APPENDIX A

ACCURACY, PRECISION, ERRORS, UNCERTAINTY, ETC.

In common speech, the words accuracy and precision are often used interchangeably. However, many scientists like to make a distinction between the meanings of the two words. Accuracy refers to the relationship between a measured quantity and the real value of that quantity. The accuracy of a single measurement can be defined as the difference between the measured value and the true value of the quantity. The accuracy of a measurement is therefore limited by such things as the calibration and sensitivity of the instruments used, your ability to read the meters, mistakes in recording the numbers, and so on.

The word precision refers to the amount scatter in a series of measurements of the same quantity. It is possible for a measurement to be very precise, but at the same time not very accurate. For example, if you measure a voltage using a digital voltmeter that is incorrectly calibrated the answer will be precise (repeated measurements will give essentially the same result to several decimal places) but inaccurate (all of the measurements will be wrong). By making a series of measurements of some quantity, we can obtain an estimate of the precision of each individual measurement. The example discussed below illustrates how this works.

The words error and uncertainty are also often used interchangeably. Nevertheless, it is important to be aware of the distinction between the actual error in a given measurement (i.e. in the amount by which the measured value differs from the true value) and the uncertainty in a measurement. The point is that in many experiments we do not know the true value of the quantity we are measuring, and therefore cannot determine the actual error in our result. However, it is still possible to make an estimate of the uncertainty (or the probable error) in the measurement based on what we know about the properties of the measuring, instruments, etc.

The following example illustrates several of these ideas. In this example the resistance of a known 1000 $\pm$ 0.01 Ω resistor is determined by measuring V and I for several different voltage settings. The results are given in the table on the following page. The average value of R in this example is 1002.4 Ω, so our final result has an error of 2.4 Ω. The precision of any individual measurement of R can be determined by calculating the standard deviation:

$$\sigma \equiv \frac{1}{\sqrt{N-1}}\left[\sum_{i=1}^{N}\left(x_i-\bar{x}\right)^2\right]^{1/2}$$

where $\bar{x}$ is the average value of x and where N is the number of measurements. In this example the standard deviation is 5.6 Ω. We could take this as an estimate of the uncertainty or probable error, since any individual measurement has a reasonable probability of being in error by at least that amount. It should be emphasized, however, that the actual error in a measurement can be much larger than the standard deviation if there are systematic errors (for example errors in the calibration of some meter) that affect all the measurements the same way.

Data for a 1000 ± 0.01 kΩ Resistor

V(a) (volts)	I(b) (mA)	R(c) (Ω)
1.000	0.99	1010
2.000	1.99	1005
3.000	3.00	1000
4.000	4.02	995
5.000	4.99	1002
	Average =	1002.4
	Standard Deviation =	5.6
	Error =	2.4
	% Error =	0.24 %

(a) Measured with digital voltmeter.
(b) Measured with Simpson VOM.
(c) Calculated from R = V/I.

Whenever possible measured values of quantities should be compared with given or theoretical values and the percent error given. In some of the experiments you will be asked to make detailed calculations of the uncertainties in your measurements. Although this is usually not required (since the calculations are often long and time consuming), it is always important for experimenters to be aware of potential sources of error.

SUGGESTIONS FOR TABULATING AND PLOTTING DATA

Much of the data you take can and should be collected into tables for clarity. Any plot you make should have an associated table, and the location of the table (page number in the notebook) should be written on the plot.

In any table you make, the columns must be labeled with the names of the quantities being tabulated and with the units being used. Ordinarily the name is given first, while the units are given directly underneath in parentheses (see the example on p.2). Remember that it must always be clear from your notebook how each quantity in the table was determined. One way to do this (if you haven't already explained elsewhere how the measurement was done) is to use footnotes as in the example.

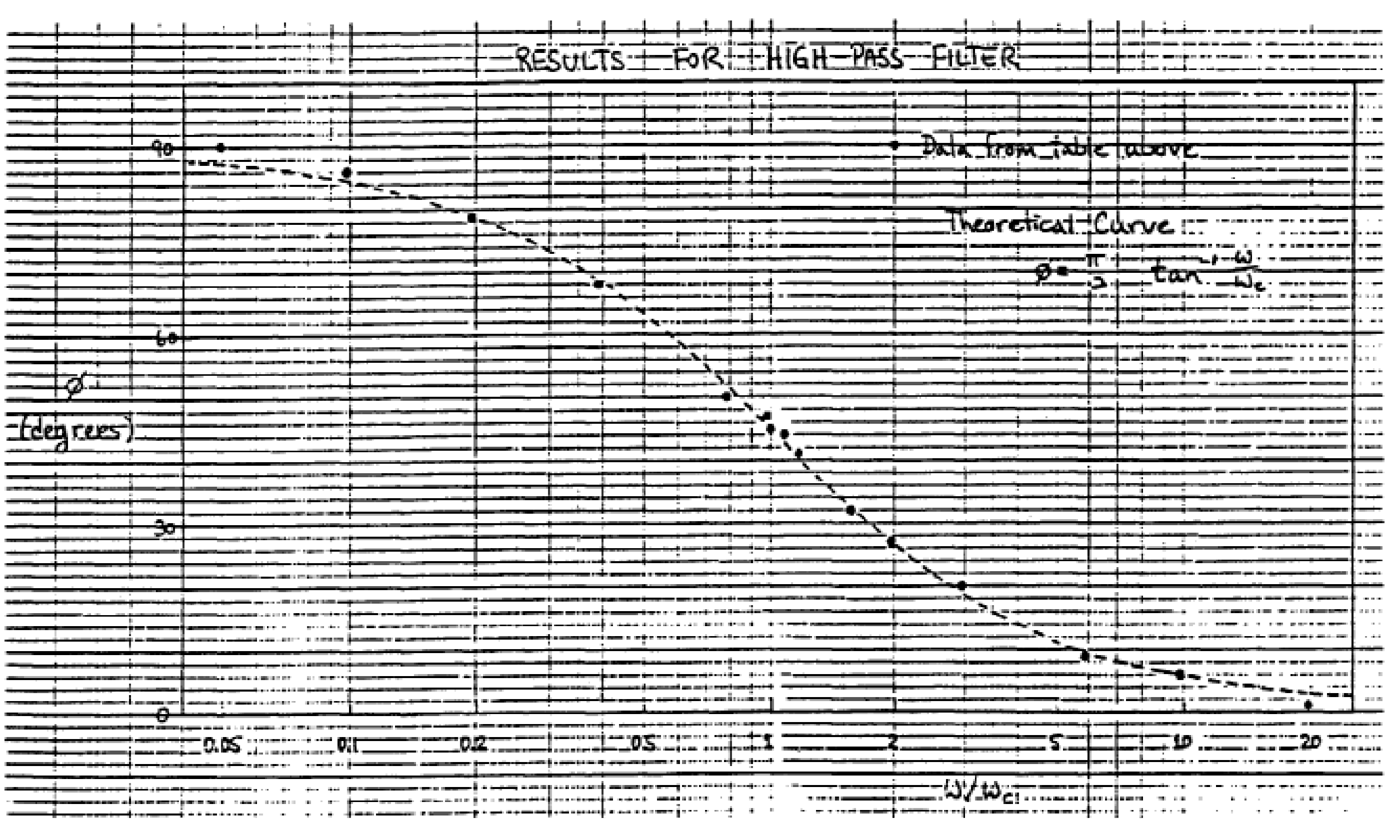
Think about how the table will look before writing anything down. For example, how many columns will the table have, how many entries, is there enough space left on the page for the table, etc. You may want to leave space for additional columns and entries just in case you need to add another calculation, such as converting data from degrees to radians, or if you need to add a few more data points. It is a good idea to start at the left with measured quantities and place calculated results to the right. It also helps to place quantities which are to be plotted against each other in adjacent columns, since this leads to fewer mistakes. Finally, record the data in an orderly fashion, with one of the important parameters monotonically increasing or decreasing.

The table on the following page contains data for a high-pass filter (experiment 4). The measured quantities are the frequency, f, the input voltage vin, the voltage across a resistor, v_R, and the phase angle ϕ between vin and v_R. The derived quantities are v_R/vin, $A_{db} = 20 \log_{10} v_R/\text{vin}$, and ω/ω_C.

The figure directly below the table shows a good example of a plot. In any plot you make, the axes should be labeled with the name of the quantity being plotted, followed in parentheses by the units (in this example ω/ω_C is dimensionless). Reference should always be made to the source of the data, and the meaning of any curves should be made clear. In this example the plot shows the measured values of ϕ vs ω/ω_C for the high-pass filter. The curve is the prediction obtained from circuit theory,

Data for a High-Pass Filter

f (Hz)	Vin (RMS V)	V_R (RMS V)	ϕ (degrees)	ω/ω_C	Adb	V_R/Vin
50	7.54	0.37	90	0.049	–26.0	0.049
100	7.55	0.74	86	0.098	–20.2	0.098
200	7.54	1.45	79	0.196	–14.3	0.192
400	7.52	2.72	68	0.392	–8.80	0.362
800	7.46	4.57	50	0.784	–4.30	0.613
1 k	7.43	5.19	47	0.980	–3.12	0.698
1.02 k	7.42	5.25	45	1.00	–3.00	0.708
1.1 k	7.42	5.43	44	1.08	–2.71	0.732
1.2 k	7.42	5.64	41	1.18	–2.40	0.760
1.6 k	7.39	6.21	32	1.57	–1.50	0.840
2.0 k	7.37	6.54	27	1.96	–1.00	0.887
3.0 k	7.34	6.94	20	2.94	–0.49	0.945
6.0 k	7.32	7.21	9	5.88	–0.13	0.985



In general, quantities that are related by a linear function (such as current vs voltage for a resistor) should be plotted on linear graph paper, although if the range of the measurements is large, it may be preferable to use log-log graph paper, which permits you to see data covering several decades. For quantities that are related by an exponential formula (for example $V = V_0 e^{-\gamma t}$) it is often best to use semi-log graph paper. Rewriting the formula as $\ln V = \ln V_0 - \gamma t$ shows that a plot of ln V (or $\log_{10} V$) vs t will make a straight line with a slope that depends on the decay constant γ. Quantities which are related by a power law ($x = Cy^{\alpha}$) will make a straight line if you plot them on log-log paper. To see this, note that the formula can be rewritten as $\log_{10} x = \log_{10} C + \alpha \log_{10} y$.