Physics 208 Final Exam Dec. 21, 2006

Print your name and section clearly above. If you do not know your section number, write your TA's name.
Your final answer must be placed in the box provided. **You must show all your work to receive full credit.** If you only provide your final answer (in the box), and do not show your work, you will receive very few points.
Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units, and also the direction of vectors.
You are allowed two 8½ x 11” sheets of notes and no other references. The exam lasts exactly 120 minutes.

| Problem 1: _____ / 20 |
| Problem 2: _____ / 20 |
| Problem 3: _____ / 20 |
| Problem 4: _____ / 20 |
| Problem 5: _____ / 20 |
| Problem 6: _____ / 20 |

**TOTAL: _____ / 120**

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**Coulomb constant** \( k_c = 9.0 \times 10^9 \, \text{N} \cdot \text{m}^2/\text{C}^2 \)

**Speed of light in vacuum:** \( c = 3 \times 10^8 \, \text{m/s} \)

**Permittivity of free space** \( \varepsilon_0 = 4\pi k_c = 8.85 \times 10^{-12} \, \text{C}^2/\text{N} \cdot \text{m}^2 \)

**Permeability of free space** \( \mu_0 = 4\pi \times 10^{-7} \, \text{T} \cdot \text{m}/\text{A} \)

**Planck’s constant** \( h = 6.626 \times 10^{-34} \, \text{J} \cdot \text{s} \)

\( = 4.1357 \times 10^{-15} \, \text{eV} \cdot \text{s} \)

\( h = h/2\pi \)

**Bohr radius** \( a_0 = 0.053 \, \text{nm} \)

**Bohr magneton** \( \mu_B = 5.788 \times 10^{-5} \, \text{eV} / \text{T} \)

**Atomic mass unit** \( 1 \, \text{u} = 1.66054 \times 10^{-27} \, \text{kg} \)

\( = 931.494 \, \text{MeV} / \text{c}^2 \)

**Electron mass** \( m_e = 9.11 \times 10^{-31} \, \text{kg} \)

\( = 0.00055 \, \text{u} = 0.51 \, \text{MeV} / \text{c}^2 \)

**Proton mass** \( m_p = 1.67262 \times 10^{-27} \, \text{kg} \)

\( = 1.00728 \, \text{u} = 938.28 \, \text{MeV} / \text{c}^2 \)

**Neutron mass** \( m_n = 1.67493 \times 10^{-27} \, \text{kg} \)

\( = 1.00866 \, \text{u} = 939.57 \, \text{MeV} / \text{c}^2 \)

**Neutron mass**

\( h_c = 1240 \, \text{eV} \cdot \text{nm} \)

\( 1 \, \text{eV} = 1.602 \times 10^{-19} \, \text{J} \)

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1) [20 pts, 4 pts each] Multiple choice.
i) A parallel-plate capacitor of capacitance 1 µF is connected to a 2 V battery.
The battery is disconnected, and the plate separation is reduced by a factor of two.
What is the final voltage on the capacitor?

- a. 1 V
- b. 2 V
- c. 4 V
- d. 0.5 V
- e. 8 V
- f. none of these

\[ Q = CV, \text{ } Q \text{ is constant, } C \text{ increases by factor of two, so potential drops by factor of two. Or, electric field remains constant, but distance drops by factor of two, so } V \text{ drops by a factor of two.} \]

\[ B \text{-field at particle from current in wire is out of page. So } vxB \text{ is down. But charge is negative, so force is up} \]

ii) A negative-charged particle is moving as shown parallel to a wire carrying a positive current \( I \) in the direction shown. The direction of the force on the particle due to the current in the wire is

- a. up
- b. down
- c. left
- d. right
- e. into page
- f. out of page

\[ E = hf = hc/\lambda, \text{ so violet photons have twice the energy. Same number of photons / sec means twice the power.} \]

\[ \text{Magnification = image dist/object distance = 0.015m/4.5m=0.00333} \]
\[ \text{So image height is 0.005 m = 5 mm.} \]

iii) The wavefunction of a quantum mechanical particle is shown at right. How do the probabilities of finding the particle at 0.25 nm (\( P(0.25 \text{ nm}) \)) and at 0.75 nm (\( P(0.75 \text{ nm}) \)) compare?

- a. \( P(0.25 \text{ nm})=P(0.75 \text{ nm}) \)
- b. \( P(0.25 \text{ nm})<P(0.75 \text{ nm}) \)
- c. \( P(0.25 \text{ nm})>P(0.75 \text{ nm}) \)
- d. only identical locations can be compared
- e. need to know mass of particle

iv) Particular red (600 nm) and violet (300 nm) lasers both shoot out the same number of photons per second. How does the power output of the two lasers compare?

- a. Both the same.
- b. Violet has 1/4 the power as red.
- c. Violet has 1/2 the power as red.
- d. Violet has 2 times the power as red.
- e. Violet has 4 times the power as red.

v) A 1.5 m tall woman stands 4.5 m in front of a friend. The friend's eye has a separation of 1.5 cm between eye lens and retina. The size of the woman's image on the retina is

- a. 0.01 mm
- b. 0.1 mm
- c. 1.1 mm
- d. 1.5 mm
- e. 5 mm
2) [20 points, 5 pts each] Short-answer questions

a) A light bulb is in series in a single-turn loop of wire as shown. The light bulb has a resistance of 0.1 Ω, and the loop resistance is negligible. The loop is near a long, straight wire that carries 10 A of current. This 10 A current produces a magnetic flux of 0.01 Tesla-m$^2$ through the loop. Calculate the time $\Delta t$ over which you must reduce the wire current to zero (at a constant rate) in order to dissipate 1 Watt of power in the light bulb.

\[ \text{Value} = \frac{0.316V}{\Delta t} \rightarrow \Delta t = 0.0316s = 31.6ms \]

b) An x-ray photon used in a dentist’s office to produce an x-ray of your teeth has an energy of 70,000 eV. Calculate its wavelength.

\[ \lambda = \frac{hc}{E} = \frac{1240eV \cdot nm}{70,000eV} = 0.0177nm \]

c) A large number of hydrogen atoms are in the $n=3$, $l=1$ state. In zero magnetic field, you center your spectrometer on a 102.6 nm spectral line. You now turn on a 1 T magnetic field. How many spectral lines do you see?

There are three values of $m_l$ for this $l$: $m_l = -1, 0, +1$. In a magnetic field these all have different energy. The transition is to the $n=1$ state, where $l=0$ and $m_l=0$. So there are now three transitions of different energy down to the $n=1$ state.
d) A very small particle with 10 µC positive charge is 1 cm away from the surface of a fixed 2 cm diameter, 1 m long insulating cylinder with 100 µC of charge distributed uniformly on its surface. What is the force on the 10 µC particle? (Ignore end effects, essentially assuming an infinitely long tube)

[Hint: Use Gauss’ law]

Use a Gaussian cylinder of 1 m length, 4 cm diameter. Electric flux through this is \((E\text{-field})\times(\text{surface area}) = E(\pi)(0.04m)(1m) = E0.1256m^2\). By Gauss law, this is \(100\mu\text{C}/\varepsilon_0\), so \(E = (100 \times 10^{-6}\text{C})/((8.85 \times 10^{-12}\text{C}^2/\text{Nm}^2) \times 0.1256m^2) = 88.92 \times 10^6\ V/m\). The force is then 889.2N
3) [20 pts, 5 pts each]
A single laser produces a light beam consisting of equal intensities of two wavelengths: blue (450 nm) and green (550 nm). It shines on two slits separated by 0.01 mm. The five interference maxima with the smallest diffraction angle are indicated as filled circles below.

a) Identify the color of the interference maxima by writing either ‘blue’, ‘green’, or ‘mixed’ in the boxes above. ‘Mixed’ means blue and green mixed together.

b) The screen is 1 m away.
Calculate the spatial separation of the top two interference maxima.

\[ \lambda = dsin\theta, \text{ and } sin\theta \approx \frac{y}{L}, \text{ where } y \text{ is the distance from the central maximum.} \]
So \( \lambda = dy/L \). y = \( \lambda L/d \). So sep is \( \Delta y = (L/d) \Delta \lambda = 1 \text{ cm} \)

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The screen is 1 m away.
Calculate the spatial separation of the top two interference maxima.

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This means it has 0.1 mW of power = \( (10^{-4} \text{ J/s})(1 \text{ eV/1.602x10^{-19} J}) \) = 6.242x10^{14} \text{ eV/s.}
This is split equally among blue and green photons, so 3.121x10^{14} \text{ eV/s each.}
One blue photon has energy 1240eV-nm/450nm=2.756eV->1.133x10^{14} \text{ blue photons/s}
One green photon has energy 1240eV-nm/550nm=2.255eV->1.384x10^{14} \text{ green photons/s}
This is a total of 2.517x10^{14} \text{ photons/s}

| # photons/sec= | Value |
d) Now two linear polarizers are added, one in front of each slit. The intensity of the central maximum is measured to be $I_{||}$ with the transmission axes of the polarizers parallel. One of the polarizers is then rotated as shown below to make the transmission axes perpendicular and the intensity $I_{\perp}$ measured. What is the ratio $I_{\perp}/I_{||}$?

$$I_{\perp}/I_{||} = 1/2.$$ 

4) [20 pts, 5 pts each] Two infinite conducting plates are separated by $10^{-4}$ m to make a parallel plate capacitor. The left plate has $+1 \mu C/m^2$ of charge, and the right plate has $-1 \mu C/m^2$ of charge. ($1 \mu C = 10^{-6} C$)

a) Calculate the electric field magnitude and direction between the plates (region II).

$$\frac{\sigma}{\varepsilon_0} + \frac{\sigma}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \left(10^{-6} C/m^2\right)/8.85 \times 10^{-12} C^2/Nm^2 = 1.13 \times 10^5 N/C$$

The field points away from positive charge, so it points to the right. Its strength has contributions from the charge on each plate:

$$E = \frac{Q}{d} = \frac{Q}{\varepsilon_0 Ad} = \frac{Q}{\varepsilon_0 A} d = \frac{Q}{A} \frac{1}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

b) Calculate the electric field magnitude and direction outside the plates (regions I and III). EXPLAIN

The field from the positive and negative charge densities are equal and opposite outside the plates, and so cancel. $E = 0.$
c) Calculate the electric potential difference between the plates, \( V_{\text{left}} - V_{\text{right}} \).

Since the electric field is constant, the potential difference has magnitude
\[
1.13 \times 10^5 \text{V/m} \times 10^{-4} \text{m} = 11.3 \text{V}
\]
\( V_{\text{left}} \) must be higher potential than \( V_{\text{right}} \) because it has positive charge.
So \( V_{\text{left}} - V_{\text{right}} = 11.3 \text{V} \)

\[
V = \begin{array}{|c|c|}
\hline
\text{Value} & \text{Units} \\
\hline
\end{array}
\]

\[
\text{d) How much work per square meter } W \text{ must you do to pull the plates apart to twice their original separation?}
\]

Think of a section of the capacitor of area 1\( \text{m}^2 \). The energy stored in that section is
\[
\frac{Q^2}{2C} = \frac{Q^2}{2(\varepsilon_0 A/d)} = \left(10^{-6} \text{C} \right)^2 / 2 \left( \frac{8.854 \times 10^{-12} \text{C}^2 / \text{Nm}^2}{1 \text{m}^2} \right) 1 \text{m}^2 / 10^{-4} \text{m} = 5.6 \times 10^{-6} \text{J}
\]
Doubling the separation doubles this number, so 0.056 \text{J/m}^2 of work are required.

Or can work directly from the energy density
\[
\frac{\varepsilon_0 E^2}{2} = \frac{1}{2} \left( 8.854 \times 10^{-12} \text{C}^2 / \text{N} \cdot \text{m}^2 \right) \left( 1.13 \times 10^5 \text{N} / \text{C} \right)^2 = 0.0565 \text{J}.
\]
Now multiply by change in volume \( 10^{-4} \text{m}^2 \) (since \( E \) is constant) to get same as above.

\[
W = \begin{array}{|c|c|}
\hline
\text{Value} & \text{Units} \\
\hline
\end{array}
\]

Or can use energy stored \( = \frac{1}{2} QV \). The change in stored energy is
\[
\frac{1}{2} Q (V_{\text{final}} - V_{\text{initial}}) = \frac{1}{2} (10^{-6} \text{C}) (22.6 \text{V} - 11.3 \text{V})
\]
\[
= \frac{1}{2} (10^{-6} \text{C})(11.3 \text{V}) = 5.65 \times 10^{-6} \text{J}
\]
5) [20 pts, 4 pts each] $^{241}\text{Am}$ is used in smoke detectors, with the alarm coming on when the alpha particles emitted from its nucleus are blocked by smoke.

a) In the $^{241}\text{Am}$ nucleus, there are 95 protons and 241 total nucleons. 
After the alpha emission, $^{241}\text{Am}$ becomes

i. $^{239}\text{Nd}$
ii. $^{237}\text{Nd}$
iii. $^{239}\text{Pa}$
iv. $^{237}\text{Pa}$
v. $^{237}\text{U}$

Np is the element with 93 protons
U is the element with 92 protons
Pa is the element with 91 protons

b) The atomic mass of $^{241}\text{Am}$ is 241.056829u. If the ejected alpha particle has a measured energy of 5.64 MeV, calculate the atomic mass of the daughter nucleus $m_{\text{daughter}}$. The atomic mass of a He atom is 4.002603u

$$241.056829u - m_{\text{daughter}} - 4.002603u = \frac{(5.64 \text{ MeV}/c^2)/(931.494\text{MeV}/c^2/u)}{241.056829u - 4.002603u - 0.00605u = 237.04818u = m_{\text{daughter}}}$$

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$c) ^{241}\text{Am}$ has a half-life of 432 years. How many alpha particles per second will be emitted from 1 micro-gram (2.5x10$^{15}$ nuclei) of $^{241}\text{Am}$?

A half-life of 432 years is 1.362x10$^{10}$ sec. The corresponding decay rate is

$$\lambda = \ln 2 / \tau_{1/2} = 5.089 \times 10^{-11} / s.$$ The total decays per second is then

$$\left(5.089 \times 10^{-11}/s\right)\left(2.5 \times 10^{15}\right) = 1.272 \times 10^5 \text{ decays/s}$$

| #/sec | |
d) If a brand-new smoke detector has $10^5$ decays per second, and needs at least $7 \times 10^4$ decays per second to operate correctly, how long would it operate correctly? (don’t worry about the batteries!)

\[ N = N_0 e^{-\lambda t}, \text{ with } \lambda = 5.089 \times 10^{-11} / s \]

Then \( t = \frac{1}{\lambda} \ln\left(\frac{N_0}{N}\right) = 7.00 \times 10^9 \text{ s} \)

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e) $^{241}\text{Am}$ is the direct product of the decay of $^{241}\text{Pu}$. Plutonium has an atomic number of 94. What particle is emitted when $^{241}\text{Pu}$ decays?

i. alpha particle
ii. gamma particle
iii. electron
iv. positron
v. neutron
vi. proton

Since $^{241}\text{Am}$ has one more proton than $^{241}\text{Pu}$ but the same total number of nucleons, a neutron must have changed into a proton in the nucleus. By charge conservation, an electron must have been emitted from the nucleus.

6) [20 pts, 5 pts each]
A one-dimensional box with a length $L=1 \text{ nm}$ has one electron in it. The energy levels are given by \( E_n = \frac{n^2 \hbar^2}{8 m_e L^2} \).

a) What is the longest wavelength photon that can be emitted by this quantum system?

The longest wavelength corresponds to the lowest energy. The lowest energy is the $n=2 \Rightarrow n=1$.

So photon energy is \( \left(2^2 - 1^2\right) \left(\frac{\hbar^2 c^2}{8 m e L^2}\right) = 3 \left(\frac{1240 \text{ eV} \cdot \text{ nm}}{0.511 \times 10^6 \text{ eV} \cdot (1 \text{ nm})^2}\right) = 1.128 \text{ eV} \)

The corresponding wavelength is \( \lambda = \frac{hc}{E} = 1240 \text{ eV} \cdot \text{ nm} / 0.0113 \text{ eV} = 1099 \text{ nm} \)

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b) Calculate the wavelength of the electron in the $n=3$ state of the quantum box.

In the $n=3$ state, 3 half-wavelengths fit inside the box. So

\( \frac{3}{2} \lambda = 1 \text{ nm} \Rightarrow \lambda = 0.667 \text{ nm} \)

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c) Draw the wavefunction $\Psi$ for the $n=3$ state and the probability $P$ of finding the electron for the $n=3$ state. Make sure the wavefunction and probability are consistent with each other, and with the markers on the axes labels where applicable.

\[ \Psi \]

\[ \Psi = 0 \]

\[ x = 0 \]

\[ x = L \]

\[ P = 1 \]

\[ P = 0 \]

\[ x = 0 \]

\[ x = L \]

d) If the size of the box is increased, the wavelength and energy of the particle in part c) change as

a. wavelength shorter, energy larger
b. wavelength longer, energy smaller
c. wavelength shorter, energy smaller
d. wavelength longer, energy larger
e. wavelength and energy unchanged