Physics 207, Lecture 11, Oct. 8

Goals:
- Chapter 9: Momentum & Impulse
  - Understand what momentum is and how it relates to forces
  - Employ momentum conservation principles
  - In problems with 1D and 2D Collisions
  - In problems having an impulse (Force vs. time)
- Chapter 8: Use models with free fall

Assignment:
- Monday, Read through Chapter 10 (1st half)
- MP HW5, due Wednesday 10/15

Locomotion: how fast can a biped walk?
How fast can a biped walk?

What about weight?

(a) A heavier person of equal height and proportions can walk faster than a lighter person

(b) A lighter person of equal height and proportions can walk faster than a heavier person

(c) To first order, size doesn’t matter
How fast can a biped walk?

What can we say about the walker’s acceleration if there is UCM (a smooth walker)?

**Acceleration is radial!**

So where does it, $a_r$, come from? (i.e., what external forces are on the walker?)

1. Weight of walker, downwards
2. Friction with the ground, sideways
How fast can a biped walk?
What can we say about the walker’s acceleration if there is UCM (a smooth walker)?

Acceleration is radial!
So where does it, \( a_r \), come from?
(i.e., what external forces are on the walker?)

1. Weight of walker, downwards
2. Friction with the ground, sideways
   (most likely from back foot)
3. Need a normal force as well
How fast can a biped walk?

Given a model then what does the physics say?

Choose a position with the simplest constraints.

If his radial acceleration is greater than $g$ then he is “in orbit”

$$F_r = m a_r = m \frac{v^2}{r} < mg$$

Otherwise you will lose contact!

$$a_r = \frac{v^2}{r} \rightarrow v_{\text{max}} = (gr)^{\frac{1}{2}}$$

$$v_{\text{max}} \approx 3 \text{ m/s}$$

(And it pays to be tall and live on Jupiter)

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Orbiting satellites $v_T = (gr)^{\frac{1}{2}}$
**Geostationary orbit**

- The radius of the Earth is \(~6000\) km, but at \(36000\) km you are \(~42000\) km from the center of the earth.

- \(F_{\text{gravity}}\) is proportional to \(r^2\) and so little \(g\) is now \(~10\) m/s\(^2\) / 50
  
  \[
  v_T = (0.20 \times 42000000)^{\frac{1}{2}} \text{ m/s} = 3000 \text{ m/s}
  \]

- At \(3000\) m/s, period \(T = \frac{2\pi r}{v_T} = \frac{2\pi 42000000}{3000} \text{ sec} = \frac{90000}{3600} \text{ s/hr} = 24 \text{ hrs}

- Orbit affected by the moon and also the Earth’s mass is inhomogeneous (not perfectly geostationary)

- Great for communication satellites
  
  (1\(^{\text{st}}\) pointed out by Arthur C. Clarke)
Impulse & Linear Momentum

- Transition from forces to conservation laws

Newton’s Laws → Conservation Laws
Conservation Laws → Newton’s Laws

They are different faces of the same physics

NOTE: We have studied “impulse” and “momentum” but we have not explicitly named them as such

Conservation of momentum is far more general than conservation of mechanical energy

Forces vs time (and space, Ch. 10)

- Underlying any “new” concept in Chapter 9 is
  1. A net force changes velocity (either magnitude or direction)
  2. For any action there is an equal and opposite reaction

- If we emphasize Newton’s 3\textsuperscript{rd} Law and look at the changes with time then this leads to the Conservation of Momentum Principle
Example 1

A 2 kg block, initially at rest on frictionless horizontal surface, is acted on by a 10 N horizontal force for 2 seconds (in 1D).

What is the final velocity?
- $F$ is to the positive & $F = ma$ thus $a = F/m = 5 \text{ m/s}^2$
- $v = v_0 + a \Delta t = 0 \text{ m/s} + 2 \times 5 \text{ m/s} = 10 \text{ m/s (+ direction)}$

Notice: $v - v_0 = a \Delta t \rightarrow m (v - v_0) = ma \Delta t \rightarrow m \Delta v = F \Delta t$

If the mass had been 4 kg … now what final velocity?

Twice the mass

- Same force
- Same time
- Half the acceleration ($a = F/m'$)
- Half the velocity! (5 m/s)
Example 1

- Notice that the final velocity in this case is inversely proportional to the mass (i.e., if thrice the mass...one-third the velocity).

- Here, mass times the velocity always gives the same value. (Always 20 kg m/s.)

\[
\text{Area under curve is still the same!}
\]

\[
\text{Force} \times \text{change in time} = \text{mass} \times \text{change in velocity}
\]

Example 1

- There many situations in which the sum of the products “mass times velocity” is constant over time
- To each product we assign the name, “momentum” and associate it with a conservation law.
  (Units: kg m/s or N s)
- A force applied for a certain period of time can be graphed and the area under the curve is the “impulse”

\[
\text{Area under curve: “impulse”}
\]

\[
\text{With: } m \Delta v = F_{\text{avg}} \Delta t
\]
Force curves are usually a bit different in the real world

Example 1 with Action-Reaction

- Now the 10 N force from before is applied by person A on person B while standing on a frictionless surface
- For the force of A on B there is an equal and opposite force of B on A

\[ M_A \times \Delta V_A = \text{Area of top curve} \]
\[ M_B \times \Delta V_B = \text{Area of bottom curve} \]
\[ \text{Area (top)} + \text{Area (bottom)} = 0 \]
Example 1 with Action-Reaction

\[ M_A \Delta V_A + M_B \Delta V_B = 0 \]

\[ M_A [V_A(\text{final}) - V_A(\text{initial})] + M_B [V_B(\text{final}) - V_B(\text{initial})] = 0 \]

Rearranging terms

\[ M_A V_A(\text{final}) + M_B V_B(\text{final}) = M_A V_A(\text{initial}) + M_B V_B(\text{initial}) \]

which is constant regardless of \( M \) or \( \Delta V \)

(Remember: frictionless surface)

Define \( MV \) to be the “momentum” and this is conserved in a system if and only if the system is \textbf{not} acted on by a \textbf{net} external force (choosing the system is key)

Conservation of momentum is a special case of applying Newton’s Laws
Applications of Momentum Conservation

Radioactive decay:

\[ ^{238}\text{U} \rightarrow ^{234}\text{Th} + ^{4}\text{He} \]

\[ \text{Alpha Decay} \quad v_2 \quad v_1 \]

Explosions

\[ v_H = 300 \text{ m/s} \]
\[ m_B = 5.00 \text{ g} \]

Collisions

Impulse & Linear Momentum

- **Definition:** For a single particle, the momentum \( p \) is defined as:

\[
\mathbf{p} \equiv m\mathbf{v}
\]

(p is a vector since \( \mathbf{v} \) is a vector)

So \( p_x = mv_x \) and so on (y and z directions)

- Newton’s 2\(^{nd}\) Law: \( \mathbf{F} = m\mathbf{a} \)

\[
= m \frac{d\mathbf{v}}{dt} = \frac{d}{dt}(m\mathbf{v}) \quad \Rightarrow \quad \mathbf{F} = \frac{d\mathbf{p}}{dt}
\]

- This is the most general statement of Newton’s 2\(^{nd}\) Law
Momentum Conservation

\[ F_{\text{ext}} = \frac{dP}{dt} \Rightarrow \frac{dP}{dt} = 0 \Rightarrow F_{\text{ext}} = 0 \]

- Momentum conservation (recasts Newton’s 2\textsuperscript{nd} Law when \( F = 0 \)) is an important principle

- It is a vector expression (\( P_x, P_y \) and \( P_z \)).

  \[ \checkmark \] And applies to any situation in which there is NO net external force applied (in terms of the x, y & z axes).

Momentum Conservation

- Many problems can be addressed through momentum conservation even if other physical quantities (e.g. mechanical energy) are not conserved

- Momentum is a vector quantity and we can independently assess its conservation in the x, y and z directions

  (e.g., net forces in the z direction do not affect the momentum of the x & y directions)
Exercise 1  
**Momentum is a Vector (!) quantity**

- A block slides down a frictionless ramp and then falls and lands in a cart which then rolls horizontally without friction
- **In regards to the** block landing in the cart is momentum conserved?

A. Yes  
B. No  
C. Yes & No  
D. Too little information given

Let a 2 kg block start at rest on a 30° incline and slide vertically a distance 5.0 m and fall a distance 7.5 m into the 10 kg cart.

What is the final velocity of the cart?
Exercise 1
Momentum is a Vector (!) quantity

- x-direction: No net force so \( P_x \) is conserved
- y-direction: \( v_y \) of the cart + block will be zero and we can ignore \( v_y \) of the block when it lands in the cart.

\[
\begin{align*}
\text{Initial} & \quad \text{Final} \\
M V_x + m v_x & = (M+m) V'_x \\
M 0 + m v_x & = (M+m) V'_x \\
V'_x & = m v_x / (M + m) \\
& = 2 \times (8.7)/ 12 \text{ m/s} \\
V'_x & = 1.4 \text{ m/s}
\end{align*}
\]

\[
\begin{align*}
a_i & = g \sin 30^\circ \\
& = 5 \text{ m/s}^2 \\
d & = 5 \text{ m} / \sin 30^\circ \\
& = \frac{1}{2} a_i \Delta t^2 \\
10 \text{ m} & = 2.5 \text{ m/s}^2 \Delta t^2 \\
2s & = \Delta t \\
v & = a_i \Delta t = 10 \text{ m/s} \\
v_x & = v \cos 30^\circ \\
& = 8.7 \text{ m/s}
\end{align*}
\]

Inelastic collision in 1-D: Example 2

- A block of mass \( M \) is initially at rest on a frictionless horizontal surface. A bullet of mass \( m \) is fired at the block with a muzzle velocity (speed) \( v \). The bullet lodges in the block, and the block ends up with a speed \( V \). In terms of \( m, M, \) and \( V \):

What is the momentum of the bullet with speed \( v \)?
Example 2
Inelastic Collision in 1-D with numbers

*Do not try this at home!*

Before: A 4000 kg bus, twice the mass of the car, moving at 30 m/s impacts the car at rest.

What is the final speed after impact if they move together?

Exercise 2
Momentum Conservation

- Two balls of equal mass are thrown horizontally with the same initial velocity. They hit identical stationary boxes resting on a frictionless horizontal surface.

- The ball hitting box 1 bounces elastically back, while the ball hitting box 2 sticks.

- Which box ends up moving fastest?

   A. Box 1
   B. Box 2
   C. same
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