Physics 207, Lecture 12, Oct. 13

Goals:
- Chapter 9: Momentum & Impulse
  - Solve problems with 1D and 2D Collisions
  - Solve problems having an impulse (Force vs. time)
- Chapter 10
  - Understand the relationship between motion and energy
  - Define Potential & Kinetic Energy
  - Develop and exploit conservation of energy principle

Assignment:
- HW5 due Wednesday
- HW6 available today
- For Wednesday: Read all of chapter 10

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Inelastic collision in 1-D: Example

- A block of mass $M$ is initially at rest on a frictionless horizontal surface. A bullet of mass $m$ is fired at the block with a muzzle velocity (speed) $v$. The bullet lodges in the block, and the block ends up with a speed $V$. In terms of $m$, $M$, and $V$:

What is the momentum of the bullet with speed $v$?
Inelastic collision in 1-D: Example

What is the momentum of the bullet with speed \( v \)? \( m\vec{V} \)

Key question: Is x-momentum conserved?

\[
\begin{align*}
\text{Before} & \quad \text{After} \\
mv + M0 &= (m + M)V
\end{align*}
\]

Home Exercise

Inelastic Collision in 1-D with numbers

Before: A 4000 kg bus, twice the mass of the car, moving at 30 m/s impacts the car at rest.

What is the final speed after impact if they move together?
**Home exercise**

Inelastic Collision in 1-D

\[ M = 2m \]

**initially**

\[ \vec{v}_0 \quad \vec{v} = 0 \]

(no friction)

\[ MV_0 = (m + M)V \quad \text{or} \quad V = \frac{M}{m + M} \frac{V_0}{2m + m} \]

**finally**

\[ 2\frac{V_0}{3} = 20 \text{ m/s} \]

\[ \vec{v}_f = ? \]

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**Exercise**

Momentum Conservation

- Two balls of equal mass are thrown horizontally with the same initial velocity. They hit identical stationary boxes resting on a frictionless horizontal surface.

- The ball hitting box 1 bounces elastically back, while the ball hitting box 2 sticks.

  - Which box ends up moving fastest?

  A. Box 1
  B. Box 2
  C. same
Exercise Momentum Conservation

- The ball hitting box 1 bounces elastically back, while the ball hitting box 2 sticks.
  - Which box ends up moving fastest?
  - Notice the implications from the graphical solution: Box 1’s momentum must be bigger because the length of the summed momentum must be the same.
  - The longer the green vector the greater the speed

<table>
<thead>
<tr>
<th>Before</th>
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<th>Before</th>
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<tbody>
<tr>
<td>Ball 1</td>
<td>Ball 1</td>
<td>Ball 2</td>
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</tr>
<tr>
<td>Box 1</td>
<td>Box 1</td>
<td>Box 2</td>
<td>Box 2</td>
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<tr>
<td>Box 1+Ball 1</td>
<td>Box 1+Ball 1</td>
<td>Box 2+Ball 2</td>
<td>Box 2+Ball 2</td>
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Exercise Momentum Conservation

- Which box ends up moving fastest?
- Examine the change in the momentum of the ball.

In the case of box 1 the balls momentum changes sign and so its net change is largest. Since momentum is conserved the box must have the largest velocity to compensate.

(A) Box 1    (B) Box 2    (C) same
A perfectly inelastic collision in 2-D

- Consider a collision in 2-D (cars crashing at a slippery intersection...no friction).

\[ \begin{align*}
\text{before} & & \text{after} \\
\vec{v}_1 & & \vec{v} \\
\vec{v}_2 & & \\
\end{align*} \]

- If no external force momentum is conserved.
- Momentum is a vector so \( p_x, p_y \) and \( p_z \) are conserved.

\[ \begin{align*}
\text{x-dir } p_x : \ m_1 \ v_1 &= (m_1 + m_2) \ V \cos \theta \\
\text{y-dir } p_y : \ m_2 \ v_2 &= (m_1 + m_2) \ V \sin \theta \\
\end{align*} \]
Elastic Collisions

- Elastic means that the objects do not stick.
- There are many more possible outcomes but, if no external force, then momentum will always be conserved.
- Start with a 1-D problem.

Billiards

- Consider the case where one ball is initially at rest.

The final direction of the red ball will depend on where the balls hit.
Billiards: All that really matters is conservation
momentum (and energy Ch. 10 & 11)

- Conservation of Momentum
- x-dir $P_x$: $m v_{\text{before}} = m v_{\text{after}} \cos \theta + m V_{\text{after}} \cos \phi$
- y-dir $P_y$: $0 = m v_{\text{after}} \sin \theta + m V_{\text{after}} \sin \phi$

Force and Impulse
(A variable force applied for a given time)

- Gravity: At small displacements a “constant” force $t$

- Springs often provide a linear force ($-kx$) towards its equilibrium position (Chapter 10)

- Collisions often involve a varying force
  \[ F(t): 0 \rightarrow \text{maximum} \rightarrow 0 \]

- We can plot force vs time for a typical collision. The impulse, $J$, of the force is a vector defined as the integral of the force during the time of the collision.
Force and Impulse
(A variable force applied for a given time)

- $\mathbf{J}$ reflects momentum transfer

\[ \mathbf{J} = \int_0^t \mathbf{F} \, dt = \int_0^t (\mathbf{d}\mathbf{p} / dt) \, dt = \int_0^\mathbf{p} \mathbf{d}\mathbf{p} \]

Impulse $\mathbf{J}$ = area under this curve!
(Transfer of momentum!)

Impulse has units of Newton-seconds

- Two different collisions can have the same impulse since $\mathbf{J}$ depends only on the momentum transfer, **NOT** the nature of the collision.
Average Force and Impulse

\[ \Delta t \text{ big, } F_{av} \text{ small} \]

\[ \Delta t \text{ small, } F_{av} \text{ big} \]

Exercise 2
Force & Impulse

Two boxes, one heavier than the other, are initially at rest on a horizontal frictionless surface. The same constant force \( F \) acts on each one for exactly 1 second.

Which box has the most momentum after the force acts?

A. heavier
B. lighter
C. same
D. can’t tell
Boxing: Use Momentum and Impulse to estimate g “force”

Back of the envelope calculation

\[ \vec{J} = \int F \, dt = \bar{F}_{\text{avg}} \Delta t \]

(1) \( m_{\text{arm}} \sim 7 \text{ kg} \)  (2) \( v_{\text{arm}} \sim 7 \text{ m/s} \)  (3) \( \text{Impact time} \Delta t \sim 0.01 \text{ s} \)

Question: Are these reasonable?

\( \Rightarrow \text{Impulse} \quad J = \Delta p \sim m_{\text{arm}} v_{\text{arm}} \sim 49 \text{ kg m/s} \)

\( \Rightarrow F \sim J/\Delta t \sim 4900 \text{ N} \)

(1) \( m_{\text{head}} \sim 6 \text{ kg} \)

\( \Rightarrow a_{\text{head}} = F / m_{\text{head}} \sim 800 \text{ m/s}^2 \sim 80 \text{ g} ! \)

- Enough to cause unconsciousness \( \sim 40\% \) of fatal blow
- Only a rough estimate!
During "collision" with a tree
\[ a_{\text{head}} \approx 600 - 1500 \, \text{g} \]

How do they survive?

- Jaw muscles act as shock absorbers
- Straight head trajectory reduces damaging rotations (rotational motion is very problematic)

Home Exercise

- The only force acting on a 2.0 kg object moving along the x-axis. Notice that the plot is force vs time.
- If the velocity \( v_x \) is +2.0 m/s at 0 sec, what is \( v_x \) at 4.0 s?

\[ \Delta p = m \Delta v = \text{Impulse} \]
\[ m \Delta v = J_{0,1} + J_{1,2} + J_{2,4} \]
\[ m \Delta v = (-8)1 \, \text{N} \, \text{s} + \frac{1}{2} (-8)1 \, \text{N} \, \text{s} + \frac{1}{2} 16(2) \, \text{N} \, \text{s} \]

\[ m \Delta v = 4 \, \text{N} \, \text{s} \]
\[ \Delta v = 2 \, \text{m/s} \]
\[ v_x = 2 + 2 \, \text{m/s} = 4 \, \text{m/s} \]
Chapter 10: Energy

- We need to define an “isolated system”?
- We need to define “conservative force”?
- Recall, chapter 9, force acting for a period of time gives an impulse or a change (transfer) of momentum
- What if a force acting over a distance: Can we identify another useful quantity?

Energy

\[ F_y = m \ a_y \] and let the force be constant
- \[ y(t) = y_0 + v_{y0} \Delta t + \frac{1}{2} a_y \Delta t^2 \Rightarrow \]
  \[ \Delta y = y(t) - y_0 = v_{y0} \Delta t + \frac{1}{2} a_y \Delta t^2 \]
- \[ v_y(t) = v_{y0} + a_y \Delta t \Rightarrow \Delta t = (v_y - v_{y0}) / a_y \]
Energy

\( F_y = m a_y \) and let the force be constant

1. \( y(t) = y_0 + v_{y0} \Delta t + \frac{1}{2} a_y \Delta t^2 \)

\[ \Delta y = y(t) - y_0 = v_{y0} \Delta t + \frac{1}{2} a_y \Delta t^2 \]

2. \( v_y(t) = v_{y0} + a_y \Delta t \)

\[ \Delta t = \frac{(v_y - v_{y0})}{a_y} \]

Now substitute out \( \Delta t \)

3. \( \Delta y = \frac{v_{y0}(v_y - v_{y0})}{a_y} + \frac{1}{2} a_y \left( v_y^2 - 2v_y v_{y0} + v_{y0}^2 \right) / a_y^2 \)
Energy

And now

$$\Delta y = \frac{1}{2} \left( v_y^2 - v_{y0}^2 \right) / a_y$$

can be rewritten as:

$$ma_y \Delta y = \frac{1}{2} m (v_y^2 - v_{y0}^2)$$

And if the object is falling under the influence of gravity then

$$a_y = -g$$

Energy

$$-mg \Delta y = \frac{1}{2} m (v_y^2 - v_{y0}^2)$$

$$-mg (y_f - y_i) = \frac{1}{2} m (v_{yf}^2 - v_{yi}^2)$$

A relationship between \textit{y}-displacement and change in the \textit{y}-speed

Rearranging to give initial on the left and final on the right

$$\frac{1}{2} m v_{yi}^2 + mgy_i = \frac{1}{2} m v_{yf}^2 + mgy_f$$

We now define \textit{mgy} as the “gravitational potential energy”
Energy

- Notice that if we only consider gravity as the external force then the x and z velocities remain constant.
- To \( \frac{1}{2} m v_{yi}^2 + mg y_i = \frac{1}{2} m v_{yi}^2 + mg y_f \)
- Add \( \frac{1}{2} m v_{xi}^2 + \frac{1}{2} m v_{zi}^2 \) and \( \frac{1}{2} m v_{xf}^2 + \frac{1}{2} m v_{zf}^2 \)

\[
\frac{1}{2} m v_i^2 + mg y_i = \frac{1}{2} m v_f^2 + mg y_f
\]

- where \( v_i^2 = v_{xi}^2 + v_{yi}^2 + v_{zi}^2 \)

\( \frac{1}{2} m v^2 \) terms are defined to be kinetic energies
  (A scalar quantity of motion)

Energy

- If only “conservative” forces are present, the total energy (sum of potential, \( U \), and kinetic energies, \( K \)) of a system is conserved.

\[
E_{mech} = K + U
\]

\( E_{mech} = K + U = \text{constant} \)

- \( K \) and \( U \) may change, but \( E_{mech} = K + U \) remains a fixed value.

\( E_{mech} \) is called “mechanical energy”
Example of a conservative system: The simple pendulum.

- Suppose we release a mass $m$ from rest a distance $h_1$ above its lowest possible point.
  - What is the maximum speed of the mass and where does this happen?
  - To what height $h_2$ does it rise on the other side?

Example: The simple pendulum.

- What is the maximum speed of the mass and where does this happen?
  - $E = K + U = \text{constant}$ and so $K$ is maximum when $U$ is a minimum.
Example: The simple pendulum.

What is the maximum speed of the mass and where does this happen?

- $E = K + U = \text{constant}$ and so $K$ is maximum when $U$ is a minimum
- $E = mgh_1$ at top
- $E = mgh_1 = \frac{1}{2}mv^2$ at bottom of the swing

Example: The simple pendulum.

To what height $h_2$ does it rise on the other side?

- $E = K + U = \text{constant}$ and so when $U$ is maximum again (when $K = 0$) it will be at its highest point.
- $E = mgh_1 = mgh_2$ or $h_1 = h_2$
Cart Exercise Revisited: How does this help?

1st Part: Find \( v \) at bottom of incline

- \( a_i \) = \( g \sin 30^\circ \)
  - \( = 5 \text{ m/s}^2 \)
- \( d = 5 \text{ m} / \sin 30^\circ \)
  - \( = \frac{1}{2} a_i \Delta t^2 \)
- \( 10 \text{ m} = 2.5 \text{ m/s}^2 \Delta t^2 \)
- \( 2s = \Delta t \)
- \( v = a_i \Delta t = 10 \text{ m/s} \)
- \( v_x = v \cos 30^\circ \)
  - \( = 8.7 \text{ m/s} \)

One step, no FBG needed

\[ E_{\text{mech}} \text{ is conserved} \]

\[ K_i + U_i = K_f + U_f \]

\[ 0 + mg_y = \frac{1}{2} mv_x^2 + 0 \]

\[ (2gy)^{\frac{1}{2}} = v = (100)^{\frac{1}{2}} \text{ m/s} \]

\[ v_x = v \cos 30^\circ \]

\[ = 8.7 \text{ m/s} \]

Home exercise

A block is shot up a frictionless 40° slope with initial velocity \( v \). It reaches a height \( h \) before sliding back down. The same block is shot with the same velocity up a frictionless 20° slope.

On this slope, the block reaches height

A. \( 2h \)
B. \( h \)
C. \( h/2 \)
D. Greater than \( h \), but we can’t predict an exact value.
E. Less than \( h \), but we can’t predict an exact value.
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