Today’s Topics

- Wave Motion
  - General Wave
    - Wave Function, wave speed.
    - Transverse And Longitudinal Waves
  - Sinusoidal Wave
  - Wave dynamics
    - Wave speed on string
    - Reflection and Transmission
    - Energy Transfer
General Waves

- **Wave:**
  Propagation of a physical quantity in space over time
  \[ q = q(x, t) \]

- **Examples of waves:**
  Water wave, wave on string, sound wave, earthquake wave, electromagnetic wave, “light”, quantum wave.

- **Mechanic wave:**
  Propagation of small motion (“disturbance”) in a medium.
  ➔ Physical quantity to be propagated: displacement.

Recall: Displacement is a vector.
Transverse and Longitudinal Waves

- If the direction of mechanic disturbance (displacement) is perpendicular to the direction of wave motion, the wave is called transverse wave.

- If the direction of mechanic disturbance (displacement) is parallel to the direction of wave motion, the wave is called longitudinal wave.

→ see demos.

- In general, a wave can be a combination of the above modes.

- The definition can be extended to other (non-mechanic) waves.
  - e.g Electromagnetic waves are always transverse.
Waves are described by wave functions in the form:

\[ y(x, t) = f(x - vt) \]

- **y**: A certain physical quantity, e.g., displacement in the y direction
- **f**: Can any forms
- **x**: Space position. Coefficient arranged to be 1
- **t**: Time. Its coefficient \( v \) is the wave speed. \( v > 0 \) moving right, \( v < 0 \) moving left
An Exercise to Explain Wave Speed

- A wave is described by function $y=f(x-vt)$.
  - At time $t_1$ in position $x_1$, how large is the quantity $y$?
    - $y= f(x_1-vt_1) = y_0$
  - At a later time $t_2=t_1+\Delta t$, what is $y$ in position $x_2=x_1+v\Delta t$?
    - $y= f(x_2-vt_2) = f(x_1+v\Delta t -v(t_1+\Delta t))= f(x_1-vt_1) = y_0$

- How to interpret the result?
  - Between $t_1$ and $t_1+\Delta t$, the value $y_0$ has transmitted from position $x_1$ to $x_1+v\Delta t$
  - Speed = $\frac{(x_1+v\Delta t - x_1 )}{(t_1+\Delta t - t_1 )} = v$
  - i.e $v>0 \iff$ moving right; $v<0 \iff$ moving left;
Practical Technique:
Identify Wave Speed in A Wave Function

- A wave function is in the form:
  \[ y(x,t) = \frac{2}{(x - 3.0t)^2 + 1} \]

- What is the wave speed:
  - Answer: 3.0 m/s to the right

- Illustrate wave form at t = 0s,1s,2s
Identify Wave Speed in A Wave Function: Another Example

- A wave function is in the form:
  \[ y(x, t) = 5.0e^{(1.8x+7.2t)^2} \]

- What is the wave speed:
  - Answer: - 4.0 m/s (to the left)

Why?

\[
y(x, t) = 5.0e^{(1.8x+7.2t)^2} = 5.0e^{1.8^2(x+\frac{7.2}{1.8}t)^2} = 5.0e^{1.8^2(x+4.0t)^2} = 5.0e^{1.8^2(x-(-4.0)t)^2}
\]
Wave On A Stretched Rope

- It is a transverse wave
  - See demos.
- The wave speed is determined by the tension and the linear density of the rope:

\[ v = \sqrt{\frac{T}{\mu}} \quad ; \quad \mu \equiv \frac{\Delta m}{\Delta l} \]
Reflection and Transmission of Waves

- **Reflection**: Waves can be reflected back at the boundary of the medium. Some time the reflected wave are inverted. See demo.

- **Transmission**: Waves can also transmit over the boundary into another medium. The transmitted waves are never inverted. See demo.
Sinusoidal Wave

- A wave describe a function \( y = A \sin(kx - \omega t + \phi) \) is called sinusoidal wave. (Harmonic wave)
- Quick Quiz: can you see that its wave speed \( v = \frac{\omega}{k} \)?
- Quick Quiz: At each fixed position \( x \), the element is undergoing a harmonic oscillation. Can you see it? What are the amplitude, frequency and phase constant of this oscillation? \( (|A|, \omega, -kx-\phi) \)
- If we fix \( t \) and take a snap shot, the wave form is periodic over space (\( x \)). The spacing period \( \lambda \) is called wavelength and \( \lambda = 2\pi/k \) (see next page)
Parameters For A Sinusoidal Wave

- **Snapshot with fixed t:**
  - wave length $\lambda = \frac{2\pi}{k}$
- **Snapshot with fixed x:**
  - angular frequency $\omega$
  - frequency $f = \frac{\omega}{2\pi}$
  - Period $T = \frac{1}{f}$
  - Amplitude $= A$
- **Wave Speed** $v = \frac{\omega}{k}$
  - $v = \lambda f$, or
  - $v = \frac{\lambda}{T}$
- **Phase angle difference between two positions**
  - $\Delta \phi = -k \Delta x$
Waves Transfer Energy

- As motion in propagating in the form of wave in a medium, energy is transmitted.

- It can be shown easily that the rate of energy transfer by a sinusoidal wave on a rope is:

  \[ P = \frac{1}{2} \mu A^2 \omega^2 v \]

- It is important to be familiar to this power dependence on \( A, \omega, v \)
Linear Wave Equation

- Wave function are solutions to linear wave equation.

\[ \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \]

- While the exact form of the solution also depends on initial conditions, wave speed is determined directly by the wave equation.
Forced (driven) Oscillation

- If in addition there is a driving force with its own frequency $\omega$: $F_0 \cos(\omega t)$, the equation becomes:

$$m \frac{d^2x}{dt^2} = -kx - b \frac{dx}{dt} + F_0 \cos(\omega t)$$

- This equation can be solved analytically.
  - At large $t$, the solution is:

$$x = A \cos(\omega t + \phi)$$

  - with

$$A = \frac{F_0 / m}{\sqrt{(\omega - \omega_0)^2 + b^2 / 2m^2}}$$

- At large $t$, the frequency is determined by driving $\omega$
- When $\omega = \omega_0$, amplitude is maximum $\rightarrow$ resonance
Resonance Amplitude

\[ A = \frac{F_0 / m}{\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + \left(\frac{b}{2m}\right)^2}} \]