Exam 3 covers
Lecture, Readings, Discussion, HW, Lab

Exam 3 is Tue. Nov. 25, 5:30-7 pm, 2103 Chamberlin (here)

Biot-Savart Law - currents produce magnetic fields
Ampere's law - shortcut to determining mag. fields from currents.
Magnetic flux, Faraday effect, Lenz' law
Electromagnetic waves:
- Wavelength, freq, speed
- E&B fields, intensity, power, rad. pressure, Poynting vec
- Polarization
Modern Physics (quantum mechanics)
- Photons & photoelectric effect
- Bohr atom: Energy levels, absorbing & emitting photons

Law of Biot-Savart
• Each element of current produces a contribution to the magnetic field.

\[
\frac{dB}{dr} = \frac{\mu_0}{4\pi} \frac{Ids \times \hat{r}}{r^2}
\]

Field from a circular loop
• Each current element produces \( dB \)
• All contributions add as vectors
• Along axis, all components cancel except for \( x \)-comp

Magnetic field from loop
Which of these graphs best represents the magnetic field on the axis of the loop?

Quick Quiz
A long wire of radius \( R \) carries a current uniformly through its cross-section. How does the \( B \)-field at \( R \) compare to that at \( R/2 \)?

- \( B_R = B_{R/2} \)
- \( B_R = 2B_{R/2} \)
- \( B_R = B_{R/2} \)
- \( B_R = 4B_{R/2} \)

\[
\int B \cdot ds = \mu_0 I
\]
- Ampere's law
Ampere’s law for the solenoid

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu I_{\text{through}} = \mu L \]

Time-dependent fields

- Faraday effect
  - Changing magnetic flux induces EMF
- Displacement current
  - Changing electric flux induces \( B \)-field
- Electromagnetic waves
  - \( E \)-field and \( B \)-field both changing in time
  - Each ‘induces’ the other
  - Propagating \( E \)- and \( B \)-fields

Magnetic flux

- Magnetic flux is defined exactly as electric flux
- (Component of \( B \) \perp \) surface) x (Area element)

\[ \Phi_B = \int \mathbf{B} \cdot d\mathbf{A} \]

Faraday’s law

\[ \mathcal{E} = -\frac{d}{dt} \Phi_B \]

If path along conducting loop, induces current \( I = \frac{\mathcal{E}}{R} \)

Quick quiz

Which of these conducting loops will have currents flowing in them?

A. Constant I  
B. Constant \( I(t) \) increases  
C. Constant I  
D. Constant \( I \)  
E. Constant V  
F. Constant V

Quick Quiz

A person moves a conducting loop with constant velocity away from a wire as shown.

The wire has a constant current

What is the direction of force on the loop from the wire?

A. Left  
B. Right  
C. Up  
D. Down  
E. Into page  
F. Out of page

\[ F_{\text{top}} = I_{\text{loop}} L_{\text{top}} x B_{\text{top}} = ILB \hat{y} \]

\[ F_{\text{bottom}} = I_{\text{loop}} L_{\text{bottom}} x B_{\text{bottom}} = -ILB \hat{y} \]

\[ F_{\text{left}} + F_{\text{left}} = 0 \] (cancel)

\( B_{\text{top}} > B_{\text{bottom}} \rightarrow \) Force is up
Quick Quiz
A square loop rotates at frequency $f$ in a 1T uniform magnetic field as shown. Which graph best represents the induced current (CW current is positive)?

- A.
- B.
- C.
- D.

Inductance: a general result

- Flux = (Inductance) X (Current)
  $$\Phi = LI$$

- Change in Flux
  $$\Delta \Phi = L \Delta I$$

- Faraday’s law:
  $$EMF = \frac{d\Phi}{dt} = -L \frac{dl}{dt}$$

Energy stored in ideal inductor

- Constant current (uniform charge motion)
  - No work required to move charge through inductor

- Increasing current:
  - Work $\Delta V_q = \Delta V_{Itd}$ required to move charge across induced EMF
  - $dW = \Delta V_{Itd} = L \frac{dl}{dt}ldl = Lldl$
  - Total work
    $$W = \int Lldl = \frac{1}{2}LI^2$$ Energy stored in inductor

RL circuits

$$\frac{dl}{dt} = -\frac{R}{L}l \Rightarrow dl = -\frac{R}{L} dt$$

- Current decreases in time
  - Slow for large inductance
    • (inductor fights hard, tries to keep constant current)
  - Slow for small resistance
    • (no inductor EMF needed to drive current)
  - Time constant $\tau = R / L$

Electromagnetic waves

In empty space: sinusoidal wave propagating along x with velocity $c$

- $E = E_{max} \cos (kx - \omega t)$
- $B = B_{max} \cos (kx - \omega t)$
- $\omega = \frac{2\pi}{\lambda}$

- $E$ and $B$ are perpendicular oscillating vectors
- The direction of propagation is perpendicular to $E$ and $B$

Quick Quiz on EM waves

A planar electromagnetic wave is propagating through space. Its electric field vector is given by $E = E_{max} \cos(kz - \omega t)$ as its magnetic field vector is:

1. $B = B_{max} \cos(kz - \omega t)$
2. $B = B_{max} \cos(ky - \omega t)$
3. $B = B_{max} \cos(kx - \omega t)$
4. $B = B_{max} \cos(ky - \omega t)$
EM wave transports energy at its propagation speed. Instantaneous power/area = |Poynting vector| = $|\mathbf{E} \times \mathbf{B}| / \mu_0 c$

Intensity = Average power/area = 

$$I = \frac{1}{2} \left( \frac{E R_0}{2} \right)^2 = \frac{c E_0^2}{2} = \frac{c E_0^2}{2}$$

Spherical wave: $I = P_{\text{source}} / 4 \pi r^2$

Radiation Pressure

EM wave incident on surface exerts a radiation pressure $p_{\text{rad}}$ (force/area) proportional to intensity $I$.

- Perfectly absorbing (black) surface: $P_{\text{rad}} = I / c$
- Perfectly reflecting (mirror) surface: $P_{\text{rad}} = 2I / c$

Resulting force = (radiation pressure) x (area)

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Power and Intensity

- **EM wave transports energy at its propagation speed.**
- **Instantaneous power/area = |Poynting vector| = $|\mathbf{E} \times \mathbf{B}| / \mu_0 c$.**
- **Intensity**: $I = \frac{1}{2} \left( \frac{E R_0}{2} \right)^2 = \frac{c E_0^2}{2} = \frac{c E_0^2}{2}$
  - Spherical wave: $I = P_{\text{source}} / 4 \pi r^2$

Radiation Pressure

- EM wave incident on surface exerts a radiation pressure $p_{\text{rad}}$ (force/area) proportional to intensity $I$.
  - Perfectly absorbing (black) surface: $P_{\text{rad}} = I / c$
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Quantum Mechanics

- **Light comes in discrete units:**
  - **Photon energy**: $E_{\text{photon}} = hf = hc / \lambda$.
  - **Demonstrated by Photoelectric Effect**
    - Photon of energy $hf$ collides with electron in metal
    - Transfers some or all of $hf$ to electron
    - If $hf > \phi$ (work function) electron escapes

Electron ejected only if $hf > \phi$

Minimum photon energy $E_{\text{photon}} = hf$ required

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Photon properties of light

- **Photon of frequency $f$ has energy** $hf$
  - $E_{\text{photon}} = hf = hc / \lambda$
  - $h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$
  - $c = 3.00 \times 10^8 \text{ m/s}$
  - $hc = 1240 \text{ eV} \cdot \text{nm}$

- **Red light made of ONLY red photons**
  - The intensity of the beam can be increased by increasing the number of photons/second.
  - $(\text{#Photons/second}) \cdot \text{Energy/photon} = \text{energy/second} = \text{power}$

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Photon energy

What is the energy of a photon of red light ($\lambda = 635 \text{ nm}$)?

- A. 0.5 eV
- B. 1.0 eV
- C. 2.0 eV
- D. 3.0 eV

**C. 2.0 eV**

$$E = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{635 \text{ nm}} = 1.95 \text{ eV}$$

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Bohr’s model of Hydrogen atom

- **Planetary model**: Circular orbits of electrons around proton.
- **Quantization**
  - Discrete orbit radii allowed: $r_n = \frac{n^2 a_0}{\hbar}$
  - Discrete electron energies: $E_n = -13.6 / n^2 \text{ eV}$
  - Each quantum state labeled by quantum number $n$

How did he get this?

Quantization of circular orbit angular mom: $|L| = L \times p = mv r = nh$

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Polarization

- **Linear polarization**:
  - E-field oscillates in fixed plane of polarization
- **Circular polarization**:
  - E-field rotates at constant magnitude
- **Linear polarizer**:
  - Transmits component of E-field parallel to transmission axis
  - Absorbs component perpendicular to transmission axis
Consequences of Bohr model

- Electron can make transitions between quantum states.
- Atom loses energy: photon emitted
  \[ E_{\text{photon}} = E_{\text{atom}}(n_{\text{initial}}) - E_{\text{atom}}(n_{\text{final}}) \]
- Photon absorbed: atom gains energy:
  \[ E_{\text{photon}} = E_{\text{atom}}(n_{\text{final}}) - E_{\text{atom}}(n_{\text{initial}}) \]

\[ E_{\text{atom}}(n) = \frac{13.6 \text{ eV}}{n^2} \]

Ephoton = hc

Wavelength is smaller for larger jump!

Spectral Question

Compare the wavelength of a photon produced from a transition from n=3 to n=1 with that of a photon produced from a transition n=2 to n=1.

- A. \( \lambda_31 < \lambda_{21} \)
- B. \( \lambda_31 = \lambda_{21} \)
- C. \( \lambda_31 > \lambda_{21} \)

E31 > E21 so \( \lambda_31 < \lambda_{21} \)

Ephoton = hc

Question

This quantum system (not a hydrogen atom) has energy levels as shown. Which photon could possibly be absorbed by this system?

- A. 1240 nm
- B. 413 nm
- C. 310 nm
- D. 248 nm