From last time...

Faraday: \( E = -\frac{d}{dt}\Phi_B \)

Lenz: induced currents oppose change in flux

Faraday’s law

\[ E = \oint E \cdot ds = -\frac{d}{dt}\Phi_B = -\frac{d}{dt}\int B \cdot dA \]

EMF around loop
Magnetic flux through surface bounded by path
EMF no longer zero around closed loop

Lenz’s law

- Induced current produces a magnetic field.
- Interacts with bar magnet just as another bar magnet
- Lenz’s law
  - Induced current generates a magnetic field that tries to cancel the change in the flux.
  - Here flux through loop due to bar magnet is increasing. Induced current produces flux to left.
  - Force on bar magnet is to left.

Question

Which way is the magnet moving if it is inducing a current as shown?

A. Up
B. Down

Current creates flux up. This must be opposing increase in flux down. So magnet must be falling down

Question

As current is increasing in the solenoid, what direction will current be induced in ring?

A. Same as solenoid
B. Opposite of solenoid
C. No current

Question

What is the direction of the current induced in the can?

A. Into page
B. Out of page
C. CW
D. CCW
E. None of the above

What is the direction of the force on the can?
A copper guillotine blade falls toward a victim. It enters field from strong magnets on way down. 
What is the direction of the current induced in the loop (blade) as it enters field?
A. CW  
B. CCW  
C. Depends on field direction

What is the direction of the magnetic force on the blade?
A. Up  
B. Down  
C. Depends on field direction.

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**Eddy current braking**

ICE 3 near the Oberhaid-er Wald Tunnel on the Cologne-Frankfurt high-speed rail line

Shinkansen

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**Back to basics: solenoid flux**

- Uniform field inside \( B_{\text{solenoid}} = \frac{\mu_0 N I}{I} \)
- Change current \( \rightarrow \) change flux

**Inductance: a general result**

- Flux = (Inductance) \( \times \) (Current) \( \Phi = LI \)
- Change in Flux  
  = (Inductance) \( \times \) (Change in Current)  
  \( \Delta \Phi = L \Delta I \)
- Faraday's law:  
  \( EMF = \frac{d\Phi}{dt} = -L \frac{dI}{dt} \)
### Changing current in inductor

- **Ideal inductor:**
  - Inductance, but zero resistance
  - Constant current: \( \mathcal{E} = 0 \)
  - Changing current: \( \mathcal{E} \neq 0 \)

- Sign of \( \mathcal{E} \) across ideal inductor
  - Opposes change in current.
  - Inductor fights to keep a constant current

### Energy stored in ideal inductor

- **Constant current (uniform charge motion)**
  - No work required to move charge through inductor

- **Increasing current:**
  - Work \( -\Delta V_{\text{EMF}} = -\Delta V_{\text{ind}} \) required to move charge across induced EMF
  - \( dW = -\Delta V_{\text{EMF}} = L \frac{di}{dt} dt = L i \frac{di}{dt} \)
  - Total work

\[
W = \int L i \frac{di}{dt} dt = \int L i I dt = L I dI
\]

\[
\frac{dW}{dt} = L I dI
\]

\[
W = L I dI = \frac{1}{2} L I^2
\]

### Magnetic energy density

- **Energy stored in inductor** \( U_L = \frac{1}{2} L I^2 \)
- **Solenoid inductance** \( L_{\text{solenoid}} = \mu_0 \frac{N^2 A}{\ell} \)
- **Energy stored in solenoid** \( U_{\text{solenoid}} = \frac{1}{2} \mu_0 \frac{N^2 A}{\ell} \frac{I}{2} \mu_0 \)

### Question

A solenoid is stretched to twice its length while keeping the same current and same cross-sectional area. The stored energy

- **A. Increases**
  - \( B_{\text{solenoid}} = \frac{N I}{\ell} \)
  - \( B \) decreases by 2
  - \( \frac{U_{\text{solenoid}}}{Al} = \frac{B_{\text{solenoid}}^2}{2\mu_0} \)
  - Energy density decr by 4
  - Volume increases by 2

- **B. Decreases**
- **C. Stays the same**