In this experiment we will measure the gain and the phase shift of some simple filter circuits. The measurements will be compared with theoretical calculations of the same quantities. Measurements will be made for a low-pass filter, a high-pass filter and a resonant filter. Some sample tables and plots for part 1 of this experiment are shown in Appendix B.

For the high-pass and low-pass filters you should choose circuit components that give critical frequencies $f$ of roughly 1 kHz. This keeps the frequencies well within the AC voltage measuring range of the DMM ($f_{\text{max}} \leq 20$ kHz). Note that on the capacitance decade boxes, MF means $10^{-6}$ F.

1. Set up the high pass filter circuit shown, with the function generator set to produce sine waves.

(a) Measure and tabulate $|v_{\text{in}}|$, $|v_R|$ and the phase $\Phi$ of $v_R$ relative to $v_{\text{in}}$

[Note that $v_R(t)$ is proportional to $I(t)$] as a function of frequency. Assume that the frequencies you read from the dial of the function generator are correct. Take measurements for something like 15 points from $f = 50$ Hz to $f = 15$ kHz, increasing $f$ by roughly a factor of 1.5 at each step. Make sure it is clear in your notebook how the various quantities in the table were determined. Use the DMM to measure the two voltages (the DMM reads the RMS voltage).

To measure phase differences, display the two sine waves simultaneously on the scope, making sure that you trigger the scope on only one of the two inputs. Phase differences can be determined by using the scope vertical cursors to measure time differences. First measure the period, and then measure the time between two identical points on each waveform, but always choose the shortest time difference. Since one period corresponds to a phase difference of $2\pi$, the phase difference between the two waveforms is just the fraction of $2\pi$ given by the time difference relative to one period.

The Zoom feature of the scope can be used to magnify the horizontal axis so you can see more detail. Push the Zoom button (the magnifier icon) and use the
horizontal SCALE control to adjust the amount of magnification. Use the horizontal POSITION control to select the portion of the waveform you want to magnify.

(b) Calculate the gain in dB \( A_{\text{dB}} = 20 \log_{10} \left| \frac{V_R}{V_{\text{in}}} \right| \) at each point, and include the results in your table. At the corner frequency \( f_C \), \( \Phi = 45^\circ \) and \( A_{\text{dB}} = -3 \). Determine the corner frequency \( f_C \) of the circuit from the measurements of \( \Phi \) and/or \( A_{\text{dB}} \), and then calculate \( \frac{f}{f_C} \) for each point, again including the results in your table. How does your value of \( \omega_C = 2\pi f_C \) compare with the expected value for your circuit, \( \omega_C = 1/RC \).

(c) Plot the measured values of \( A_{\text{dB}} \) and \( \Phi \) (see example in Appendix B) as a function of \( \omega/\omega_C \). Use semi-log paper with \( \omega/\omega_C \) plotted along the logarithmic axis. Determine the asymptotic behavior of \( A_{\text{dB}} \) (i.e. the slope of \( A_{\text{dB}} \) vs \( \log \omega/\omega_C \) in decibels per decade) for \( \omega \ll \omega_C \) and \( \omega \gg \omega_C \).

(d) Compare your measurements with theory by plotting the theoretical predictions for \( A_{\text{dB}} \) and \( \Phi \):

\[
A_{\text{dB}} = 20 \log_{10} \left\{ \frac{(\omega/\omega_C)}{[1 + (\omega/\omega_C)^2]^{1/2}} \right\}
\]

\[
\Phi = \pi/2 - \tan^{-1}(\omega/\omega_C)
\]

(e) For one frequency (take \( f = 2 f_C \)) measure the phase and amplitude of \( v_{\text{in}} \), \( v_R \), and \( v_C \), and draw a phasor diagram including \( v_{\text{in}} \), \( v_R \), \( v_C \) and \( I \). (NOTE: To measure the phase of the voltage across C, switch R and C in the circuit so that C is adjacent to ground.) Be sure you determine the sign as well as the magnitude of the phase difference.

2. Change the circuit to a low-pass filter by replacing the capacitor with an inductor. As in part 1 choose L and R to give \( f_C = 1 \) kHz. Repeat steps (a) - (d), but to save time skip the parts that involve measuring and plotting \( \Phi \) (you should also skip part (e)). For part (d) you will need to derive the theoretical formula for \( A_{\text{dB}} \).

3. Now set up the resonant filter circuit shown below. Use \( L = 40 \) mH, \( R = 160 \) Ω and \( C = 25 \) nF (or \( L = 10 \) mH, \( R = 40 \) Ω, \( C = 100 \) nF).
LCR CIRCUITS

(a) Measure $|v_{in}|$, $|v_R|$ and $\Phi$ for frequencies from 200 Hz to 25 kHz. You will need to take quite a few points near the resonant frequency, so that you can map out the shape of the resonance curves.

(b) Determine the resonant frequency and plot the measured values of $A_{\text{dB}}$ and $\Phi$ as a function of $\omega/\omega_0$.

(c) Determine the Q of the resonant circuit ($Q = \omega_0 L/R$) by measuring $|v_L|$ and $|v_R|$ at the resonant frequency. How does the measured value ($Q = |v_L| / |v_R|$) compare with the expected value ($Q = \omega_0 L/R$) where $\omega_0 = 1/\sqrt{LC}$.

(d) Set the function generator for a point about 500 Hz below the resonance. Measure the amplitudes and relative phases of $v_{in}$, $v_R$, $v_L$ and $v_C$. To measure the phases you will need to interchange the components to put the one of interest adjacent to ground. Draw a phasor diagram. If you do everything right, $v_{in}$ should be equal to the vector sum of $v_L$, $v_R$ and $v_C$. How close do your results come?