Lecture 10

- Goals:
  - Employ Newton’s Laws in 2D problems with circular motion

Assignment: HW5, (Chapters 8 & 9, due 10/15, Wednesday)
For this Wednesday: Read Chapter 9

Uniform Circular Motion

For an object moving along a curved trajectory, with non-uniform speed
\( \mathbf{a} = \mathbf{a}_r \) (radial only)

\[ |\mathbf{a}_r| = \frac{v_r^2}{r} \]

Perspective is important
Non-uniform Circular Motion

For an object moving along a curved trajectory, with non-uniform speed

\[ \mathbf{a} = \mathbf{a}_r + \mathbf{a}_t \]  
(radial and tangential)

\[ |\mathbf{a}_r| = \frac{v_T^2}{r} \]

\[ |\mathbf{a}_T| = \frac{d|\mathbf{v}|}{dt} \]

Circular motion

- Circular motion implies one thing

\[ |a_{\text{radial}}| = \frac{v_T^2}{r} \]
Key steps

- Identify forces (i.e., a FBD)
- Identify axis of rotation
- Apply conditions (position, velocity, acceleration)

Example
The pendulum

Consider a person on a swing:

When is the tension on the rope largest?
And at that point is it:
  (A) greater than
  (B) the same as
  (C) less than
the force due to gravity acting on the person?
Example
Gravity, Normal Forces etc.

at top of swing $v_T = 0$

$F_r = m \frac{v_T^2}{r} = 0 = T - mg \cos \theta$

$T = mg \cos \theta$

$T < mg$

at bottom of swing $v_T$ is max

$F_r = ma_c = m \frac{v_T^2}{r} = T - mg$

$T = mg + m \frac{v_T^2}{r}$

$T > mg$

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Conical Pendulum (very different)

- Swinging a ball on a string of length $L$ around your head
  ($r = L \sin \theta$)

$\Sigma F_r = ma_r = T \sin \theta$

$\Sigma F_z = 0 = T \cos \theta - mg$

so

$T = mg / \cos \theta \ (> mg)$

$ma_r = mg \sin \theta / \cos \theta$

$a_r = g \tan \theta = \frac{v_T^2}{r} \rightarrow v_T = (gr \tan \theta)^{1/2}$

Period:

$t = 2\pi \frac{r}{v_T} = 2\pi \left( \frac{r \cot \theta}{g} \right)^{1/2}$
A match box car is going to do a loop-the-loop of radius $r$. What must be its minimum speed $v_t$ at the top so that it can manage the loop successfully?

To navigate the top of the circle its tangential velocity $v_T$ must be such that its centripetal acceleration at least equals the force due to gravity. At this point $N$, the normal force, goes to zero (just touching).

$$F_r = ma_r = mg = m\frac{v_T^2}{r}$$

$$v_T = (gr)^{1/2}$$
The match box car is going to do a loop-the-loop. If the speed at the bottom is $v_B$, what is the normal force, $N$, at that point?

**Hint:** The car is constrained to the track.

\[ F_r = ma_r = \frac{mv_B^2}{r} = N - mg \]

\[ N = \frac{mv_B^2}{r} + mg \]

Once again the car is going to execute a loop-the-loop. What must be its minimum speed at the bottom so that it can make the loop successfully?

This is a difficult problem to solve using just forces. We will skip it now and revisit it using energy considerations later on...
Example Problem

Swinging around a ball on a rope in a “nearly” horizontal circle over your head. Eventually the rope breaks. If the rope breaks at 64 N, the ball’s mass is 0.10 kg and the rope is 0.10 m. How fast is the ball going when the rope breaks? (neglect mg contribution, 1 N << 40 N)

\[ T = 40\ N \]

\[ F_r = m \frac{v_T^2}{r} \approx T \]

\[ v_T = \left( r \frac{F_r}{m} \right)^{1/2} \]

\[ v_T = (0.10 \times 64 / 0.10)^{1/2} \ m/s \]

\[ v_T = 8 \ m/s \]

Example, Circular Motion Forces with Friction

(recall \( ma_r = m \left| \frac{v_T}{r} \right|^2 / r \ F_f \leq \mu_s N \))

- How fast can the race car go?
  (How fast can it round a corner with this radius of curvature?)

\[ m_{car} = 1600 \ kg \]

\[ \mu_s = 0.5 \text{ for tire/road} \]

\[ r = 80 \ m \]

\[ g = 10 \ m/s^2 \]
Example

- Only one force is in the horizontal direction: static friction

$$x\text{-dir: } F_r = ma_r = -m |v_T|^2 / r = F_s = -\mu_s N \ (\text{at maximum})$$

$$y\text{-dir: } ma = 0 = N - mg \quad N = mg$$

$$v_T = (\mu_s \ m \ g \ r / m)^{1/2}$$

$$v_T = (\mu_s \ g \ r)^{1/2} = (0.5 \times 10 \times 80)^{1/2}$$

$$v_T = 20 \text{ m/s}$$

Another Example

- A horizontal disk is initially at rest and very slowly undergoes constant angular acceleration. A 2 kg puck is located a point 0.5 m away from the axis. At what angular velocity does it slip (assuming $a_T \ll a_T$ at that time) if $\mu_s=0.8$?

- Only one force is in the horizontal direction: static friction

$$x\text{-dir: } F_r = ma_r = -m |v_T|^2 / r = F_s = -\mu_s N \ (\text{at } \omega)$$

$$y\text{-dir: } ma = 0 = N - mg \quad N = mg$$

$$v_T = (\mu_s \ m \ g \ r / m)^{1/2}$$

$$v_T = (\mu_s \ g \ r)^{1/2} = (0.8 \times 10 \times 0.5)^{1/2}$$

$$v_T = 2 \text{ m/s} \rightarrow \omega = v_T / r = 4 \text{ rad/s}$$
Zero Gravity Ride

A rider in a “0 gravity ride” finds herself stuck with her back to the wall.

Which diagram correctly shows the forces acting on her?
Banked Curves

In the previous car scenario, we drew the following free body diagram for a race car going around a curve on a flat track.

What differs on a banked curve?

Banked Curves

Free Body Diagram for a banked curve.
Use rotated x-y coordinates
Resolve into components parallel and perpendicular to bank

For very small banking angles, one can approximate that $F_f$ is parallel to $ma_r$. This is equivalent to the small angle approximation $\sin \theta = \tan \theta$, but very effective at pushing the car toward the center of the curve!!
Banked Curves, Testing your understanding

Free Body Diagram for a banked curve.
Use rotated x-y coordinates
Resolve into components parallel and perpendicular to bank

At this moment you press the accelerator and, because of the frictional force (forward) by the tires on the road you accelerate in that direction.
How does the radial acceleration change?

Navigating a hill

Knight concept exercise: A car is rolling over the top of a hill at speed $v$.
At this instant,

A. $n > w$.
B. $n = w$.
C. $n < w$.
D. We can’t tell about $n$ without knowing $v$.

At what speed does the car lose contact?

This occurs when the normal force goes to zero or, equivalently, when all the weight is used to achieve circular motion.

$$F_c = mg = m \frac{v^2}{r} \Rightarrow v = \sqrt{gr} \frac{1}{2} \text{ (just like an object in orbit)}$$

Note this approach can also be used to estimate the maximum walking speed.
Exercise

When a pilot executes a loop-the-loop (figure on the right) the aircraft moves in a vertical circle of radius R=2.70 km at a constant speed of v=225 m/s. Is the force exerted by the seat on the pilot:

(A) Larger
(B) Same
(C) Smaller

than the pilot’s weight (mg) at

(I) the bottom and
(II) at the top of the loop?

Locomotion: how fast can a biped walk?
How fast can a biped walk?

What about weight?
(a) A heavier person of equal height and proportions can walk faster than a lighter person
(b) A lighter person of equal height and proportions can walk faster than a heavier person
(c) To first order, size doesn’t matter

What about height?
(a) A taller person of equal weight and proportions can walk faster than a shorter person
(b) A shorter person of equal weight and proportions can walk faster than a taller person
(c) To first order, height doesn’t matter
How fast can a biped walk?

What can we say about the walker's acceleration if there is UCM (a smooth walker)?

**Acceleration is radial!**

So where does it, $a_r$, come from?
(i.e., what external forces are on the walker?)

1. Weight of walker, downwards
2. Friction with the ground, sideways

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First exam (mean 71, high 100, low 20)