Physics 202 Midterm Exam 1
September 29th 2008

Name: ........................................... ID#: ...........................................

Section: .................................

TA (please circle):

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Instructions:

1. Don’t forget to write down your name and section number.

2. Show your work. A reasonable amount of work is required to receive full credit.

3. Be aware that intermediate steps earn points even if the final answer is incorrect.

4. Erase (or cross out) any mistakes or you will be marked down. Grading is based on everything you have written down.

5. Both the magnitude and direction of vector quantities need to be specified for full credit.

Fundamental constants:

\[ \varepsilon_0 = (4\pi k_c)^{-1} = 8.85 \times 10^{-12} \text{C}^2/(\text{N} \cdot \text{m}^2) \quad \mu_0 = 4\pi \times 10^{-7} \text{T} \cdot \text{m} / \text{A} \quad c = 3 \times 10^8 \text{m/s} \]

\[ m_p = 1.67 \times 10^{-27} \text{kg} \quad m_e = 9.11 \times 10^{-31} \text{kg} \quad q_p = -q_e = 1.6 \times 10^{-19} \text{C} \]

Scores:

Problem 1  Problem 2  Problem 3  Problem 4
Problem 1 (25 Points)
For the following distribution of charges, with \( Q = 5 \times 10^{-6} \text{ C} \) and \( a = 10 \text{ cm} \) compute:

\[
\begin{align*}
\phi_1 &= 225^\circ \\
\phi_2 &= 90^\circ \\
\phi_3 &= -45^\circ \\
E_1 &= E_3 = \frac{kQ}{(\sqrt{2}a)^2} = \frac{kQ}{2a^2} \\
E_2 &= \frac{2kQ}{a^2} \\
\end{align*}
\]

a) The vector of the electric field \( \vec{E} \) at point P. (remember to compute both magnitude and direction)

\[
\begin{align*}
\vec{E} &= \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \\
E_x &= E_{1x} + E_{2x} + E_{3x} \\
E_y &= E_{1y} + E_{2y} + E_{3y} \\
E_x &= E_1 \cos 225^\circ + E_2 \cos 90^\circ + E_3 \cos (-45^\circ) = 0 \\
E_y &= E_1 \sin 225^\circ + E_2 \sin 90^\circ + E_3 \sin (-45^\circ) = -E_1 \frac{\sqrt{2}}{2} + E_2 - E_3 \frac{\sqrt{2}}{2} = \\
&= \frac{kQ}{a^2} \left( 2 - \frac{\sqrt{2}}{2} \right) > 0 \\
E_y &= 5.8 \times 10^6 \text{ V/m} \\
\vec{E} &= (0, E_y)
\end{align*}
\]

b) The vector of the force exerted on a charge \( Q' = 2 \times 10^{-6} \text{ C} \) at point P. (remember to compute both magnitude and direction)

\[
\vec{F} = Q' \vec{E} = (0, \frac{kQ'Q}{a^2} \left( 2 - \frac{\sqrt{2}}{2} \right)) = (0, 11.6 \text{ N})
\]
c) The potential $V$ at point P (assume $V=0$ at infinity).

$$V_p = V_1 + V_2 + V_3 = -\frac{kQ}{12a} + \frac{2kQ}{a} - \frac{kQ}{12a} = kQ \left( \frac{1}{2} - T_2 \right) = 2.64 \times 10^5 \text{ V}$$

d) The energy required to bring a charge $Q'=2 \times 10^{-6} \text{ C}$ from infinity to point P.

$$U = Q'V_p = 2 \times 10^{-6} \times 2.65 \times 10^5 = 0.527 \text{ J}$$
Problem 2 (30 Points)
Consider two coaxial cylindrical conductors (see figures), where \( R_a = 5\,cm \), \( R_b = 2\,cm \) and \( l = 1m \) with a linear charge density \( \lambda = 10^{-6}\,C/m \) on both cylinders (see graph). Ignore edge effects and the thickness of the outer cylinder.

(a) What are the three electric fields inside the inner cylinder, in between the two cylinders and outside the outer cylinder? Sketch the electric field as a function of the radius.

1) \( E = 0 \) for \( r < R_b \) (\( q_{\text{ind}} = 0 \))

2) \( \oint E \, dA = q_{\text{ind}} \, \pi l \, R_b \)
   \[ E = \frac{q_{\text{ind}}}{\pi l R_b} = \frac{\lambda \ell}{2 \pi \varepsilon_0} \]
   \[ E = \frac{1.8 \times 10^{-5}}{2 \pi \varepsilon_0} \]

3) \( E = E \hat{r} \), where \( \hat{r} \) is a unit vector along the radius.

4) \( E = 0 \) for \( r > R \) (\( q_{\text{ind}} = 0 \))
b) What is the difference of potential between the two cylinders?

\[ V_f - V_i = - \int E \, dl \quad \text{if} \quad \Delta V = \int_{R_b}^{R_a} E \, dr = - \int_{R_b}^{R_a} \frac{\lambda}{2\pi \varepsilon_0} \, \frac{dl}{r} = - \frac{\lambda}{2\pi \varepsilon_0} \ln \left( \frac{R_a}{R_b} \right) \]

\[ \Delta V = -1.6 \cdot 10^5 \, V \]

c) Compute the capacitance of the system

\[ C = \frac{Q}{\Delta V} = \frac{\lambda \ell}{2\pi \varepsilon_0 \ln \left( \frac{R_a}{R_b} \right)} = \frac{2\pi \varepsilon_0 \ell}{\ln \left( \frac{R_a}{R_b} \right)} \]

\[ C = 6.2 \cdot 10^{-11} \, F \]
Problem 3 (20 points)
An electron moves along the x axis with a velocity $v=100$ m/s. It enters an electric field, $\vec{E} = -E\vec{y}$, where $\vec{y}$ is the unitary vector of the y axis and $E=5\times 10^{-10}$ V/m (see graph). Ignore the edge effects.

After traversing a distance $L=50$ cm along the x axis.

a) What is the vector of velocity? (remember to compute both magnitude and direction)

\[
\vec{V}(V_x, V_y) ; \quad V_x = V_{ox} + a_x t ; \quad V_y = V_{oy} + a_y t
\]

\[
a_y = \frac{eE}{m} = -\frac{eE}{m} > 0 ; \quad V_y = -\frac{eE}{m} t = -\frac{eE}{m} \frac{L}{V_{ox}} > 0
\]

\[t = 0.005 \text{s} \quad \therefore a = 87.81 \text{ m/s}^2 \quad \therefore V_y = 4.39 \text{ m/s}
\]

b) What is the kinetic energy of the electron when leaving the field?

\[
k = \frac{1}{2} m (V_{ox}^2 + \left(\frac{eE}{m} \frac{L}{V_{ox}}\right)^2) = 4.56 \times 10^{-27} \text{ J}
\]

c) What is the displacement along the y axis?

\[
y = \frac{1}{2} a_y t^2 = -\frac{1}{2} \frac{eE}{m} \left(\frac{L}{V_{ox}}\right)^2 > 0
\]

\[y = 0.0011 \text{ m}
\]
Problem 4 (25 points)
Consider the following circuit, where $C=10^{-9}$ F (evaluated for permittivity in vacuum) and $V=5$ Volts

![Circuit Diagram]

a) Compute the effective capacitance of the circuit

$$\frac{1}{C_1} = \frac{1}{C} + \frac{1}{2C} \Rightarrow C_1 = \frac{2C}{3}$$

$$C_2 = C_1 + \frac{C}{3} \Rightarrow C_2 = C$$

$$C_k = k \frac{C}{2} = \frac{3C}{2}$$

$$\frac{1}{C_3} = \frac{1}{C_2} + \frac{1}{C_k} = \frac{1}{C} + \frac{2}{3C} = \frac{5}{3C}$$

$$C_3 = 6 \cdot 10^{-10} F$$

b) Compute the energy stored in the capacitor with the dielectric

$$V = \frac{1}{2} \frac{Q^2}{C_k}$$

$$Q = C_3 \ V = \frac{3C}{5} \cdot V = 3 \cdot 10^{-9} C$$

$$V = \frac{1}{2} \frac{Q^2}{k \frac{C}{2}} = \frac{1}{2} \frac{2}{3C} Q^2 = 3 \cdot 10^{-9} J$$