Today’s Topics

- Inductance (Ch 30 1-4)
- Reminder of Faraday’s and Lenz’s Laws
- Mutual Inductance
- Self Inductance (Inductor L)

- Energy Stored in magnetic Field

- LR Circuit
Review: Who is Prof. Yibin Pan?

- It’s me. (whoever stands here at this moment!)
- Research field: High Energy experiments.
  (Heard of the “Big Bang” machine, LHC, in Geneva?)
- Coordinates:
  - Office: 4283 Chamberlin
  - Email: pan@hep.wisc.edu
- Office hours: any time I am in. (and I shall be in often)

- Priority passes (optional):
  - Appointments before visits
  - “202” somewhere in email subject line.
Review: Faraday’s Law of Induction

- Faraday’s Law in plain words: When the magnetic flux through an area is changed, an emf is produced along the closed path enclosing the area.

Quantitatively:

\[ \mathcal{E} = - \frac{d\Phi_B}{dt} \]

Note the - sign
Review: Lenz’s Law

Lenz’s law in plain words: the induced emf always tends to work against the original cause of flux change.

<table>
<thead>
<tr>
<th>Cause of $\frac{d\Phi_B}{dt}$</th>
<th>“Current” due to Induced $\varepsilon$ will:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increasing $B$</td>
<td>generate $B$ in opposite dir.</td>
</tr>
<tr>
<td>Decreasing $B$</td>
<td>generate $B$ in same dir.</td>
</tr>
<tr>
<td>Relative motion</td>
<td>subject to a force in opposite direction of relative motions</td>
</tr>
</tbody>
</table>

Note: “Current” may not actually produced if no circuit.
Examples of Lenz’s Law
Magnetically Coupled Coils (Transformers)

Question: Why use iron core?
Mutual Inductance

- For coupled coils:
  \[ \varepsilon_2 = - M_{12} \frac{dI_1}{dt} \]
  \[ \varepsilon_1 = - M_{21} \frac{dI_2}{dt} \]

Can prove (not here):
\[ M_{12} = M_{21} = M \]

\( M \): mutual inductance
(unit: Henry)

\[ \varepsilon_2 = - M \frac{dI_1}{dt} \]
\[ \varepsilon_1 = - M \frac{dI_2}{dt} \]
Self Inductance

- When the current in a conducting device changes, an induced emf is produced in the opposite direction of the source current. → self inductance.

\[ \Phi_B = LI \]

- The magnetic flux due to self inductance is proportional to \( I \): \( \Phi_B = LI \)

- The induced emf is proportional to \( \frac{dI}{dt} \):

\[ \varepsilon_L = -L \frac{dI}{dt} \]

\( L \): Inductance, unit: Henry (H)
Exercise: Calculate Inductance of a Solenoid

(Text example 30-3)
show that for an ideal solenoid:

\[ L = \frac{\mu_0 N^2 A}{\ell} \]

(see board)

Area: \( A \)

\# of turns: \( N \)

\[ B = \mu_0 n I \quad \text{(ideal case)} \]
Inductors

- Inductance is intrinsic to a conductor circuit.
- Two factors that determine the inductance:
  - Geometric configuration of the circuit
  - Filling of magnetic material.
- Specifically configured inductance devices (inductors) are very useful in electronic and electrical applications:
Energy in an Inductor

- When an inductor of inductance $L$ is carrying a current changing at a rate $dI/dt$, the power supplied is

$$P = I\varepsilon = LI \frac{dI}{dt}$$

- The work needed to increase the current in an inductor from zero to some value $I$

$$W = \int dW = \int_0^I LI \, dI = \frac{1}{2} LI^2$$
Energy in a Magnetic Field

- \( U = \frac{1}{2} LI^2 \)
  - Solenoid: \( B = \mu_0 \frac{N}{l} I \) and \( L = \mu_0 N^2 A/l \)
  - \( U = \frac{1}{2} B^2/\mu_0 (A\ell) \)

- The energy is in the form of B field:
  - energy density: \( u_B = \frac{1}{2} B^2/\mu_0 \)
  - (recall: \( u_E = \frac{1}{2} \varepsilon_0 E^2 \))

- Compare:
  - Inductor: energy stored \( U = \frac{1}{2} LI^2 \) \( \Rightarrow \) \( \frac{1}{2} B^2/\mu_0 \)
  - Capacitor: energy stored \( U = \frac{1}{2} C(\Delta V)^2 \) \( \Rightarrow \) \( \frac{1}{2} \varepsilon_0 E^2 \)
  - Resistor: no energy stored, (all energy converted to heat)
## Basic Circuit Components

<table>
<thead>
<tr>
<th>Component</th>
<th>Symbol</th>
<th>Behavior in circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal battery, emf</td>
<td>![Symbol]</td>
<td>$\Delta V = V_+ - V_- = \varepsilon$</td>
</tr>
<tr>
<td>Resistor</td>
<td>![Symbol]</td>
<td>$\Delta V = -IR$</td>
</tr>
<tr>
<td>Realistic Battery</td>
<td>![Symbol]</td>
<td>$\Delta V = 0 \ (\Rightarrow R=0, L=0, C=0)$</td>
</tr>
<tr>
<td>(Ideal) wire</td>
<td>![Symbol]</td>
<td>$\Delta V = 0 \ (\Rightarrow R=0, L=0, C=0)$</td>
</tr>
<tr>
<td>Capacitor</td>
<td>![Symbol]</td>
<td>$\Delta V = V_- - V_+ = -\frac{q}{C}, \frac{dq}{dt} = I$</td>
</tr>
<tr>
<td>Inductor</td>
<td>![Symbol]</td>
<td>$\Delta V = -L\frac{dI}{dt}$</td>
</tr>
<tr>
<td>(Ideal) Switch</td>
<td>![Symbol]</td>
<td>$L=0, C=0, R=0 \ (on), R=\infty \ (off)$</td>
</tr>
<tr>
<td>Transformer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diodes, Transistors,...</td>
<td></td>
<td>Future Topics</td>
</tr>
</tbody>
</table>
LR Circuit

- Current as a function of time after switching on: \( I(t) \)

\[
I(t) = \frac{\varepsilon}{R} \left( 1 - e^{-\frac{t}{L/R}} \right)
\]

- Switching on at \( t=0 \)

- \( \varepsilon \) is the voltage source
- \( R \) is the resistor
- \( L \) is the inductor

- \( \tau = \frac{L}{R} \)

Note: the time constant is \( \tau = \frac{L}{R} \)

Quiz: What is the current when \( t=\infty \)?

Homework: “Switching off”
Turn on LR Circuit: Algebra Details

Apply Kirchhoff loop rule

\[ V_0 dt - IR dt - LdI = 0 \]

\[ \frac{dI}{V_0 - IR} = \frac{dt}{L} \]

\[ \int_0^I \frac{dI}{V_0 - IR} = \int_0^t \frac{dt}{L} \]

\[ -\frac{1}{R} \ln \left( \frac{V_0 - IR}{V_0} \right) = \frac{t}{L} \]

\[ I = \frac{V_0}{R} \left(1 - e^{-\frac{t}{L/R}}\right) \]
LR Circuit: Time Constant

turning on

\[ I = \frac{V_0}{R} \left( 1 - e^{-\frac{t}{L/R}} \right) \]

\[ I_{\text{max}} = \frac{V_0}{R} \]

\[ 0.63 I_{\text{max}} \]

\[ t = \frac{L}{R} \text{ Time} \]

turning off

\[ I = I_0 e^{-\frac{t}{L/R}} \]

\[ 0.37 I_0 \]

Time Constant of the LR circuit: \( \tau = \frac{L}{R} \)

Quiz: What is \( \tau \) for RC circuit?