Lecture 26, Dec. 1

Goals:

- Chapter 19
  - Understand the relationship between work and heat in a cycling process
  - Follow the physics of basic heat engines and refrigerators.
  - Recognize some practical applications in real devices.
  - Know the limits of efficiency in a heat engine.

- Assignment
  - HW11, Due Friday, Dec. 5th
  - HW12, Due Friday, Dec. 12th
  - For Wednesday, Read through all of Chapter 20

Heat Engines and Refrigerators

- Heat Engine: Device that transforms heat into work (Q → W)
- It requires two energy reservoirs at different temperatures
  - An thermal energy reservoir is a part of the environment so large with respect to the system that its temperature doesn’t change as the system exchanges heat with the reservoir.
  - All heat engines and refrigerators operate between two energy reservoirs at different temperatures $T_H$ and $T_C$. 

![Diagram of a heat engine cycle with energy exchanges and temperature reservoirs.](image)
Heat Engines

For practical reasons, we would like an engine to do the maximum amount of work with the minimum amount of fuel. We can measure the performance of a heat engine in terms of its thermal efficiency \( \eta \) (lowercase Greek eta), defined as

\[
\eta = \frac{W_{\text{out}}}{Q_H} = \frac{\text{what you get}}{\text{what you had to pay}}
\]

We can also write the thermal efficiency as

\[
\eta = 1 - \frac{Q_C}{Q_H}
\]

**Exercise Efficiency**

- Consider two heat engines:
  - Engine I:
    - Requires \( Q_{\text{in}} = 100 \text{ J} \) of heat added to system to get \( W = 10 \text{ J} \) of work (done on world in cycle)
  - Engine II:
    - To get \( W = 10 \text{ J} \) of work, \( Q_{\text{out}} = 100 \text{ J} \) of heat is exhausted to the environment

- Compare \( \eta_1 \), the efficiency of engine I, to \( \eta_{II} \), the efficiency of engine II.

\[
\eta = \frac{W_{\text{cycle}}}{Q_H} = \frac{|Q_h| - |Q_C|}{Q_H} = 1 - \frac{|Q_C|}{|Q_h|}
\]
Exercise Efficiency

- Compare $\eta_I$, the efficiency of engine I, to $\eta_{II}$, the efficiency of engine II.
  - Engine I:
    - Requires $Q_{in} = 100$ J of heat added to system to get $W = 10$ J of work (done on world in cycle)
    - $\eta = 10 / 100 = 0.10$
  - Engine II:
    - To get $W = 10$ J of work, $Q_{out} = 100$ J of heat is exhausted to the environment
    - $Q_{in} = W + Q_{out} = 100 + 10 = 110$ J
    - $\eta = 10 / 110 = 0.09$

\[ \eta = \frac{W_{cycle}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} \]

Refrigerator (Heat pump)

- Device that uses work to transfer heat from a colder object to a hotter object.

\[ K = \frac{Q_{Cold}}{W_{in}} = \frac{\text{What you get}}{\text{What you pay}} \]
The best thermal engine ever, the Carnot engine

- A perfectly reversible engine (a **Carnot engine**) can be operated either as a heat engine or a refrigerator between the same two energy reservoirs, by reversing the cycle and with no other changes.

- A **Carnot cycle** for a gas engine consists of two isothermal processes and two adiabatic processes.

- A **Carnot engine** has maximum thermal efficiency, compared with any other engine operating between \( T_H \) and \( T_C \).

\[
\eta_{\text{Carnot}} = 1 - \frac{T_{\text{Cold}}}{T_{\text{Hot}}}
\]

- A **Carnot refrigerator** has a maximum coefficient of performance, compared with any other refrigerator operating between \( T_H \) and \( T_C \).

\[
K_{\text{Carnot}} = \frac{T_{\text{Cold}}}{T_{\text{Hot}} - T_{\text{Cold}}}
\]

The Carnot Engine

- Carnot showed that the thermal efficiency of a Carnot engine is:

\[
\varepsilon_{\text{Carnot cycle}} = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}}
\]

- All real engines are less efficient than the Carnot engine because they operate irreversibly due to the path and friction as they complete a cycle in a brief time period.
Problem

- You can vary the efficiency of a Carnot engine by varying the temperature of the cold reservoir while maintaining the hot reservoir at constant temperature.

Which curve that best represents the efficiency of such an engine as a function of the temperature of the cold reservoir?

\[ \eta_{\text{Carnot}} = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}} \]

The Carnot Engine (the best you can do)

- No real engine operating between two energy reservoirs can be more efficient than a Carnot engine operating between the same two reservoirs.

A. A→B, the gas expands isothermally while in contact with a reservoir at \( T_h \)
B. B→C, the gas expands adiabatically \( (Q=0, \Delta U=W_{B\rightarrow C}, T_h \rightarrow T_c) \), \( PV'=\text{constant} \)
C. C→D, the gas is compressed isothermally while in contact with a reservoir at \( T_c \)
D. D→A, the gas compresses adiabatically \( (Q=0, \Delta U=W_{D\rightarrow A}, T_c \rightarrow T_h) \)
Carnot Cycle Efficiency

\[ \varepsilon_{\text{Carnot}} = 1 - \frac{Q_c}{Q_h} \]

\[ Q_{A \rightarrow B} = Q_h = W_{AB} = nRT_h \ln(V_B/V_A) \]

\[ Q_{C \rightarrow D} = Q_c = W_{CD} = nRT_c \ln(V_D/V_C) \]

(here we reference work done by gas, \( dU = 0 = Q - P \, dV \))

But \( P_A V_A = P_B V_B = nRT_h \) and \( P_C V_C = P_D V_D = nRT_c \)

so \( \frac{P_B}{P_A} = \frac{V_A}{V_B} \) and \( \frac{P_C}{P_D} = \frac{V_D}{V_C} \)

as well as \( P_B V_B^\gamma = P_C V_C^\gamma \) and \( P_D V_D^\gamma = P_A V_A^\gamma \)

with \( P_B V_B^\gamma P_A V_A^\gamma = P_C V_C^\gamma P_D V_D^\gamma \) thus

\[ (\frac{V_B}{V_A}) = (\frac{V_D}{V_C}) \]

\[ Q_c/Q_h = T_c/T_h \]

Finally

\[ \varepsilon_{\text{Carnot}} = 1 - \frac{T_c}{T_h} \]

Other cyclic processes: Turbines

- A turbine is a mechanical device that extracts thermal energy from pressurized steam or gas, and converts it into useful mechanical work. 90% of the world electricity is produced by steam turbines.

- Steam turbines & jet engines use a Brayton cycle.
Steam Turbine in Madison

- MG&E, the electric power plan in Madison, boils water to produce high pressure steam at 400°C. The steam spins the turbine as it expands, and the turbine spins the generator. The steam is then condensed back to water in a Monona-lake-water-cooled heat exchanger, down to 20°C.

- Carnot Efficiency?

\[ \eta_{\text{Carnot}} = 1 - \frac{T_{\text{Cold}}}{T_{\text{Hot}}} = 1 - \frac{293K}{673K} = 0.56 \]

The Sterling Cycle

- Return of a 1800’s thermodynamic cycle
Sterling cycles

- 1 Q, V constant → 2 Isothermal expansion (\( W_{\text{on system}} < 0 \)) →
- 3 Q, V constant → 4 Q out, Isothermal compression (\( W_{\text{on sys}} > 0 \))

1. \( Q_1 = nR C_V (T_H - T_C) \)
2. \( W_{\text{on2}} = -nR T_H \ln \left( \frac{V_b}{V_a} \right) = -Q_2 \)
3. \( Q_3 = nR C_V (T_C - T_H) \)
4. \( W_{\text{on4}} = -nR T_L \ln \left( \frac{V_a}{V_b} \right) = -Q_4 \)

\( Q_{\text{Cold}} = - (Q_3 + Q_4) \)
\( Q_{\text{Hot}} = (Q_1 + Q_2) \)

\[ \eta = 1 - \frac{Q_{\text{Cold}}}{Q_{\text{Hot}}} \]

Power from ocean thermal gradients... oceans contain large amounts of energy

Carnot Cycle Efficiency

\[ \eta_{\text{Carnot}} = 1 - \frac{Q_c}{Q_h} = 1 - \frac{T_c}{T_h} \]

See: http://www.nrel.gov/otec/what.html
Ocean Conversion Efficiency

\[ \eta_{\text{Carnot}} = 1 - \frac{T_c}{T_h} = 1 - \frac{275}{300} \approx 0.083 \] (even before internal losses and assuming a REAL cycle)

Still: “This potential is estimated to be about \(10^{13}\) watts of base load power generation, according to some experts. The cold, deep seawater used in the OTEC process is also rich in nutrients, and it can be used to culture both marine organisms and plant life near the shore or on land.”

“Energy conversion efficiencies as high as 97% were achieved.”


So \[ \eta = 1 - \frac{Q_c}{Q_h} \] is always correct but \[ \eta_{\text{Carnot}} = 1 - \frac{T_c}{T_h} \] only reflects a Carnot cycle

Internal combustion engine: gasoline engine

- A gasoline engine utilizes the Otto cycle, in which fuel and air are mixed before entering the combustion chamber and are then ignited by a spark plug.
Internal combustion engine: Diesel engine

• A Diesel engine uses compression ignition, a process by which fuel is injected after the air is compressed in the combustion chamber causing the fuel to self-ignite.

Thermal cycle alternatives

• Fuel Cell Efficiency (from wikipedia)

Fuel cells do not operate on a thermal cycle. As such, they are not constrained, as combustion engines are, in the same way by thermodynamic limits, such as Carnot cycle efficiency. The laws of thermodynamics also hold for chemical processes (Gibbs free energy) like fuel cells, but the maximum theoretical efficiency is higher (83% efficient at 298K) than the Otto cycle thermal efficiency (60% for compression ratio of 10 and specific heat ratio of 1.4).

• Comparing limits imposed by thermodynamics is not a good predictor of practically achievable efficiencies

• The tank-to-wheel efficiency of a fuel cell vehicle is about 45% at low loads and shows average values of about 36%. The comparable value for a Diesel vehicle is 22%.

• Honda Clarity
(now leased in CA and gets ~70 mpg equivalent)

This does not include H₂ production & distribution
Fuel Cell Structure

1. Hydrogen fuel is channeled through flow plates to the anode on one side of the fuel cell, while oxidant (oxygen or air) is channeled to the cathode on the other side of the cell.

2. At the anode, a platinum catalyst causes the hydrogen to split into positive hydrogen ions (protons) and negatively charged electrons.

3. The polymer electrolyte membrane (PEM) allows only the positively charged ions to pass through it to the cathode. The negatively charged electrons must travel along an external circuit to the cathode, creating an electrical current.

4. At the cathode, the electrons and positively charged hydrogen ions combine with oxygen to form water, which flows out of the cell.

Problem-Solving Strategy: Heat-Engine Problems

**VISUALIZE** Draw the $pV$ diagram of the cycle.

**SOLVE** There are several steps in the mathematical analysis.

- Use the ideal-gas law to complete your knowledge of $n$, $p$, $V$, and $T$ at one point in the cycle.
- Use the ideal-gas law and equations for specific gas processes to determine $p$, $V$, and $T$ at the beginning and end of each process.
- Calculate $Q$, $W$, and $\Delta E_{\text{th}}$ for each process.
- Find $W_{\text{net}}$ by adding $W_i$ for each process in the cycle. If the geometry is simple, you can confirm this value by finding the area enclosed within the $pV$ curve.
- Add just the positive values of $Q$ to find $Q_{\text{th}}$.
- Verify that $(\Delta E_{\text{th}})_{\text{net}} = 0$. This is a self-consistency check to verify that you haven’t made any mistakes.
- Calculate the thermal efficiency $\eta$ and any other quantities you need to complete the solution.

**ASSESS** Is $(\Delta E_{\text{th}})_{\text{net}} = 0$? Do all the signs of $W_i$ and $Q$ make sense? Does $\eta$ have a reasonable value? Have you answered the question?
Going full cycle

- 1 mole of an ideal gas and $PV = nRT \rightarrow T = PV/nR$
  
  $T_1 = 8300 \quad 0.100 / 8.3 = 100 \text{ K} \quad T_2 = 24900 \quad 0.100 / 8.3 = 300 \text{ K}$
  
  $T_3 = 24900 \quad 0.200 / 8.3 = 600 \text{ K} \quad T_4 = 8300 \quad 0.200 / 8.3 = 200 \text{ K}$

  
  $(W_{net} = 16600 \times 0.100 = 1660 \text{ J})$

1→2

  
  $\Delta E_{th} = 1.5nR \Delta T = 1.5 \times 8.3 \times 200 = 2490 \text{ J}$
  
  $W_{by} = 0 \quad Q_{in} = 2490 \text{ J} \quad Q_{H} = 2490 \text{ J}$

2→3

  
  $\Delta E_{th} = 1.5nR \Delta T = 1.5 \times 8.3 \times 300 = 3740 \text{ J}$
  
  $W_{by} = 2490 \text{ J} \quad Q_{in} = 3740 \text{ J} \quad Q_{H} = 6230 \text{ J}$

3→4

  
  $\Delta E_{th} = 1.5nR \Delta T = -1.5 \times 8.3 \times 400 = -4980 \text{ J}$
  
  $W_{by} = 0 \quad Q_{in} = -4980 \text{ J} \quad Q_{C} = 4980 \text{ J}$

4→1

  
  $\Delta E_{th} = 1.5nR \Delta T = -1.5 \times 8.3 \times 100 = -1250 \text{ J}$
  
  $W_{by} = 830 \text{ J} \quad Q_{in} = -1240 \text{ J} \quad Q_{C} = 2070 \text{ J}$

  
  $Q_{H(total)} = 8720 \text{ J} \quad Q_{C(total)} = 7060 \text{ J} \quad \eta = 1660 / 8720 = 0.19$ (very low)

Exercise

- If an engine operates at half of its theoretical maximum efficiency ($\eta_{max}$) and does work at the rate of $W \text{ J/s}$, then, in terms of these quantities, how much heat must be discharged per second.

- This problem is about process (Q and W), specifically $Q_{C}$?

  
  $\eta_{max} = 1 - Q_{C} / Q_{H}$ and $\eta = 1/2 \eta_{max} = 1/2 (1 - Q_{C} / Q_{H})$

  
  
  $W = \eta Q_{H} = \eta_{max} Q_{H} \rightarrow 2W / \eta_{max} = Q_{H}$

  
  $-Q_{H} (\eta_{max} - 1) = Q_{C} \rightarrow Q_{C} = 2W / \eta_{max} (1 - \eta_{max})$