Physics 202 Review Lectures

- Exam 1&2 materials: today
- Optics: Reviewed Dec 10, 2009. (available on Web)
- Exam 3 materials: Reviewed on Nov. 19/20 (available on web).
- Next Tuesday: More optics & General Q&A

Exam 1 and Exam 2 were also reviewed by Prof. Mellado
Chapters Covered In Exam 1&2

Exam 1 Coverage
- Chapter 23: Electric Fields
- Chapter 24: Gauss’s Law
- Chapter 25: Electric Potential
- Chapter 26: Capacitance

Exam 2 Coverage
- Chapter 27: Electric Currents
- Chapter 28: DC Circuit
- Chapter 29: Magnetism
- Chapter 30: Source of Magnetic Field
Exam Topics (1)

- Basic Quantities:
  - Electric Charge
  - Electric Force
  - Electric Field, Field Lines, Electric Flux
  - Electric Potential
  - Electric Potential Energy
  - Capacitance
  - Electrical Current
  - Resistance
  - Magnetic Force
  - Magnetic Field, Magnetic Field Lines, Magnetic Flux
Exam Topics (2)

- Electric charge
  - Two types
  - Total charge is conserved.

- Electric force
  - Can be attractive/repulsive
  - Coulomb’s Law

- Electric field
  - E. field is a form of matter, it carries energy.
  - E. field is independent of test charge.
  - E. field is a vector quantity.
  - Three ways to calculate E. field
    - direct vector sum, Gauss’s Law, derivative of $V$
  - $F = qE$ (note: $E$ does not include the one created by $q$)
Exam Topics (3)

- Electric potential energy.
  - E. force is a conservative force
  - E. potential energy depends on both the source and test charge.
  - Like all potential energies, the E. potential energy is relative to a certain reference state. (Usually, an “infinity” state is taken as U=0.)
  - Use of E. potential energy with energy conservation, work-kinetic energy theorem.
Exam Topics(4)

- Electric potential
  - Electric potential depends only on the source.
  - E. Potential and E. Field are closely related.
    - From $\mathbf{E}$ to $V$
    - From $V$ to $\mathbf{E}$
  - E. potential and E. potential energy are different quantities.
    $\Rightarrow$ higher $V$ does not necessarily mean higher $U$.
  - E. potential and E. potential energy are related: $U=qV$
Exam Topics(5)

- Capacitance
  - $C = \frac{Q}{\Delta V}$
  - Connection in parallel and in series
  - $C$ of typical configurations (parallel plates, ....)

- Current and resistance.
  - Ohm’s Law
  - Power consumption on $R$

- DC circuit
  - Kirchhoff’s Rules
  - Simple 1-loop, 2-loop circuit of $R$’s and $\varepsilon$’s
Exam Topics(6)

- Magnetic Force
  - Magnetic force has a form of $qv \times B$.
    - always perpendicular to $v$ and $B$.
    - never does work
    - charged particle moves in circular/helix path in uniform $B$ field ($\omega = qB/m, r = mv/qB$)
    - On current segment, it has the form $IL \times B$
      - Uniform $B$, closed loop $\rightarrow \Sigma F = 0, \Sigma \tau = \mu B$

- Magnetic Field:
  - Field lines, “north” and “south”.
  - $B$ field contains energy, $u = \frac{1}{2} B^2/\mu_0$
  - $B$ field never does work.
Exam Topics(7)

- Magnetic Fields can be produced by:
  - moving charge (Biot-Savart law)
  - change of $E$ field
    (displacement current, conceptual level only)
- Ampere’s Law
  - Ampere’s law simplifies the calculation of $B$ field in some symmetric cases.
    - (infinite) straight line, (infinite) current sheet, Solenoid, Toroid
- Gauss’s Law in Magnetism → no magnetic charge.
- Forces between two currents
  - Can be attractive/repulsive
  - No force if perpendicular
Topics With More Weights

- In this final exam, the following topics will carry more weights among Exam 1&2 topics:
  - Electric field and electric force.
  - Electric potential and potential energy
  - Magnetic field and magnetic force
  - Capacitors and energy storage in capacitors

- Other Exam 1&2 topics are also important, but will be de-emphasized to make your preparation easier.

- The following exercises reflect this special treatment.
Reminder:
Three Ways to Calculate Electrostatic Field

- Superposition with Coulomb's Law (first principle):

\[ \vec{E} = k_e \sum \frac{q_i}{r_i^2} \hat{r}_i = k_e \int \frac{dq}{r^2} \hat{r} \]

- Apply Gauss’s Law:
  (Practical only for cases with high symmetry)

\[ \Phi_E = \oint E \cdot dA = \sum \frac{q_{in}}{\varepsilon_0} \]

- From a known potential:

\[ E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z} \]
Exercise 1: Seven Point Charges

- Six point charges are fixed at corners of a hexagon as shown. A seventh point charge \( q_7 = 2Q \) is placed at the center.

1. What is the force on \( q_7 \)?
2. What is the minimum energy required to bring \( q_7 \) from infinity to its current position at the center?

- Solutions: (See board)
  - First thoughts:
    - Point charges: Coulomb’s Law
    - Be aware of symmetry
    - Energy: \( \Delta U = q \Delta V \)
Exercise 2: Three shells of charge

- As shown below three thin sphere shells have radius \( R, 2R, 4R \), and charges \( Q, -Q, 2Q \), respectively.
  - Use Gauss’s law, find the electric field distribution.

**Solution:**
- The setting is highly symmetrical
  - Gauss’ surface will be concentric sphere of radius \( r \).

\[
\oint_S \vec{E} \cdot d\vec{A} = 4\pi r^2 E = \frac{1}{\varepsilon_0} q_{\text{enclosed}}
\]

- \( 0 < r < R \): \( q_{\text{enclosed}} = 0 \) \( \rightarrow \) \( E = 0 \)
- \( R < r < 2R \): \( q_{\text{enclosed}} = Q \) \( \rightarrow \) \( E = \frac{Q}{4\pi \varepsilon_0 r^2} \)
- \( 2R < r < 4R \): \( q_{\text{enclosed}} = Q - Q = 0 \) \( \rightarrow \) \( E = 0 \)
- \( 4R < r \): \( q_{\text{enclosed}} = Q - Q + 2Q = 2Q \) \( \rightarrow \) \( E = \frac{2Q}{4\pi \varepsilon_0 r^2} \)
Field lines always point towards lower electric potential.
Field lines and equal-potential lines are always at a normal angle.
In an electric field:
- a +q is always subject a force in the same direction of field line. (i.e. towards lower V)
- a -q is always subject a force in the opposite direction of field line. (i.e. towards higher V)
Exercise 3: Potential, Field, and Energy

- The equal potential lines surrounding two conductors, +10V and +15V, are shown below.
  - Draw on the figure the direction of electric field at point C
  - If a charge of $Q=+0.5\text{C}$ is to be moved from point B to C, how much work is required?

$$W_{B\to C} = U_C - U_B = Q(V_C - V_B) = 0.5 \times (12 - 9) = 1.5\text{J}$$
Exercise 4: Potential and Field

- The electric potential of a field is described by

\[ V = 3x^2y + y^2 + yz. \]

- Find the force on a test charge \( q = 1 \text{C} \) at \((x,y,z) = (1,1,1)\)

Solution:

First Thoughts:

- \( F = qE \)

- \( E_x = -\frac{dV}{dx} = -6x = -6 \) @ \((1,1,1)\)

- \( E_y = -\frac{dV}{dy} = -(3x^2 + 2y + z) = -6 \) @ \((1,1,1)\)

- \( E_z = -\frac{dV}{dz} = -y = -1 \) @ \((1,1,1)\)

\[
(F_x, F_y, F_z) = qE = (-6, -6, -1) \text{ N}
\]
Exercise 5: Electric Potential Energy

- A charged non-conducting ring of radius $R$ has charge $-Q$ distributed on the left side and $2Q$ distributed on the right side.
  - How much energy is required to bring a point charge $q=0.5Q$ from infinity to the center of the ring?
  - What if $q=-0.5Q$ instead?

Solution: See board

First Thought:
Energy required = external work required to bringing in $q$.

Follow up: How to calculate work?

- integral $dw=Fd$ (won’t work, too complicated)
- use energy conservation & the idea of electric potential
Reminder: Capacitors

- Capacitance: \( C = \frac{Q}{V} \) (Q=CV)
- Two capacitors in series:
  \( Q = Q_1 = Q_2 \), \( V = V_1 + V_2 \) \( \Rightarrow \) \( \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \)
- Two capacitors in parallel:
  \( V = V_1 = V_2 \), \( Q = Q_1 + Q_2 \) \( \Rightarrow \) \( C = C_1 + C_2 \)
- Energy storage: \( U = \frac{1}{2} CV^2 \)
Exercise 6: Two Capacitors

- Two capacitors, $C_1 = 1 \mu F$ and $C_2 = 2 \mu F$, are connected in series as shown. An voltage of 30V is applied across the two capacitors.
  - What is the combined capacity?
  - What is the charge on $C_1$?
  - What is the voltage on $C_1$?
  - What is the energy stored on $C_1$?

Solution:
- $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C = 0.67 \mu F$
- $Q = Q_1 = Q_2 = CV = 0.67 \mu F \times 30V = 2 \times 10^{-5} \text{ Coulomb}$
- $V_1 = \frac{Q_1}{C_1} = 2 \times 10^{-5} / 1 \times 10^{-6} = 20 \text{ V}$
- $U_1 = \frac{1}{2} C_1 V_1^2 = 2 \times 10^{-4} \text{ J}$
Reminder:
Two Ways to Calculate Magnetic Field

- **Biot-Savart Law:**

  \[
  \mathbf{B} = \frac{\mu_0 I}{4\pi} \int \frac{\mathbf{d}s \times \mathbf{\hat{r}}}{r^2}
  \]

- **Ampere’s Law:**

  \[
  \oint \mathbf{B} \cdot \mathbf{d}s = \mu_0 I
  \]

  *(Practical only for cases with high symmetry)*
Exercise 7: Biot-Savart Law

- Use Biot-Savart to find the magnetic field at the point P.

- Solutions: (See board)

\[ \vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{ds \times \hat{r}}{r^2} \]

Answer:

segment 1 contribution: B=0

segment 3 contribution: B=0

segment 2: \[ \vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{ds \times \hat{r}}{R^2} = \frac{\mu_0 I}{8R} \]

into page
Exercise 8: Ampere’s Law

- An infinite straight thin wire is at the center of two concentric conducting cylinders of radius $R$ and $2R$.

The currents are $I$ (into the page), $2I$ (out), and $I$ (in), respectively for the center wire and the two cylinders. (as color coded).

Find $B$ as function of $r$.

Solution:

\[ \int \vec{B} \cdot d\vec{s} = 2\pi rB = \mu_0 I_{\text{enclosed}} \]

\[ B = \frac{\mu_0 I_{\text{enclosed}}}{2\pi r} \]

Answers:

- $r<R$, $B = \mu_0 I/2\pi r$ (Clockwise)
- $R<r<2R$, $B = \mu_0 I/2\pi r$ (counter-clockwise)
- $r>2R$, $B = 0$