Chapter 2: Motion in One Dimension
The first exam will be 5:45pm-7pm on Tuesday, September 29

Please let us know as soon as possible (and certainly by 9/15) if

(1) you are enrolled in another class that meets at that time, or

(2) you have a McBurney VISA and need accommodation for a disability

so that we can make the appropriate accommodations.
Displacement: change in position

\[ \Delta x = x_f - x_i \]
Average velocity and speed

Average velocity

\[
\text{Average velocity} = \frac{\text{net displacement}}{\text{total time}} = \frac{s}{\Delta t}
\]

Average speed

\[
\text{Average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{s}{\Delta t}
\]
Distance and displacement of a dog (p 29)

- Time 0: $x_0 = 0$
- Time 2: $x_2 = 5$ ft
- Time 1: $x_1 = 20$ ft

Total distance traveled = 35 ft
Net displacement = 5 ft
Average velocity and speed

Average velocity = \frac{\text{net displacement}}{\text{total time elapsed}}

Average speed = \frac{\text{total distance traveled}}{\text{total time elapsed}}

Velocity includes *direction*.

(It is fundamental for the laws of motion; speed is not.)
The average velocity for a time interval \((t_1, t_2)\) is the slope of the straight line connecting the points \((t_1, x_1)\) and \((t_2, x_2)\) on a graph of \(x\) versus \(t\).

\[
\frac{\Delta x}{\Delta t} = \text{slope} = v_{av} \times
\]

The average velocity of particle between times \(t_1\) and \(t_2\) is greater than the average velocity of particle between times \(t_1\) and \(t_2\).
Average velocity and average speed of a dog (p 31)

Average velocity = \((\text{net displacement})/\text{(total time)}\) = \((5 \text{ m})/(2.5 \text{ s})\) = 2 m/s

Average speed = \((\text{total distance traveled})/\text{(total time)}\) = \((35 \text{ m})/(2.5 \text{ s})\) = 14 m/s
The instantaneous velocity along $x$, $v_x$, is the limit of the ratio $\Delta x/\Delta t$ as $\Delta t$ approaches zero.

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}.$$
Approaching the limit $\Delta t \to 0$ to find the instantaneous velocity.
Instantaneous velocity at a given time is the derivative of the x-versus-t curve.
Displacement versus time when \( x = 5t^2 \), with \( x \) in meters and \( t \) in seconds (p 34).

velocity = derivative of displacement
\[
\frac{dx}{dt} = 10 \, t \, \text{m/s}.
\]
Acceleration

Average acceleration along $x$

\[
\text{net change in velocity along } x = \frac{\Delta v_x}{\Delta t}.
\]

Instantaneous acceleration along $x$:

\[
a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}.
\]

= derivative of $v_x$-versus-$t$ curve.
Instantaneous acceleration along x is first derivative of $v_x$ with respect to time t, and second derivative of x with respect to t:

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}$$

$$= \frac{d v_x}{d t}$$

$$= \frac{d^2 x}{d t^2}.$$
A skydiver jumps out

A skydiver is falling straight down, along the negative y direction. During the initial part of the fall, her speed increases from 16 m/s to 28 m/s in 1.5 s. Which of the following is correct?

1. \(v > 0, a > 0\)
2. \(v > 0, a < 0\)
3. \(v < 0, a > 0\)
4. \(v < 0, a < 0\)
A skydiver jumps out

A skydiver is falling straight down, along the negative y direction. During the initial part of the fall, her speed increases from 16 m/s to 28 m/s in 1.5 s. Which of the following is correct?

1. \( v>0, a>0 \)
2. \( v>0, a<0 \)
3. \( v<0, a>0 \)
4. \( v<0, a<0 \)  \hspace{1cm} \text{correct}

\[
\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{-28 \text{ m/s} - (-16 \text{ m/s})}{1.5 \text{ s}} = -8 \text{ m/s}^2
\]
Parachute opens

During a later part of the fall, after the parachute has opened, her speed decreases from 48 m/s to 26 m/s in 11 s. Which of the following is correct?

1. $v>0$, $a>0$
2. $v>0$, $a<0$
3. $v<0$, $a>0$
4. $v<0$, $a<0$
During a later part of the fall, after the parachute has opened, her speed decreases from 48 m/s to 26 m/s in 11 s. Which of the following is correct?

1. \( v > 0, a > 0 \)
2. \( v > 0, a < 0 \)
3. \( v < 0, a > 0 \)  \( \text{correct} \)
4. \( v < 0, a < 0 \)

If speed is increasing, \( v \) and \( a \) are in same direction.
If speed is decreasing, \( v \) and \( a \) are in opposite directions.

\[
\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{-26 \text{ m/s} - (-48 \text{ m/s})}{11 \text{ s}} = 2 \text{ m/s}^2
\]
Special case: constant acceleration
If acceleration is constant, velocity increases linearly with time.
Motion with constant acceleration

When acceleration is constant, the average velocity is the velocity halfway through the time interval

(THIS IS NOT ALWAYS TRUE IF THE ACCELERATION IS NOT CONSTANT!)
Motion with constant acceleration: find velocity and position versus time

Consider a particle that at time $t_0$ has velocity along $x$ of $v_{x0}$ and is at position $x_0$. If the acceleration $a_x$ is constant, the velocity at time $t$ is:

$$v_x(t) = v_{x0} + a_x(t-t_0)$$

and the particle’s position at time $t$, $x(t)$, is

$$x(t) = x_0 + v_{x0}(t-t_0) + \frac{1}{2}a_x(t-t_0)^2.$$
Motion with constant acceleration: find velocity versus position

Recall velocity is: \[ v_x(t) = v_{x0} + a_x(t-t_0) \]

\[ \Rightarrow t-t_0 = (v_x(t)-v_{x0})/a_x. \]

Since the particle's position at time \( t \), \( x(t) \), is

\[ x(t) = x_0 + v_{x0}(t-t_0) + (1/2)a_x(t-t_0)^2, \]

this means

\[ x(t) = x_0 + v_{x0}(v_x(t)-v_{x0})/a_x + (1/2)a_x((v_x(t)-v_{x0})/a_x)^2 \]

\[ x(t)-x_0 = v_{x0}(v_x(t)-v_{x0})/a_x + (1/2)(v_x(t)-v_{x0})^2/a_x. \]

Multiply both sides of equation by \( 2a_x \) and simplifying yields

\[ 2a_x(x(t)-x_0) = v_x(t)^2 - v_{x0}^2. \]
Free fall: If no air, all objects in free-fall with same initial velocity move identically.

acceleration due to gravity
= \( g \approx 9.81 \text{ m/s}^2 \approx 32 \text{ ft/s}^2 \), directed downwards (same for all objects)

The acceleration results in a widening of the spaces between images.
What is $x_f$? (Ex. 2-10, stopping distance)

$v_0 = 15 \text{ m/s}$

$a_x = -5.0 \text{ m/s}^2$

$x_0 = 0$

$v_f = 0$

$x_f$
What is $x_f$? (Ex. 2-10, stopping distance)

Let $\Delta t$ be the time it takes the car to come to a stop:

$\Delta t = \Delta v/a_x = (-15 \text{ m/s})/(-5.0 \text{ m/s}^2))$.

Distance traveled in 3.0 s = $v_0\Delta t + (1/2) a_x(\Delta t)^2$

$= (15 \text{ m/s})(3.0 \text{ s}) + (1/2) (-5.0 \text{ m/s}^2)(3.0 \text{ s})^2$

$= 45 \text{ m} - 22.5 \text{ m}$

$= 23 \text{ m}$ (since we are keeping 2 significant figures)
Example 2-13, The Flying Cap (p 44)

A cap is tossed in the air. It starts at height $y_0$ with a velocity $v_{0y} = 14.7 \text{ m/s}$ upward.

How long does it take for the cap to reach its highest point?

$\mathbf{y} = -9.81 \text{ m/s}^2$

How high does the cap go?
Example 2-13, The Flying Cap (p 44)

A cap is tossed in the air. It starts at height $y_0$ with a velocity $v_{0y}=14.7 \text{ m/s}$ upward.

How long does it take for the cap to reach its highest point?

Highest point is when the velocity is zero.

$$v_y(t) = v_{0y} - a_y t_f \implies t_f = \frac{(v_y(t)-v_{0y})}{a_y}$$

$$= \frac{(14.7 \text{ m/s})}{(-9.81 \text{ m/s}^2)} = 1.50 \text{ s}$$

How high does the cap go?

The cap goes distance $v_{0y} t_f + \frac{1}{2}a_y t_f^2$

$$= (14.7 \text{ m/s})(1.50 \text{ s}) + \frac{1}{2}(-9.81 \text{ m/s}^2)(1.50 \text{ s})^2$$

$$= 11.0 \text{ m}.$$
Example 2-13; Height and velocity of the flying cap

![Graph showing height and velocity over time]

- **Height**
  - Y-axis: $y(t)$, m
  - X-axis: $t$, s
  - Graph shows a parabolic shape with a peak at $t = 1.5$ s.

- **Velocity**
  - Y-axis: $v_x(t)$, m/s
  - X-axis: $t$, s
  - Graph shows a linear decrease from $v_x(t) = 15$ m/s at $t = 0$ s to $v_x(t) = 0$ m/s at $t = 3$ s.
Getting velocity from acceleration by integration

\[ a_x(t) = \frac{d v_x(t)}{d t} \]

\[ \Rightarrow v_x(t) = \int dt \ a_x(t) \]

(The only tricky part is putting in the limits of integration.)
Getting position from velocity by integration

\[ v_x(t) = \frac{dx(t)}{dt} \]

\[ \Rightarrow x(t) = \int dt \ v_x(t) \]

(The only tricky part is putting in the limits of integration.)
The position is the integral of the velocity (the limit of the sum of areas of the rectangles).
The displacement is given by the area under the $v_x$-versus-$t$ curve.

The area under the curve is given by the integral:

$$\int v_x(t) \, dt$$

And the velocity is:

$$v_x = v_{0x} + at$$
Problem 2.31  

Reginald and Josie go the same distance. What is the relationship between $v_j$ and $v_{\text{max}}$?
Problem 2.31  Reginald and Josie go the same distance. What is relationship between $v_J$ and $v_{\text{max}}$?

Distance traveled is area under $v$-versus-$t$ curve.

For Josie, this area is $v_J t_f$.

For Reginald, this area is $t_{\text{max}} v_{\text{max}}/2 + (t_f - t_{\text{max}}) v_{\text{max}}/2$.

The areas are equal when $v_{\text{max}} = 2v_J$. 
Problem 2.106  write algebraic expressions for $x(t)$, $v_x(t)$, and $a_x(t)$
Problem 2.106  write algebraic expressions for $x(t)$, $v_x(t)$, and $a_x(t)$

**Reading from graph,** $v_x(t) = v_0 + At$, with $v_0 = 50$ m/s and $A = -10$ m/s$^2$.

**Acceleration is derivative of velocity:** $a_x(t) = A = -10$ m/s$^2$.

**Position is integral of velocity:** $x(t) = x(0) + v_0 t + \frac{1}{2} At^2 = 50t - 5t^2$. 
Motion with time-varying acceleration

An object undergoes a linearly increasing acceleration $a(t) = 10 t \text{ m/s}^2$ along a straight line. It starts at the origin $x(0)=0$ with a velocity of 2 m/s. Where is its position at time $t=4$ seconds?
Motion with time-varying acceleration

An object undergoes a linearly increasing acceleration \( a(t) = 10t \text{ m/s}^2 \) along a straight line. It starts at the origin \( x(0)=0 \) with a velocity of 2 m/s. What is its position at time \( t=4 \) seconds?

In this problem the acceleration is NOT constant. So must do the integrals explicitly.

\[
a(t) = 10t
\]

\[
v(t) = \int a(t)dt = 10 \frac{t^2}{2} + v_0 = 5t^2 + v_0
\]

\[
x(t) = \int v(t)dt = 5 \frac{t^3}{3} + v_0t + x_0
\]

\[
x(4) = 5 \times \frac{64}{3} + 2 \times 4 + 0 = 114.7 \text{ m}
\]
Which graph of $v$ versus $t$ best describes the motion of a particle with positive velocity and negative acceleration?
Which graph of $v$ versus $t$ best describes the motion of a particle with positive velocity and negative acceleration?
A car accelerates uniformly from a velocity of 10 km/h to 30 km/h in one minute. Which graph best describes the motion of the car?
A car accelerates uniformly from a velocity of 10 km/h to 30 km/h in one minute. Which graph best describes the motion of the car?
The acceleration of a vehicle is given by \( a(t) = At \) where \( A \) is a constant. Its velocity as a function of time is \( (v_o \) is a constant)

- A. \( v(t) = \frac{A}{2} t^2 + v_o \)
- B. \( v(t) = At^2 + v_o \)
- C. \( v(t) = At + v_o \)
- D. \( v(t) = \frac{A}{3} t^3 + v_o \)
- E. None of the above answers is correct.
The acceleration of a vehicle is given by \( a(t) = At \) where \( A \) is a constant. Its velocity as a function of time is \( (v_o \) is a constant)

- A. \( v(t) = \frac{A}{2} t^2 + v_o \)
- B. \( v(t) = At^2 + v_o \)
- C. \( v(t) = At + v_o \)
- D. \( v(t) = \frac{A}{3} t^3 + v_o \)
- E. None of the above answers is correct.
Concept question: velocity and acceleration

A ball is thrown vertically upward. At the very top of its trajectory, which of the following statements is true:

a. velocity is zero and acceleration is zero
b. velocity is not zero and acceleration is zero
c. velocity is zero and acceleration is not zero
d. velocity is not zero and acceleration is not zero
Concept question: velocity and acceleration

A ball is thrown vertically upward. At the very top of its trajectory, which of the following statements is true:

a. velocity is zero and acceleration is zero
b. velocity is not zero and acceleration is zero
c. velocity is zero and acceleration is not zero  

Correct

d. velocity is not zero and acceleration is not zero

At the top of the path, the velocity of the ball is zero, but the acceleration is not zero. The velocity at the top is changing, and the acceleration is the rate at which velocity changes.

Acceleration is the change in velocity. Just because the velocity is zero does not mean that it is not changing.

Acceleration is not zero since it is due to gravity and is always a downward-pointing vector.