Physics 207, Lecture 4, Sept. 15

- Goals for Chaps. 3 & 4
  - Perform vector algebra (addition & subtraction) graphically or by $x,y \text{ & } z$ components
  - Interconvert between Cartesian and Polar coordinates
  - Distinguish position-time graphs from particle trajectory plots
  - Obtain particle velocities and accelerations from trajectory plots
    - Determine both magnitude and acceleration parallel and perpendicular to the trajectory path
  - Solve problems with multiple accelerations in 2-dimensions (including linear, projectile and circular motion)
  - Discern different reference frames and understand how they relate to particle motion in stationary and moving frames

Assignment: Read Chapter 5 (sections 1-4 carefully)

- MP Problem Set 2 due Wednesday (should have started)
- MP Problem Set 3, Chapters 4 and 5 (available today)
Vector addition

• The sum of two vectors is another vector.

\[ \mathbf{A} = \mathbf{B} + \mathbf{C} \]

Vector subtraction

• Vector subtraction can be defined in terms of addition.

\[ \mathbf{B} - \mathbf{C} = \mathbf{B} + (-1)\mathbf{C} \]
Unit Vectors

- A **Unit Vector** is a vector having length 1 and no units.
- It is used to specify a direction.
- Unit vector \( \mathbf{u} \) points in the direction of \( \mathbf{U} \):
  - Often denoted with a “hat”: \( \mathbf{u} = \hat{\mathbf{u}} \)

- Useful examples are the cartesian unit vectors \([ \mathbf{i}, \mathbf{j}, \mathbf{k} ]\):
  - Point in the direction of the \( x, y \) and \( z \) axes.
  - \( \mathbf{R} = r_x \mathbf{i} + r_y \mathbf{j} + r_z \mathbf{k} \)

Vector addition using components:

- Consider, in 2D, \( \mathbf{C} = \mathbf{A} + \mathbf{B} \).
- \( \mathbf{A} = (A_x \mathbf{i} + A_y \mathbf{j}) \) and \( \mathbf{B} = (B_x \mathbf{i} + B_y \mathbf{j}) \) then \( \mathbf{C} = (A_x + B_x) \mathbf{i} + (A_y + B_y) \mathbf{j} \)

- Comparing components of (a) and (b):
  - \( C_x = A_x + B_x \)
  - \( C_y = A_y + B_y \)
  - \( |\mathbf{C}| = \sqrt{(C_x)^2 + (C_y)^2} \)
**Exercise 1**

**Vector Addition**

- Vector \( \mathbf{A} = \{0,2,1\} \)
- Vector \( \mathbf{B} = \{3,0,2\} \)
- Vector \( \mathbf{C} = \{1,-4,2\} \)

What is the resultant vector, \( \mathbf{D} \), from adding \( \mathbf{A} + \mathbf{B} + \mathbf{C} \)?

A. \( \{3,-4,2\} \)
B. \( \{4,-2,5\} \)
C. \( \{5,-2,4\} \)
D. None of the above

---

**Converting Coordinate Systems**

- In **polar** coordinates the vector \( \mathbf{R} = (r, \theta) \)
- In Cartesian the vector \( \mathbf{R} = (r_x, r_y) = (x, y) \)
- We can convert between the two as follows:

\[
\begin{align*}
        r_x &= x = r \cos \theta \\
        r_y &= y = r \cos \theta \\
     R &= x \hat{i} + y \hat{j} \\
     r &= \sqrt{x^2 + y^2} \\
\theta &= \tan^{-1} \left( \frac{y}{x} \right)
\end{align*}
\]

- In 3D cylindrical coordinates \( (r, \theta, z) \), \( r \) is the same as the magnitude of the vector in the \( x-y \) plane [\( \sqrt{x^2 + y^2} \)]
Exercise: Frictionless inclined plane

- A block of mass $m$ slides down a frictionless ramp that makes angle $\theta$ with respect to horizontal. What is its acceleration $a$?

Resolving vectors, little $g$ & the inclined plane

- $g$ (bold face, vector) can be resolved into its $x,y$ or $x',y'$ components
  - $g = -g \hat{j}$
  - $g = -g \cos \theta \hat{j'} + g \sin \theta \hat{i'}$

- The bigger the tilt the faster the acceleration..... along the incline
Dynamics II: Motion along a line but with a twist
(2D dimensional motion, magnitude and directions)

- Particle motions involve a path or trajectory
- Recall instantaneous velocity and acceleration

\[
\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt} \\
\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}
\]

- These are vector expressions reflecting x, y & z motion

\[
\mathbf{r} = \mathbf{r}(t) \quad \mathbf{v} = \frac{d\mathbf{r}}{dt} \quad \mathbf{a} = \frac{d^2\mathbf{r}}{dt^2}
\]

Instantaneous Velocity

- But how we think about requires knowledge of the path.
- The direction of the instantaneous velocity is along a line that is tangent to the path of the particle’s direction of motion.

\[
\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt}
\]

- The magnitude of the instantaneous velocity vector is the speed, s.
  (Knight uses \(v\))
  \[
s = (v_x^2 + v_y^2 + v_z^2)^{1/2}
\]
Average Acceleration

- The average acceleration of particle motion reflects changes in the instantaneous velocity vector (divided by the time interval during which that change occurs).

\[ \bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t} \]

- The average acceleration is a vector quantity directed along \( \Delta v \) (a vector!)

Instantaneous Acceleration

- The instantaneous acceleration is the limit of the average acceleration as \( \Delta v/\Delta t \) approaches zero

\[ a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \]

- The instantaneous acceleration is a vector with components parallel (tangential) and/or perpendicular (radial) to the tangent of the path

- Changes in a particle’s path may produce an acceleration
  - The magnitude of the velocity vector may change
  - The direction of the velocity vector may change
    (Even if the magnitude remains constant)
  - Both may change simultaneously (depends: path vs time)
Motion along a path
(displacement, velocity, acceleration)

- 2 or 3D Kinematics: vector equations:
  \[ \vec{r} = \vec{r}(\Delta t) \]
  \[ \vec{v} = d\vec{r} / dt \]
  \[ \vec{a} = d^2\vec{r} / dt^2 \]

Velocity:
\[ \vec{v}_{av} = \Delta\vec{r} / \Delta t \]
\[ \vec{v} = d\vec{r} / dt \]

Acceleration:
\[ \vec{a}_{av} = \Delta\vec{v} / \Delta t \]
\[ \vec{a} = d\vec{v} / dt \]

Generalized motion with non-zero acceleration:
\[ \vec{a}_t \equiv \vec{a}_\parallel \]
\[ \vec{a}_r \equiv \vec{a}_\perp \]
\[ \vec{a} \neq 0 \text{ with } |\vec{a}| = \sqrt{a_\parallel^2 + a_\perp^2} \]

need both path & time
\[ \vec{a} = \vec{a}_\parallel + \vec{a}_\perp \]

Two possible options:
- Change in the magnitude of \( \vec{v} \) \( \vec{a}_\parallel \neq 0 \)
- Change in the direction of \( \vec{v} \) \( \vec{a}_\perp \neq 0 \)

Animation
Q1: What is the time sequence in this particle’s motion?
Q2: Is the particle speeding up or slowing down?
Examples of motion: Chemotaxis

**An example…..**

- Q1: How do single cell animals locate their “food”?
  - Possible mechanism: Tumble and run paradigm.

- Q2: How does a fly find its meal?
  - Possible mechanism: If you smell it fly into the wind, if you don’t fly across the wind.

---

**Kinematics**

- The position, velocity, and acceleration of a particle in 3-dimensions can be expressed as:

  \[
  \begin{align*}
  \mathbf{r} &= x \mathbf{i} + y \mathbf{j} + z \mathbf{k} \\
  \mathbf{v} &= v_x \mathbf{i} + v_y \mathbf{j} + v_z \mathbf{k} \\
  \mathbf{a} &= a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathbf{k}
  \end{align*}
  \]

  \[
  (i, j, k \text{ unit vectors})
  \]

  \[
  \begin{align*}
  x &= x(\Delta t) \\
  y &= y(\Delta t) \\
  z &= z(\Delta t)
  \end{align*}
  \]

  \[
  \begin{align*}
  v_x &= \frac{dx}{dt} \\
  v_y &= \frac{dy}{dt} \\
  v_z &= \frac{dz}{dt}
  \end{align*}
  \]

  \[
  \begin{align*}
  a_x &= \frac{d^2x}{dt^2} \\
  a_y &= \frac{d^2y}{dt^2} \\
  a_z &= \frac{d^2z}{dt^2}
  \end{align*}
  \]

- All this complexity is hidden away in

  \[
  \begin{align*}
  \mathbf{r} &= \mathbf{r}(\Delta t) \\
  \mathbf{v} &= \frac{d\mathbf{r}}{dt} \\
  \mathbf{a} &= \frac{d^2\mathbf{r}}{dt^2}
  \end{align*}
  \]

with, if constant accel., e.g. \( x(\Delta t) = x_0 + v_0 \Delta t + \frac{1}{2} a_x \Delta t^2 \)
**Special Case**

Throwing an object with $x$ along the horizontal and $y$ along the vertical.

$x$ and $y$ motion both coexist and $t$ is common to both

Let $g$ act in the $-y$ direction, $v_{0x} = v_0$ and $v_{0y} = 0$

---

**Another trajectory**

Can you identify the dynamics in this picture?

How many distinct regimes are there?

Are $v_x$ or $v_y = 0$? Is $v_x >, <$ or $= v_y$?
Another trajectory
Can you identify the dynamics in this picture?

How many distinct regimes are there?

\[ 0 < t < 3 \quad 3 < t < 7 \quad 7 < t < 10 \]

- I. \( v_x = \text{constant} = v_0 \); \( v_y = 0 \)
- II. \( v_x = v_y = v_0 \)
- III. \( v_x = 0 \); \( v_y = \text{constant} < v_0 \)

What can you say about the acceleration?

Exercise 2 & 3
Trajectories with acceleration

- A rocket is drifting sideways (from left to right) in deep space, with its engine off, from A to B. It is not near any stars or planets or other outside forces.
- Its “constant thrust” engine (i.e., acceleration is constant) is fired at point B and left on for 2 seconds in which time the rocket travels from point B to some point C
  - Sketch the shape of the path from B to C.
- At point C the engine is turned off.
  - Sketch the shape of the path after point C
Exercise 2
Trajectories with acceleration

From B to C?

A. A
B. B
C. C
D. D
E. None of these

Exercise 3
Trajectories with acceleration

After C?

A. A
B. B
C. C
D. D
E. None of these
Trajectory with constant acceleration along the vertical

How does the trajectory appear to different observers?
What if the observer is moving with the same x velocity (i.e. running in parallel)?

This observer will only see the y motion

In an inertial reference frame all see the same acceleration
**Trajectory with constant acceleration along the vertical**

What do the velocity and acceleration vectors look like?

Velocity vector is always tangent to the curve!

Acceleration may or may not be!

---

**Home Exercise**

**The Pendulum**

Which statement best describes the motion of the pendulum bob at the instant of time drawn?

i. when the bob is at the top of its swing.

ii. which quantities are non-zero?

A) \( v_r = 0 \quad a_r = 0 \quad v_T = 0 \quad a_T \neq 0 \)

B) \( v_r = 0 \quad a_r \neq 0 \quad v_T = 0 \quad a_T = 0 \)

C) \( v_r = 0 \quad a_r \neq 0 \quad v_T = 0 \quad a_T \neq 0 \)
Home Exercise
The Pendulum Solution

\[ \theta = 30^\circ \]

NOT uniform circular motion:
If circular motion then \( a_r \) not zero,
If speed is increasing so \( a_T \) not zero

However, at the top of the swing the bob temporarily comes to rest, so \( v = 0 \) and the net tangential force is \( mg \sin \theta \)

C) \( v_r = 0 \quad a_r = 0 \)
\[ v_T = 0 \ a_T \neq 0 \]

Everywhere else the bob has a non-zero velocity and so then
(except at the bottom of the swing)

\( v_r = 0 \ a_r \neq 0 \)
\( v_T \neq 0 \ a_T \neq 0 \)

Assignment: Read Chapter 5 (sections 1-4 carefully)

• MP Problem Set 2 due Wednesday (should have started)
• MP Problem Set 3, Chapters 4 and 5 (available today)