Physics 208 Exam 1  
Oct. 3, 2007

Print your name and section clearly above. If you do not know your section number, write your TA’s name.
Your final answer must be placed in the box provided. **You must show all your work to receive full credit.** If you only provide your final answer (in the box), and do not show your work, you will not receive very many points.
Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units, and also the direction of vectors.
You are allowed one 8½ x 11” sheet of notes and no other references. The exam lasts exactly 90 minutes.

**Speed of light in vacuum:**
\[ c = 3 \times 10^8 \text{ m/s} \]

**Permittivity of free space**
\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2 \]

**Coulomb constant**
\[ k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \]

**Magnitude of electron charge**
\[ e = 1.6 \times 10^{-19} \text{ C} \]

**Problem 1:** ______ / 20

**Problem 2:** ______ / 15

**Problem 3:** ______ / 15

**Problem 4:** ______ / 15

**Problem 5:** ______ / 15

**Problem 6:** ______ / 15

**TOTAL:** ______ / 95
1) [20 points, 4 points each].

i) Two canoes are 10 m apart on a lake. Each bobs up and down with a period $T=4.0$ s. When one is at the highest point, the other canoe is at its lowest. Both canoes are less than one water wavelength apart. What is the speed of the water wave?

- a) 2.5 m/s
- b) 5.0 m/s
- c) 14 m/s
- d) 40 m/s
- e) 80 m/s

The period is 4s, and the canoes are $\frac{1}{2}$ wavelength apart. Then the wavelength is 20m. Since $(\text{wavelength})(\text{period})=\text{wave speed}$, wave speed is $20m/4s = 5m/s$

ii) Total internal reflection occurs

- a) at angles of incidence greater than that for which the angle of refraction = $90^\circ$
- b) at angles of incidence less than that for which the angle of refraction = $90^\circ$
- c) at angles of incidence equal to $90^\circ$
- d) when the refractive indices of the two media are matched
- e) none of the above

iii) Below is an electric dipole. What direction is the electric field at the black dot?

- a) A
- b) B
- c) C
- d) D
- e) E
- f) F

iv) A 300 nm wavelength laser is shined vertically on a blu-ray disc (similar to a CD or DVD). Aside from the directly reflected beam, the only laser spots detected are at an angle of $70^\circ$ from the incident laser beam. What is the track separation on the blu-ray disc?

- a) 0.32 microns
- b) 0.0043 microns
- c) 0.25 microns
- d) 0.28 microns
- e) 0.05 microns

$d \sin \theta = m \lambda$ with $m=1$ gives

\[ d = \frac{\lambda}{\sin \theta} = \frac{0.300\mu m}{\sin (70^\circ)} = 0.319\mu m \]

v) Which of the following is NOT required of electric field lines

- a) every field line starts on a positive charge and ends on a negative charge.
- b) no two field lines cross
- c) the density of field lines is proportional to the local electric field intensity
- d) the local electric field is tangent to the electric field line at that point
- e) all of these are true.
2) **[15 points total]** White light of uniform intensity in the wavelength range 400-690 nm is perpendicularly incident on the surface of a suspended water film of thickness \(L=320\) nm and refraction index \(n_2=1.33\). The film is surrounded on all sides air with index \(n_1=1.0\).

a) **[10 points]** What wavelength \(\lambda\) of light reflected by the film appears bright to an observer?

Reflection from top surface is phase shifted by \(\pi\). Reflection from bottom surface is not. So for constructive interference, need path length difference of \((m+1/2)\lambda_{\text{water}}\). \(\lambda_{\text{water}} = \lambda / n\). Path length difference is twice the thickness, so

\[
2(320\text{nm}) = (m + 1/2)(\lambda / 1.33) \Rightarrow \lambda = \frac{1.33(640\text{nm})}{m + 1/2}
\]

\(m = 0 \Rightarrow 1700\text{nm}\)
\(m = 1 \Rightarrow 567\text{nm}\)
\(m = 2 \Rightarrow 340\text{nm}\)

So the water film appears bright at 567 nm wavelength (yellow)

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3) [15 points total] Light incident from air (air index of refraction $n_{\text{air}} = 1.0$) onto glass (glass index of refraction $n_{\text{glass}} = 1.5$) is refracted at an angle of 35°. Underwater (water index of refraction $n_{\text{water}} = 1.33$), the refracted angle is $\theta$.

The light is incident at the same angle in both cases.

a) [5 points] Argue in words (no equations) whether the underwater refracted angle $\theta$ is greater than, less than, or equal to 35°.

The refracted ray is bent less underwater because the indices of refraction of the two media are closer underwater than in air. Since in air it is bent toward the normal, the refracted angle is larger than 35° underwater to make it closer to the incident direction.

b) [10 points] Calculate the value of the underwater refracted angle $\theta$.

Use Snell’s law to find the incident angle in air:

$1.0 \sin \theta_1 = 1.50 \sin 35^\circ \Rightarrow \sin \theta_1 = 1.50 \sin 35^\circ$

Then do it underwater:

$1.33 \sin \theta_2 = 1.50 \sin \theta_2$

$1.33(1.5 \sin 35^\circ) = 1.5 \sin \theta_2$

$\sin \theta_2 = 1.33 \sin 35^\circ = 0.763$

$\theta_2 = 49.7^\circ$

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4) **[15 points total]** A projector with a single lens is to be used inside a closed room to project an image of a 20 mm X 30mm LCD (liquid crystal display) chip onto the wall of a room. The image needs to be at least 2 m X 3 m. The room is 5 m long.

a) **[5 points]** What is the minimum required magnification?

\[
\text{The minimum required magnification is } \frac{2\text{m}}{20\text{mm}} = \frac{2\text{m}}{0.02\text{m}} = 100.
\]

b) **[10 points]** You have available five different projector lenses with focal lengths 20mm, 40mm, 80mm, 120mm, 200mm. Which of these can make the required image size in the 5 m long room?

\[
\text{Smallest magnification } = \frac{2\text{m}}{0.02\text{m}} = 100. \text{ This is also image distance over object distance. So the image distance is 100 times bigger than the object distance. Then } \frac{1}{f} \sim \frac{1}{s}, \text{ and the object distance equals the focal length. Then the image distance must be at least 100 times the focal length. So the longest focal length lens that would work is } \frac{5\text{m}}{100} = 50\text{mm}. \text{ Lenses of 20mm and 40mm will work.}
\]

Possible lenses =
5) **[15 points total]** A molecule has a dipole moment of 1.21 Debyes (1 Debye = 3.336 × 10^{−30} Coulomb-meters). The charge density was displaced 0.0021 nm in order to create this dipole moment.

a) **[5 points]** How many electrons were moved to create this dipole moment (assume they were all moved the same distance)?

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<th>Dipole moment</th>
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<td>$p = 4.036 \times 10^{−30} \text{C} \cdot \text{m} = sq = (0.0021 \text{nm})q$</td>
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<td>$q = 1.92 \times 10^{−18} \text{C}$</td>
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<tr>
<td># electrons = $1.92 \times 10^{−18} \text{C} / 1.6 \times 10^{−19} \text{C/electron} = 12 \text{electrons}$</td>
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**# electrons =**

b) **[5 points]** Calculate the force on this dipole from an ion 10 nm away as shown below. The ion has a net positive charge of +2e. The dipole vector is as indicated.

Field on-axis far from dipole is same direction as dipole moment with magnitude

$$\frac{1}{4\pi\varepsilon_0} \frac{2p}{r^2} = \left(9 \times 10^9 \text{N} \cdot \text{m}^2 / \text{C}^2\right) \frac{2(4.036 \times 10^{−30} \text{C} \cdot \text{m})}{(10^{-8} \text{m})^3} = 7.26 \times 10^4 \text{N/C}$$

The force from the dipole on the ion is then this electric field multiplied by the charge 2e.

$$F = 2(1.6 \times 10^{−19} \text{C})(7.26 \times 10^4 \text{N/C}) = 2.32 \times 10^{−14} \text{N to the left. By Newton’s law the force on the dipole is equal and opposite, so directed to the right.}$$

**F =**

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c) **[5 points]** Now suppose the ion is directly below the dipole as shown below. Describe in words the force on the dipole and the motion of the dipole as a function of time due to its interaction with the ion. Assume the ion to be fixed in space.

The net force on the dipole is to the left, equal and opposite to the force on the ion, causing an initial acceleration to the left. There is also a torque about the center of mass that will accelerate the dipole clockwise, tending to align the dipole with the field from the ion. As the rotation starts, the net force will rotate toward the ion, eventually causing the dipole to accelerate toward the ion.
6) **[15 points total]** An insulating sphere of radius 1 cm has a uniform surface charge density of 1 µC/m². There is charge only on its surface, and nowhere else. Neglect any induced dipoles.

a) **[5 points]** Calculate the magnitude of the electric field at the surface of the sphere.

Can use Gauss’ law with a Gaussian sphere of same radius as insulating sphere. Charge enclosed is \(4\pi R^2 \sigma\). Total flux through surface is \(4\pi R^2 E\). Then Gauss’ law says that \(E = \frac{\sigma}{\varepsilon_0} = \left(10^{-6} \text{C}/\text{m}^2\right)/8.85 \times 10^{-12} \text{C}^2/\text{N} \cdot \text{m}^2 = 1.13 \times 10^5 \text{N}/\text{C}\)

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b) **[10 points]** Calculate the magnitude of the force on a 10 µC charge 2 cm from the center of the sphere.

Can use Gauss’ law again, but now with a Gaussian sphere of radius 2 cm. Charge enclosed is \(4\pi(0.01m)^2 \sigma\). Total flux through surface is \(4\pi(0.02m)^2 E\). Then Gauss’ law says that the electric field 2 cm from the center of the sphere is \(E = \frac{\sigma(0.01m)^2}{\varepsilon_0(0.02m)^2} = \left(10^{-6} \text{C}/\text{m}^2\right)/\left(4 \cdot 8.85 \times 10^{-12} \text{C}^2/\text{N} \cdot \text{m}^2\right) = 2.82 \times 10^4 \text{N}/\text{C}\)

The force on the 10µC charge is \(F = qE = \left(10^{-5} \text{C}\right)\left(2.82 \times 10^4 \text{N}/\text{C}\right) = 0.282\text{N}\)

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