Print your name and section clearly above. If you do not know your section number, write your TA’s name. Your final answer must be placed in the box provided. **You must show all your work to receive full credit.** If you only provide your final answer (in the box), and do not show your work, you will not receive very many points. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units, and also the direction of vectors.

You are allowed one handwritten 8½ x 11" sheet of notes and no other references. The exam lasts exactly 90 minutes.

**Problem 1:** _____ / 20

**Problem 2:** _____ / 20

**Problem 3:** _____ / 20

**Problem 4:** _____ / 20

**Problem 5:** _____ / 20

**TOTAL:** _____ / 100

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**Speed of light in vacuum:**
\[ c = 3 \times 10^8 \text{ m/s} \]

**Permittivity of free space**
\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2 \]

**Permeability of free space**
\[ \mu_0 = 4\pi \times 10^{-7} \text{ T m/A} \]

**Coulomb constant**
\[ k = 1/4\pi\varepsilon_0 = 9 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \]

**Magnitude of electron charge**
\[ e = 1.6 \times 10^{-19} \text{ C} \]

**Electron mass**
\[ m_e = 9.1 \times 10^{-31} \text{ kg} \]
1) [20 points, 4 points each] Multiple choice questions. Explain for full credit.

i) You move a point charge \( q = 1 \) \( \mu \)C from a very distant location to a distance \( d = 1 \) cm from a negative point charge of \( Q = -2 \) \( \mu \)C. How much work do you need to do?

\[
W = \boxed{\text{(value)} \quad \text{(units)}}
\]

Explanation/work:

ii) The voltage between an incandescent cathode (negative voltage) and the screen (positive voltage) of a television set is 22 kV. If we assume that the initial speed of an electron released by the cathode is zero, what is the final velocity when it hits the screen? Ignore any relativistic effect.

A) \( 8.8 \times 10^7 \) m/s
B) \( 5.2 \times 10^6 \) m/s
C) \( 9.1 \times 10^5 \) m/s
D) \( 1.1 \times 10^5 \) m/s
E) \( 2.4 \times 10^4 \) m/s

Explanation/work:
iii) The two spherical conductors in the figure are connected by a conducting wire and are in electrostatic equilibrium. Consider that the distance between the two spheres is much larger than the radii of the 2 spheres (neglect any inductance effect between the two spheres). Which of the following statements is true? Explain.

a) \( q_1 > q_2 \) and \( E_1 > E_2 \).
b) \( q_1 = q_2 \) and \( E_1 < E_2 \)
c) \( q_1 < q_2 \) and \( E_1 < E_2 \)
d) \( q_1 > q_2 \) and \( E_1 < E_2 \).

where \( q_1 \) and \( q_2 \) are the charges on the larger and smaller spheres, respectively, and \( E_1 \) and \( E_2 \) are the electric fields at the surface of the spheres.

iv) Which of the following statements is/are true? Explain.

A) A good ammeter should have a very small resistance.
B) A good ammeter should have a very large resistance.
C) A good voltmeter should have a very large resistance.
D) A good voltmeter should have a very small resistance.
E) A real battery maintains a potential difference at its terminal that is independent of any load resistor connected to its terminals.

Explanation/work:
v) Rank the equivalent resistance for each of the circuits below from lowest to highest assuming that all resistors are equal.

A) 3) = 4) < 1) < 2)
B) 1) < 4) < 3) < 2)
C) 3) = 4) < 2) < 1)
D) 1) < 3) < 2 < 4)
E) 1) = 2) < 3) = 4)

2) [20 points, 5 points each].
A cell membrane can be treated as a parallel-plate capacitor where the two plates are separated by 8 nm (1 nm = 10^{-9} m).
A) Consider a 175 μm x 175 μm square section of the cell wall and calculate its capacitance. Assume that the plasma membrane acts only as a spacer between the plates and consider the capacitor empty.

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C = | Value | Units |
| --- | --- | --- |
B) Now consider that the capacitance of the cell section is 350 pF (1 pF = 10^{-12} \text{ F}). Calculate the potential difference $\Delta V = V_{\text{in}} - V_{\text{out}}$ between the cell interior and exterior (resting potential) considering that outside the cell there are $N = 150$ millions more singly ionized sodium ions ($\text{Na}^+$, charge of an ion = $1.6 \times 10^{-19}$ C) than inside the cell.

**Explanation/work:**

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$\Delta V =$
C) During the potential change shown below (action potential), ions flow through the cell membrane in channels that behave as resistors between the plates of the capacitor. Assuming that the current through these channels is constant in time, calculate the total current flowing through the cell membrane during the depolarization phase indicated below (always assume that the cell capacitance is 350 pF).

[Hint: estimate the values you need for the calculation using the plot.]

\[
I = \text{ } \\
\begin{array}{|c|c|}
\hline
\text{Value} & \text{Units} \\
\hline
\end{array}
\]

Explanation/work:
D) During an action potential the cell can be modeled as an RC circuit (see figure below). The capacitance of the cell is always \( C = 350 \text{ pF} \). Consider that 2 ms correspond to 5 time constants of this circuit. What is the resistance of the cell? Consider that the ion channels together act as an equivalent resistor \( R \).

\[
\begin{array}{|c|c|}
\hline
\text{Value} & \text{Units} \\
\hline
R & \\
\hline
\end{array}
\]
3) Problem on Gauss’ Law [20 points, 5 each]
A sphere with radius R = 2 cm has a charge Q = 3mC uniformly distributed throughout its volume. The sphere is surrounded by a neutral concentric conducting shell with inner radius a = 4 cm and outer radius b = 5 cm.

A) What are the surface charge densities \( \sigma(a) \) and \( \sigma(b) \) on the inner and outer surfaces of the spherical conducting shell?

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B) What is the electric field as a function of the distance from the center of the system \( r \) in the region \( R < r < a \) between the sphere and the shell? Explain.

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C) In what region/regions of space the electric field is zero? Circle your answer.
   I. \( r < R \);
   II. \( R < r < a \)
   III. \( a < r < b \)
   IV. \( r > b \)

Explain:

D) Suppose now that the inner sphere is exchanged with a conducting sphere of equal radius \( R \) and charge \( Q \). Sketch the plot of the electric field and of the electric potential as a function of the distance from the centre of the system (sphere and shell).
4) [20 points, 10 each] Consider the RC Circuit in the figure below, where $\varepsilon = 10 \text{ V}$, $R = 2 \text{ M}\Omega$ and $C = 1 \mu\text{F}$. The capacitor is initially uncharged ($t=0$). When the switch $S$ is in the horizontal position the circuit is in charging mode.

A) What is the charge on the capacitor one time constant after the charging phase has started?

Explain/work:

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B) What is the energy stored in the capacitor after the charging process is complete ($t = \infty$)?

Explain/work:
5) Short problems and multiple choices on Magnetic Fields [20 points, 5 each]

A) What is the direction and magnitude of the magnetic field given the magnetic force of magnitude $F_m = 2 \times 10^{-4}$ N in the figure acting on the negative charge $q = -1$ C with velocity of $v = 1$ m/s?
1) To the left, 2 Gauss;
2) To the right, 4 Gauss;
3) Into the page, 2 Gauss;
4) Out of the page, 4 Gauss;
5) this situation is not possible.

Explanation/work:

B) You hold a segment of a wire of length $L = 1$ m vertically in a magnetic field of 3T that is directed horizontally toward you (see figure). If the segment carries an upward current $I = 1$ mA, what is the magnitude and the direction of the magnetic force on the wire?

Explanation/work:

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C) How long would an electron moving perpendicularly to the earth’s magnetic field take to complete one orbit? Assume that the magnetic field of the Earth is uniform and that its magnitude is 0.5 Gauss (1 Gauss = 10⁻⁴ T).

**Explanation/work:**

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D) The square loop in the figure in the plane xy carries a counterclockwise current and it is placed in a uniform magnetic field as shown. The loop is free to rotate around any axis. How will the loop respond to the magnetic field?

**Explanation/work:**