

## BRIDGE CIRCUITS

### EXPERIMENT 5: DC AND AC BRIDGE CIRCUITS

10/9/07

This experiment demonstrates the use of the Wheatstone Bridge for precise resistance measurements and the use of error propagation to determine the uncertainty of a measurement. The bridge circuit will be used to measure an unknown resistance to an accuracy of about 0.1%. We will also construct an AC bridge, and use it to determine the inductance of an unknown inductor.

Before coming to the lab, read the information on error propagation in the appendix to this experiment. You can also do step 1 in both the Wheatstone Bridge and AC Bridge sections.

#### THE WHEATSTONE BRIDGE

The Wheatstone Bridge circuit can be used to measure an unknown resistance in terms of three known resistances by adjusting one or more of the known resistors to obtain a zero signal (i.e. a “null” reading) on a meter. Such a measurement permits high precision since a very sensitive meter can be used to determine the null condition. The null method also reduces or eliminates sensitivity to a variety of effects (for example, fluctuations in the power supply voltage) which could lead to errors in more conventional measurements.

1. Derive the balance condition for the Wheatstone Bridge.
2. Construct the bridge circuit using the following components:

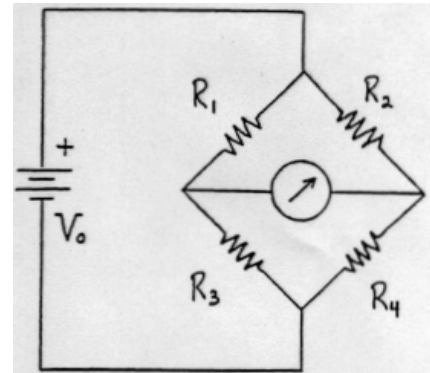
$R_1 = \pm 0.05\%$  accuracy General Radio resistance box ( $1\Omega$  steps).

$R_2 = \pm 0.5\%$  accuracy Eico resistance box.

$R_3 = \pm 0.05\%$  accuracy General Radio resistance box ( $0.1\Omega$  steps).

$R_4 = \pm 0.5\%$  accuracy Eico box ( $R_E$ ) in parallel with a  $\pm 10\%$  Heathkit resistance box ( $R_H$ ).

$V_0 =$  Lambda power supply, set to about 4 V.



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For the null detector use a DMM (as a voltmeter). Start by setting  $R_1 = R_E = 950 \Omega$ ,  $R_2 = 1000 \Omega$ ,  $R_3 = 900 \Omega$  and  $R_H = 1 \text{ M} \Omega$ . Then adjust any or all of the resistors (make only small changes, and keep  $R_H > 200 \text{ k}\Omega$ ) to balance the bridge as accurately as possible. Record all the final resistance values and the DMM reading.

3. Now calculate the ratios  $R_1/R_3$  and  $R_2/R_4$  from the resistor settings. In addition, calculate the uncertainty in each of the two ratios, assuming that the accuracies listed in part 2 are correct. To find the uncertainty in  $R_2/R_4$  you will first need to calculate the uncertainty in  $R_4$  (see appendix).

According to the balance equation  $R_1/R_3$  and  $R_2/R_4$  should be equal. Do your calculated ratios agree to within their calculated uncertainties?

4. For this part we need to make a distinction between the calculated ratios (the values you just obtained) and the true ratios (the values you would obtain for  $R_1/R_3$  and  $R_2/R_4$  if you somehow knew all the resistances exactly). What we want to do in this step is estimate how accurately the bridge has been balanced; in other words, how nearly equal are the true values of  $R_1/R_3$  and  $R_2/R_4$ , once you have balanced the bridge as accurately as possible. To answer this question what we will do is change one of the ratios ( $R_1/R_3$ ) by a small amount and see what effect this has on the null reading. The procedure is as follows. Starting with the bridge balanced change the value of  $R_3$  by a small amount (e.g. a few tenths of an ohm) and observe what effect this has on the DMM reading. By how much did you change  $R_1/R_3$ ? How large was the change in the DMM reading? Roughly how well do you think you can balance the bridge (in other words, how close to zero do you actually get when the bridge is balanced)? Call this the “null error”. How large a change in  $R_1/R_3$  does the null error correspond to? From this information estimate how nearly equal  $R_1/R_3$  and  $R_2/R_4$  are at the balance point. How does the difference between the true values of  $R_1/R_3$  and  $R_2/R_4$  compare with the uncertainties in the calculated values of  $R_1/R_3$  and  $R_2/R_4$  (from part 3)?

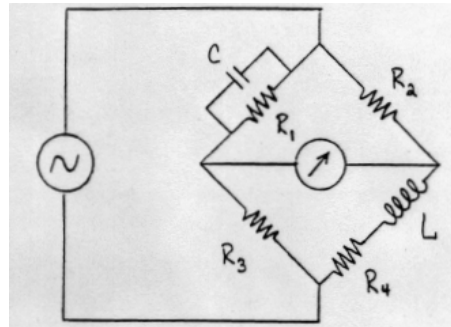
How accurately do you know the ratio  $R_2/R_4$ ?

5. Now substitute an “unknown” resistor  $R_U = 680 \Omega$  for  $R_1$  (use a second Heathkit  $\pm 10\%$  resistance box for the unknown). Rebalance the bridge by adjusting  $R_3$ . Do not change  $R_2$  or  $R_4$ . Calculate  $R_U$  from the balance equation,  $R_U = R_3 \times (R_2/R_4)$ . Then calculate the uncertainty in  $R_U$ .
6. Finally, measure  $R_U$  directly with a DMM. Compare all your results including the estimated errors in a table. The accuracy of the DMM measurement can be found in Appendix C.

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Next we will use the AC bridge circuit shown to measure an unknown inductor (with  $L$  somewhere between 15 and 25 mH).

1. Show that when the bridge is balanced the resistance  $R_4$  and inductance  $L$  are given by  $R_4 = R_2 R_3 / R_1$  and  $L = R_2 R_3 C$ . Although these results are independent of  $\omega$  the determination of  $L$  is not very precise for low frequencies because the voltage across  $L$  is too small. We will use a value of  $\omega$  that makes the magnitudes of the impedances of  $R_4$  and  $L$  comparable.
2. Construct the bridge with  $R_1 \approx R_2 \approx 2000 \Omega$  and  $R_3 \approx R_4 \approx 150 \Omega$ . Use the high precision (General Radio) resistors for  $R_2$  and  $R_3$  and the Eico resistors for  $R_1$  and  $R_4$ . Use the function generator with  $f \approx 1 \text{ kHz}$  and with the amplitude adjusted for the maximum output as the voltage source. Set the DMM null meter to read AC voltage. Now adjust  $C$  and  $R_1$  to minimize the DMM reading. (Because of noise pickup you may only be able to null the bridge to a few mV. You can test whether you have nulled the  $f = 1 \text{ kHz}$  signal by turning the function generator amplitude to zero and observing what happens to the null reading.) Record the results and calculate  $L$ . Also, record the number on the inductor board.
3. Change  $f$  by a factor of 2 and rebalance the bridge to verify that the balance equations do not depend on  $\omega$ .
4. The last step is to estimate the uncertainty in  $L$ . With  $f$  back at 1 kHz, vary  $C$  from the null setting and make a rough estimate of how accurately  $C$  can be set. Now estimate the uncertainty in  $L$  taking into account the accuracy of the capacitance box itself ( $\pm 1\%$ ) as well as the accuracy with which  $C$  can be set (the uncertainties in  $R_2$  and  $R_3$  are negligible).
5. Check with your lab instructor to get the actual value of  $L$  for the board you used. How close was your measurement to the actual value?



## APPENDIX

In physics experiments we often encounter situations in which we want to determine some quantity  $Q$  which depends in a known way on two or more separately measured quantities,  $x_1, \dots, x_n$ . Now suppose we know (or can estimate) the uncertainties in the  $n$  measured quantities and want to calculate the uncertainty in  $Q$ . The correct way to do this is to use the formula:

$$\Delta Q = \left[ \sum_{i=1}^n \left( \frac{\partial f}{\partial x_i} \Delta x_i \right)^2 \right]^{1/2}$$

where  $f$  is the function we use to calculate  $Q$  from  $x_1, \dots, x_n$  [i.e.  $Q = f(x_1, \dots, x_n)$ ].

As an example, suppose the quantity  $Q$  is the ratio of two measured quantities ( $Q = A/B$ ). Then the formula for the uncertainty in  $Q$  turns out to be:

$$\Delta Q = Q \left[ \left( \frac{\Delta A}{A} \right)^2 + \left( \frac{\Delta B}{B} \right)^2 \right]^{1/2}.$$

Next let's consider a slightly more complicated example. In the present experiment  $R_4$  in the Wheatstone Bridge consists of  $R_E$  (0.5% accuracy) in parallel with  $R_H$  (10% accuracy), so that:

$$R_4 = \frac{R_E R_H}{R_E + R_H}.$$

Applying the formula for  $\Delta Q$  to  $R_4$ , it is easy to show that:

$$\left( \frac{\Delta R_4}{R_4} \right) = \left[ \left( \frac{R_4}{R_E} \right)^2 \left( \frac{\Delta R_E}{R_E} \right)^2 + \left( \frac{R_4}{R_H} \right)^2 \left( \frac{\Delta R_H}{R_H} \right)^2 \right]^{1/2}.$$

Now typically  $R_E = 1 \text{ k}\Omega$  and  $R_H = 200 \text{ k}\Omega$ . Note that although the fractional error in  $R_H$  is large, its error doesn't contribute much to the total error in  $R_4$  since it is multiplied by the square of a factor ( $R_4/R_H \approx 1/200$ ) that is small compared to 1.