From last time…

Continuous charge distributions

Motion of charged particles

No Discussion Wed. Oct 1 — Exam grading

The story so far…

- Charges
- Forces
- Electric fields
- Electric field lines

Now something a little different

Electric flux: ‘flow’ of electric field through a surface

Electric Flux

- Electric flux $\Phi_E$ through a surface:
  (component of E-field $\perp$ surface) $\times$ (surface area)
- Proportional to # E-field lines penetrating surface

Why perpendicular component?

- Suppose surface make angle $\theta$ surface normal
- $\Phi_E = EA \cos \theta$
- $\Phi_E = 0$ if $E \parallel A$
- $\Phi_E = EA$ (max) if $E \perp A$
- Flux SI units are $\text{N} \cdot \text{m}^2/\text{C}$

Total flux

- $E$ not constant
  - add up small areas where it is constant
- Surface not flat
  - add up small areas where it is $\sim$ flat

Quick quiz

Two spheres of radius $R$ and $2R$ are centered on the positive charges of the same value $q$. The electric flux through the spheres compare as

- $\text{A)} \ \Phi(R) = \Phi(2R)$
- $\text{B)} \ \Phi(R) = 2\Phi(2R)$
- $\text{C)} \ \Phi(R) = (1/2)\Phi(2R)$
- $\text{D)} \ \Phi(R) = 4\Phi(2R)$
- $\text{E)} \ \Phi(R) = (1/4)\Phi(2R)$
Flux through spherical surface

- Closed spherical surface, positive point charge at center
- E-field \( \perp \) surface, directed outward
- E-field magnitude on sphere:
  \[ E = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2} \]
- Net flux through surface:
  \[ \Phi = \oint \vec{E} \cdot d\vec{A} = \oint E dA = E \oint dA = \oint \left( \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2} \right) 4\pi R^2 = \frac{q}{\varepsilon_0} \]

Quick quiz

Two spheres of the same size enclose different positive charges, \( q \) and \( 2q \). The flux through these spheres compare as

A) Flux(A) = Flux(B)
B) Flux(A) = 2Flux(B)
C) Flux(A) = \( \frac{1}{2} \)Flux(B)
D) Flux(A) = 4Flux(B)
E) Flux(A) = \( \frac{1}{4} \)Flux(B)

Why Gauss’ law?

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0} \]

Is a relation between charge and electric field.
Can be used to calculate electric field from charge distribution.

Using Gauss’ law

- Use to find E-field from known charge distribution.
- **Step 1:** Determine direction of E-field
  - Field direction: radially out from charge
  - Gaussian surface:
    - Sphere of radius \( r \)
    - Surface area where \( \vec{E} \cdot d\vec{A} = 0 \):
      - \( 4\pi r^2 \)
    - Value of \( \vec{E} \cdot d\vec{A} \) on this area:
      - \( E \)
  - Flux thru Gaussian surface:
    - \( E \cdot 4\pi r^2 \)
  - Charge enclosed:
    - \( Q \)
    - Gauss’ law: \( E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \)

Field outside uniformly-charged sphere

- Field direction: radially out from charge
- Gaussian surface:
  - Sphere of radius \( r \)
  - Surface area where \( \vec{E} \cdot d\vec{A} = 0 \):
    - \( 4\pi r^2 \)
  - Value of \( \vec{E} \cdot d\vec{A} \) on this area:
    - \( E \)
  - Flux thru Gaussian surface:
    - \( E \cdot 4\pi r^2 \)
  - Charge enclosed:
    - \( Q \)
    - Gauss’ law: \( E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \)
Quick Quiz
Which Gaussian surface could be used to calculate E-field from infinitely long line of charge?

A.  
B.  
C.  
D.  
None of these

Quick Quiz
Which Gaussian surface could be used to calculate E-field from infinite sheet of charge?

A.  
B.  
C.  
D.  
Any of these

E-field from line of charge
- Field direction: radially out from line charge
- Gaussian surface:
  - Cylinder of radius r
  - Area where \( E \cdot dA \neq 0 \):
    - \( 2 \pi r L \)
  - Value of \( E \cdot dA \) on this area:
    - \( E \)
  - Flux thru Gaussian surface:
    - \( E \cdot 2 \pi r L \)
  - Charge enclosed:
    - \( \lambda L \)

  - Gauss’ law: \( E \cdot 2 \pi r L = \lambda L / \varepsilon_0 \Rightarrow E = \frac{\lambda}{2 \pi \varepsilon_0 r} \)

Field from infinite plane of charge
- Field direction: perpendicular to plane
- Gaussian surface:
  - Cylinder of radius r
  - Area where \( E \cdot dA \neq 0 \):
    - \( 2 \pi r^2 \)
  - Value of \( E \cdot dA \) on this area:
    - \( E \)
  - Flux thru Gaussian surface:
    - \( E \cdot 2 \pi r^2 \)
  - Charge enclosed:
    - \( \eta \pi r^2 \)

  - Gauss’ law: \( E \cdot 2 \pi r^2 = \eta \pi r^2 / \varepsilon_0 \Rightarrow E = \frac{\eta}{2 \varepsilon_0 r} \)

Conductor in Electrostatic Equilibrium
In a conductor in electrostatic equilibrium there is no net motion of charge
- \( E = 0 \) everywhere inside the conductor
- Conductor slab in an external field \( E \); if \( E = 0 \) free electrons would be accelerated
- These electrons would not be in equilibrium
- When the external field is applied, the electrons redistribute until they generate a field in the conductor that exactly cancels the applied field.

Charge distribution on conductor
- What charge is induced on sphere to make zero electric field?
Conductors: charge on surface only

- Choose a gaussian surface inside (as close to the surface as desired)
- There is no net flux through the gaussian surface (since $E=0$)
- Any net charge must reside on the surface (cannot be inside!)

E-Field Magnitude and Direction

E-field always $\perp$ surface:
- Parallel component of $E$ would put force on charges
- Charges would accelerate
- This is not equilibrium
- Apply Gauss’s law at surface

$\varepsilon_0 E = \frac{Q_{encl}}{A}$

Summary of conductors

- $E = 0$ everywhere inside a conductor
- Charge in conductor is only on the surface
- $E \parallel$ surface of conductor