Planck’s constant  
\[ h = 6.626 \times 10^{-34} \text{ m}^2 \text{ kg/s} \]

Planck’s constant x velocity of light \( hc = 1240 \text{ eV nm} \)

Reduced Planck constant:  
\[ \hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34} \text{ J} \cdot \text{s} \]

Bohr radius \( a_o = 0.053 \text{ nm} \)

electron mass: \( 9.11 \times 10^{-31} \text{ kg} \)

electron mass squared: \( m_{\text{electron}} c^2 = 511,000 \text{ eV} \)

Speed of light in vacuum:  
\[ c = 3 \times 10^8 \text{ m/s} \]

Permittivity of free space  
\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2 \]

\[ k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \]

Permeability of free space  
\[ \mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \]

Magnitude of electron charge:  
\[ e = 1.6 \times 10^{-19} \text{ C} \]

<table>
<thead>
<tr>
<th>Problem 1:</th>
<th>_____ / 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 2:</td>
<td>_____ / 20</td>
</tr>
<tr>
<td>Problem 3:</td>
<td>_____ / 20</td>
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<tr>
<td>Problem 4:</td>
<td>_____ / 20</td>
</tr>
<tr>
<td>Problem 5:</td>
<td>_____ / 20</td>
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</table>

**TOTAL:** _____ / 100
1) [20 points, 4 points each]. Explain your reasoning for full credit. **Multiple choice/short answer questions.**

i) A wire of radius $R$ carries a uniform current density of $j=1 \text{ A/cm}^2$. The magnitude of the magnetic field inside the wire at distance $r$ from the center ($r<R$)

a. increases with $r$

b. decreases with $r$

c. is equal to zero

d. is a non-zero constant

e. none of the above

**Explanation/work:**

ii) A large coil of wire has a time-dependent current as shown below. A positive value in the plot means the current is flowing in the direction of the arrow, negative in the opposite direction. Plot the induced current in the small coil of wire at the center of the large coil. The vertical scale is arbitrary, but you need to show the correct sign and relative magnitudes.

**Explanation:**
iii) A helium-neon laser emits light of wavelength 632.8 nm \( (632.8 \times 10^{-9} \text{ m}) \) and has a power output of 1.5 milli-Watts \( (1.5 \times 10^{-3} \text{ W}) \). How many photons per second does this laser emit?

a. \( 1.34 \times 10^{14} \)
b. \( 2.56 \times 10^{15} \)
c. \( 3.09 \times 10^{15} \)
d. \( 4.22 \times 10^{15} \)
e. \( 4.78 \times 10^{15} \)

Work/explanation:

iv) Light is an electromagnetic wave. When visible light is linearly polarized, which of the following statements is/are true? Circle all that are true.

a. The electric field vector is parallel to the magnetic field vector.
b. The electric field vector is parallel to the direction of propagation.
c. The electric field vector is perpendicular to the direction of propagation, but the magnetic field vector may be in any direction.
d. The electric field vector is perpendicular to the direction of propagation, and the magnetic field vector is perpendicular to the electric field vector.
e. The direction of propagation is \( \vec{E} \times \vec{B} \).

Work/Explanation:

v) Potassium has a work function of 2.3 eV for photoelectric emission. Which of the following wavelengths is the longest wavelength for which photoemission occurs?

a. 400 nm
b. 450 nm
c. 500 nm
d. 540 nm
e. 600 nm

Work/Explanation:
2) [20 pts, 4 pts each] Short calculations.

i) The electric field in a capacitor with circular plates of radius \( R = 3 \) m varies linearly with time according to \( E = (2 \times 10^7 N/C \cdot s) t \) with time \( t \) in seconds. Calculate the conduction current flowing onto the positive plate of the capacitor.

\[
E = 2 \times 10^7 \text{ N/C} \cdot \text{s} \cdot t
\]

ii) The Sun radiation of intensity \( I = 1350 \text{ W/m}^2 \) is incident normally on a perfectly reflecting sail of a spacecraft of total mass \( m = 5 \times 10^4 \text{ kg} \) of area \( A = 10^4 \text{ m}^2 \). What is the acceleration in free space of the spacecraft?

\[
I = \text{Value} \quad \text{Units}
\]

\[
a = \text{Value} \quad \text{Units}
\]

iii) The electric field of an electromagnetic wave is \( \vec{E} = (25 \text{ V/m}) \sin(kx - \omega t) \hat{y} \). What is the amplitude and direction of magnetic field of the electromagnetic wave?

\[
\vec{B} = \text{Amplitude} \quad \text{Direction} \quad \text{Units}
\]
iv) A proton is accelerated through a potential difference of 10,000 V. How does its de Broglie wavelength compare to that of a football of mass 0.4 kg launched by Brett Favre (emeritus) at a velocity of 30 m/s?

The mass of the proton is $1.67 \times 10^{-27} \text{ kg} = 938.27 \text{ MeV/c}^2$ ($938.27 \times 10^6 \text{ eV/c}^2$).

<table>
<thead>
<tr>
<th>Ratio $\lambda_p / \lambda_{\text{football}}$ =</th>
</tr>
</thead>
</table>

$\psi(x)$

v) The probability density of a quantum particle is shown in the figure. It is zero except in the region $-10 \text{ nm} < x < 10 \text{ nm}$.

Calculate the probability that the particle will be found in the region $5 \text{ nm} < x < 10 \text{ nm}$

Probability =
3) [20 pts, 5 pts each]

Unpolarized light is incident on a series of polarizers as shown below. The transmission axis of each polarizer is indicated by a heavy line.

a) In what direction(s) does the electric field vector of the electromagnetic wave in region 1 point? Explain.

Direction(s) =

b) What is the ratio of the electric field amplitude in region 2 to that in region 1?

\[ \frac{E_2}{E_1} = \]
c) What percentage of the power incident on polarizer 3 is absorbed?

\[
\% \text{ power absorbed } = \boxed{\text{___}}
\]

d) Suppose that the middle polarizer rotates while the first and last polarizers stay fixed at the angles shown in the figure. Draw the intensity after the last polarizer (in Region 3) as a function of middle polarizer angle (defined as in the figure). **Explain**
4) [20 pts, 4 pts each]
A rectangular coil of 10,000 turns of total resistance 0.01Ω is below an infinitely long straight wire. The infinitely-long wire carries a constant current $I = 1000A$, producing a field $B = \mu_0 I / 2\pi r$ with $r$ the distance from the wire.

![Diagram of a rectangular coil and an infinitely long wire with a current of 1000A.] (Diagram not shown)

a) Assume that the 10,000-turn coil is narrow enough that the magnetic field from the long wire can be approximated as constant everywhere across the area of the coil. Calculate the magnetic flux through one turn of the coil when its center is 10 cm from the wire.

\[
\text{Flux} = \left( \frac{\mu_0 I}{2\pi r} \right) A
\]

b) You push the coil toward the long wire in the direction perpendicular to it at a constant speed of 1 cm/s. What is the direction of the current induced in the coil? Explain your reasoning in words.

Direction =
c) You are still pushing the coil toward the long wire at constant speed of 1 cm/s. Assume that the 10,000-turn coil is narrow enough that the magnetic field from the long wire can be approximated as constant everywhere across the area of the coil. By how much does the flux through one loop of the coil change when you move it from 10 cm to 9.9 cm?

\[ \Delta \Phi = \-box{\text{Value}} \times \text{Units} \]

d) You are still pushing the coil toward the long wire at constant speed of 1 cm/s. Calculate the EMF induced around the entire 10,000 turn coil when it is 10 cm from the long wire. For this part, use \( \Delta \Phi = 10^{-8} \text{Wb} \) as the answer for part c).

\[ \text{EMF} = \-box{\text{Value}} \times \text{Units} \]

e) With what force must you push the coil toward the infinitely long wire to keep it moving at 1 cm/s when it is 10 cm from the long wire? For this part, assume that the induced current in the coil is 0.2 A.

\[ \text{Force} = \-box{\text{Value}} \times \text{Units} \]
5) [20 pts, 4 pts each]

Four possible transitions for a Hydrogen atom are as follows:
(i) \( n_i = 2, n_f = 5 \);
(ii) \( n_i = 5, n_f = 3 \);
(iii) \( n_i = 7, n_f = 4 \);
(iv) \( n_i = 4, n_f = 7 \);

\( n_i \) is the quantum number of the initial state of the electron and \( n_f \) is the quantum number of the final state.

A) Label each transition as to whether a photon is emitted or absorbed. Explain

<table>
<thead>
<tr>
<th>Transition</th>
<th>Emit or Absorb</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_i = 2, n_f = 5 )</td>
<td>______________</td>
</tr>
<tr>
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<tr>
<td>( n_i = 4, n_f = 7 )</td>
<td>______________</td>
</tr>
</tbody>
</table>

B) In which transition(s) does the photon have the longest wavelength? Circle the correct answers(s). Explain

\( n_i = 2, n_f = 5 \)
\( n_i = 5, n_f = 3 \)
\( n_i = 7, n_f = 4 \)
\( n_i = 4, n_f = 7 \)

C) In which transition(s) does the atom gain the most energy? Circle the correct answer(s). Explain

\( n_i = 2, n_f = 5 \)
\( n_i = 5, n_f = 3 \)
\( n_i = 7, n_f = 4 \)
\( n_i = 4, n_f = 7 \)
D) In which transition(s) does the atom lose energy? Circle the correct answer(s).

Explain

\[
\begin{array}{|c|}
\hline
n_i = 2, n_f = 5 \\
n_i = 5, n_f = 3 \\
n_i = 7, n_f = 4 \\
n_i = 4, n_f = 7 \\
\hline
\end{array}
\]

E) Calculate the velocity of the electron in the \( n = 2 \) state.

\[
V = 
\begin{array}{|c|c|}
\hline
\text{Value} & \text{Units} \\
\hline
\end{array}
\]