and transient events, such as pulses and steps, the bandwidth can be limited by the oscilloscope’s sample rate. Please refer to the Sample Rate section under Performance Terms and Considerations for a more detailed discussion.
Fig. 14.20 Generation of a circular Lissajous figure by two sine waves 90 degrees out of phase.

Fig. 14.21 (a) Resultant patterns when the phase difference between the unknown and standard signals varies in steps of 22.5 degrees. (b) Figure used in calculating phase difference.

**Figure 6.16** Lissajous figures for frequency determination. Shape of figure is dependent on ratio ($F$) of vertical to horizontal frequency. Upper left, $F = 1$; upper center, $F = 2$; upper right, $F = 5$; lower left, $F = 0.6$; lower center, $F = 0.33$. 
PERPENDICULAR MOTIONS WITH DIFFERENT FREQUENCIES; LISSAJOUS FIGURES

It is a simple exercise, and a quite entertaining one, to extend the above analysis to motions with different frequencies. We give a few examples to illustrate the kind of results obtained.

In Fig. 2–12 we show the construction that one can make if \( \omega_2 = 2\omega_1 \) and \( \delta = \pi/4 \). We have chosen to divide the reference circle for the motion of frequency \( \omega_2 \) into eight equal time intervals, i.e., into arcs subtending 45° each. During one complete cycle of \( \omega_2 \), we go through only a half-cycle of \( \omega_1 \), and the points on the reference circles are marked accordingly, taking account of the assumed initial phase difference of 45°. To obtain one complete period of the combined motion it is, of course, necessary to go through a complete cycle of \( \omega_1 \); this requires that, after reaching the point marked “9,” we retrace our steps along the lower semicircle and proceed for a second time through all the points corresponding to a complete tour around the \( \omega_2 \) circle. In this way we end up with a closed path which crosses itself at one point and would be indefinitely repeated. Such a curve is known as a Lissajous figure, after J. A. Lissajous (1822–1880), who made an extensive study of such motions. If one introduces

\[
\omega_2 = 2\omega_1
\]

\[
\delta = 0 \quad \delta = \pi/4 \quad \delta = \pi/2 \quad \delta = 3\pi/4 \quad \delta = \pi
\]

Fig. 2–13 Lissajous figures for \( \omega_2 = 2\omega_1 \) with various initial phase differences.

35 Perpendicular motions and Lissajous figures
a slow decay of amplitude with time the patterns become still more exotic and have an esthetic appeal all their own. In Fig. 2–13 we show a set of such curves, all for $\omega_2 = 2\omega_1$, with initial phase differences of various sizes.

As one goes to more complicated frequency ratios, the resulting curves tend to become more bizarre, and Fig. 2–14 shows an assortment of examples. Such patterns are readily generated, with flexible control over amplitudes, frequencies, and phases, by applying different sinusoidal voltages to the $x$ and $y$ deflection plates of the Lissajous figure. Each figure is a result made by inspection of the oscillograph, as described.

![Lissajous figures](image)

Fig. 2–14 Lissajous figures: various frequency ratios with various differences of phase. (After J. H. Poynting, J. J. Thomson, and W. S. Tucker, Sound, Griffin, London, 1949.)

The superposition of periodic motions