Physics 202 Midterm Exam 3
November 23rd, 2009

Name: Yibin Pan  Section:  

TA (please circle):
Zack DeLand (301,311)  Walter Pettus (302,307)  Camilo Garcia (303,329)
Alex Carr (304,328)  Kenny Rudinger (305,312)  Dan Dhokarh (306,321)
Amanda Kruse (322,324)  Zhen Liu (323,325)  Kara Maller (327,330)

Instructions:
1. Don’t forget to write down your name and section number.

2. Show your work. A reasonable amount of work is required to receive full credit.

3. Be aware that intermediate steps earn points even if the final answer is incorrect.

4. Erase (or cross out) any mistakes or you will be marked down. Grading is based on everything you have written down.

5. Both the magnitude and direction of vector quantities need to be specified for full credit.

Fundamental constants:
\[ \varepsilon_0 = \left(\frac{4\pi k_e}{c}\right)^{-1} = 8.85 \times 10^{-12} \text{C}^2/(\text{N}\cdot\text{m}^2) \]
\[ \varepsilon_0 = 4\pi \times 10^{-7} \text{T}\cdot\text{m}/\text{A} \]
\[ c = 3 \times 10^8 \text{m/s} \]
\[ m_p = 1.67 \times 10^{-27} \text{kg} \]
\[ m_e = 9.11 \times 10^{-31} \text{kg} \]
\[ q_p = -q_e = 1.6 \times 10^{-19} \text{C} \]

Scores:
Problem 1 _______/25  Problem 2 _______/25  Problem 3 _______/25
Problem 4 _______/25
Total _______/100
Problem 1 (25 points): Apply Lentz’s law to each case. 
(Mark direction, arguments not necessary, no partial credits for incorrect answers).

a) What is the direction of the induced emf along the loop? (5 points)

```
  I increasing
```

b) c) What is the direction of the current passing through R? (5 points each)

```
  I Decreasing
```

d) Indicate in the following “jumping ring” configurations the direction in which the ring tends to move when the switch S is closing. Note that the battery is reversed in case d.2 as compared in case d.1. (5 points)

```
d.1
  Iron core
  Metal ring
  S  

  Iron core
  Metal ring
  S  

d.2
```

e) What is the direction of the induced current through R? (5 points)
**Problem 2 (25 points):** As shown in the figure, a circular circuit loop of area $A=100\text{cm}^2$ connected with a resistor $R=1000\Omega$ and an inductor $L=0.1\text{H}$ is placed inside an external uniform magnetic field $B$. The plane of the loop is normal to the field. At $t=0$, the magnetic field starts to increase at a constant rate $\frac{dB}{dt}=10^3\text{T/s}$.

a) What is the direction of the induced current through $R$? Indicate on figure. (5 points)

b) What is the maximum current? (5 points)

c) How long does it take to ramp up the current to 30% of the maximum? (5 points)

d) At 30% maximum current, what is the voltage across the inductor? (5 points)

e) At 30% maximum current, how much energy is stored in the inductor? (5 points)

![Diagram](image)

**Solutions:**

a) See fig.

b) $\varepsilon = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = 100 \times 10^{-4} \times 10^3 = 10\text{ V}$, $I_{\text{max}} = \frac{\varepsilon}{R} = 0.01\text{ A}$

c) $\tau = \frac{L}{R} = 0.1/1000 = 10^{-4}\text{ s}$, $i(t) = I_{\text{max}} (1 - \exp(-t/\tau)) = 0.3 I_{\text{max}} \Rightarrow \exp(-t/\tau) = 0.7$

$\Rightarrow t = 0.357\tau = 3.6 \times 10^{-5}\text{ s}$

d) $\Delta V_L = L\frac{di}{dt} = L^2 I_{\text{max}}/\tau \exp(-t/\tau) = 0.1 \times 0.01/10^{-4} \times 0.7 = 7\text{ V}$

(or a clever method: $\Delta V_L = \varepsilon - \Delta V_R = \varepsilon - iR = 10 - 0.3 \times 0.01 \times 1000 = 7\text{ V}$)

e) $U = \frac{1}{2}LI^2 = \frac{1}{2} \times 0.1 \times (0.3 \times 0.01)^2 = 4.5 \times 10^{-7}\text{ J}$
Problem 3 (25 points): A RCL series AC circuit has $R=100\,\Omega$, $L=0.42\,\text{H}$, and $C=8.00\,\mu\text{F}$. The AC power has amplitude of 220 V and frequency of 60.0 Hz.

a) The expression of AC source can be written in the form of $\Delta V=\Delta V_{\text{max}}\sin\omega t$, what is $\Delta V_{\text{max}}$ and $\omega$, respectively? (3 points)

b) Given the $\Delta V$ in a), what is function form of the current? (Please determine all unknown parameters in your answer.) (7 points)

c) What is the maximum voltage across the inductor? (5 points)

d) What is the average power consumed, respectively, by the resistor, the Inductor, and the whole circuit? (5 points)

e) A phasor diagram as shown above is used for this circuit, label all phasors. (5 points)

Solutions:

a) $\Delta V_{\text{max}}=220\,\text{V}$, $\omega=2\pi f=377\,\text{rad/s}$

b) $i=I_{\text{max}}\sin(\omega t-\phi)$, $I_{\text{max}}=\Delta V_{\text{max}}\sqrt{R^2+(\omega L-1/(\omega C))^2}=220/\sqrt{100^2+(158.3-331.6)^2}=1.1\,\text{A}$, $\phi=\tan^{-1}((\omega L-1/(\omega C))/R)=-60^\circ$ (i.e. current is 60 degree ahead of voltage) (you can pre-calculate $\omega L=158.3$ and $1/(\omega C)=331.6$ for reuse)

c) $\Delta V_{\text{max}} L = I_{\text{max}} \omega L = 1.1 \times 158.3 = 174.1\,\text{V}$

d) $P_R=\frac{1}{2} I_{\text{max}}^2 R = 60.5\,\text{W}$, $P_L=0$, $P_{\text{total}}= P_R=60.5\,\text{W}$ (or less elegantly, $P_{\text{total}} = \frac{1}{2} I_{\text{max}} \Delta V_{\text{max}} \cos \phi = 0.5 \times 1.1 \times 220 \times \cos 60^\circ = 60.5\,\text{W}$)

e) See fig.
Problem 4 (25 points): A 40 kW radio station broadcasts a radio waves with a 3.4m wavelength uniformly in all directions.

a) What is the frequency of the wave? (5 points)

b) What is intensity of the signal 1.0 km from the station? (5 points)

c) What is the average density of the energy carried by magnetic field 9.0km from the station? (5 points)

d) What is the rms magnitude of the electric field at 1.0km from the station? (5 points).

e) What is the total energy passing through a sphere of r=3.0 km (with the station being the center of the sphere) during a period of 1 second? (5 points)

Solutions:

a) \[ f = \frac{c}{\lambda} = \frac{3 \times 10^8}{3.4} = 88\text{MHz} \]

b) \[ I_{1\text{km}} = \frac{P}{A} = \frac{40000}{(4 \pi R^2)} = \frac{40000}{(4 \times 3.14 \times 1000^2)} = 3.2 \times 10^{-3} \text{W/m}^2 \]

c) \[ u_{9\text{km}} = I_{9\text{km}} / c = (I_{1\text{km}} / 81) / c = 1.3 \times 10^{-13} \text{J/m}^3 \] (or one can use \( I = P / A \) to get \( I_{9\text{km}} \))

\[ u_{9\text{km}} \text{ magnetic} = \frac{1}{2} U_{9\text{km}} = 6.5 \times 10^{-14} \text{W/m}^2 \]

d) \[ u = \varepsilon_0 E^2 \Rightarrow E_{\text{rms}} = (u_{1\text{km}} / \varepsilon_0)^{1/2} = 1.1 \text{V/m} \]

(you may use \( u_{1\text{km}} = u_{9\text{km}} \times 81 \), or repeat step c to get \( u_{1\text{km}} \))

e) Per energy conservation, \( P_{3\text{km}} = P_{\text{source}} = 40\text{kW} \rightarrow \text{in 1 s, } U = 40\text{kW} \times 1 = 40\text{KJ} \)

(Alternatively, one can use \( U = I_{3\text{km}} \times \text{Area} \times \Delta t \) to get it,

or one can use \( U = u_{3\text{km}} c \times \text{Area} \times Dt = 40\text{KJ. Compare the simplicity of methods} \)