Today’s Topics

- Wave Nature of Lights: Interference
  - Light as Waves
  - Double-Slit Interference
  - Multi-Slit Interference
Exam 3 Result

Phy202 Exam 3 Curve

Average 76.6
Median: 78
A: [89-100]
AB, B: [77-88]
BC, C: [53-76]
D: [<53]

Note: Boundaries for letter grades are for reference only. (A~15%, B~50%...) (subject to final curve at the end of semester)
Reminder: Light and Optics

- **Nature of Lights**
  - Lights as rays
  - Lights as EM waves: f, λ, φ, v, A, interference …
  - Lights as group of photons

- **Optics: Physics of lights**
  - Geometric Optics: Treat light as rays (Ch. 35,36)
    - Ray approximation.
  - Wave Optics: Wave properties becomes important. Interferences, diffraction…(Ch. 37,38)
Reminder: Light Waves

- Nature of Lights:
  Rays (classical), \(\rightarrow\) EM waves\(\leftarrow\), \(\rightarrow\) Photons\(\leftarrow\).

- Review: Electromagnetic plane waves
  \[ E = E_{\text{max}} \sin(\omega t - kx + \phi), \quad B = B_{\text{max}} \sin(\omega t - kx + \phi), \quad E/B = c \]

  As the E component and B component of an EM wave are 100% correlated, we can use just one of them to represent an EM wave.
Quick Reviews

- Superposition Principle (ch. 18).
  - when two waves, \( y_1(x,t) \) and \( y_2(x,t) \) meet, the resulting wave is the algebraic sum of the two waves: \( y(x,t) = y_1(x,t) + y_2(x,t) \)

- Intensity of an EM wave (ch. 34) \( I = \frac{S}{c} \propto E_{\text{max}}^2 \propto B_{\text{max}}^2 \)
Useful Math Formulas

- $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$
- $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$
- Small angle approximations:
  - $\sin \theta \sim \theta$, $\tan \theta \sim \theta$, when $\theta \sim 0$
- Long distance approximation: $d \sim L \theta$, when $(L \gg d)$
Interference of Light Waves

- When two light waves meet at certain location, the resulting effect is determined by the superposition (i.e. sum) of the two individual waves.
  - e.g. Two light waves with same color and amplitude.
    \[ E_1 = E_0 \sin(\omega t - kx + \phi_1) = E_0 \sin(\omega t + \phi_1) \]
    \[ E_2 = E_0 \sin(\omega t - kx + \phi_2) = E_0 \sin(\omega t + \phi_2) \]
  \[ \Delta \phi = \phi_1 - \phi_2 \]
  \[ \phi = (\phi_1 + \phi_2)/2 \]

\[ E = E_1 + E_2 = 2E_0 \cos(\Delta \phi/2) \sin(\omega t + \phi/2) \]

\[ E_{\text{max}} = 2E_0 \cos(\Delta \phi/2) \]

- Constructive interference: \( \Delta \phi = 0, 2\pi, 4\pi, \ldots \) \( E_{\text{max}} = 2E_0 \)
- Destructive interference: \( \Delta \phi = \pi, 3\pi, 5\pi, \ldots \) \( E_{\text{max}} = 0 \)

- It all depends on \( \Delta \phi \)!

Quiz: If the intensity of each incoming light is \( I \), what is the resulting intensity when (1): constructive, (2): destructive?
Constructive and Destructive Interference

Resulting amplitude: $E_{\text{max}} = 2E_0 \cos(\Delta \phi / 2)$

Constructive, $\Delta \phi = 0\pi, 2\pi, 4\pi, \ldots$

Destructive, $\Delta \phi = \pi, 3\pi, 5\pi, \ldots$
Test of the Wave Nature of Light: Double-Slit Experiment

- Rays or Waves:

If lights behave as rays

If lights behave as waves

Diffraction & interference
Young’s Famous Double-Slit Experiment
Thomas Young (1803)

See demo
Remember a Phy201 problem like this?

For two interfering waves coming through different paths the phase difference:
\[ \phi = k(r_1 - r_2) + \Delta \phi_{at\_the\_source} \] (why? see board)
where \( r_1 \) and \( r_2 \) are path lengths,
\( \Delta r = (r_1 - r_2) \) is called path length difference.
Double-Slit Experiment Explained

- The experiment can be easily explained by interference.

**Constructive,** $\Delta\phi=0\pi, 2\pi, 4\pi,..$  
**Destructive,** $\Delta\phi=\pi, 3\pi, 5\pi,..$
Quantitatively

Path length difference
\[ \delta = dsin\theta \sim d\theta \sim d \frac{y}{l} \]

\[ \Delta \phi = k(s_1 - s_2) = kd \sin \theta = \frac{2\pi d}{\lambda} \sin \theta \]

\[ I = I_o \cos^2 \left( \frac{\pi d \sin \theta}{\lambda} \right) \]
Double-Slit Experiment Explained

- **Constructive:** $\Delta \phi = 0\pi, 2\pi, 4\pi, \ldots$, or $2m\pi$, $m=0,1,2\ldots$

  \[
  \frac{2\pi d}{\lambda} \sin \theta = 2m\pi \quad \rightarrow \quad d \sin \theta = m\lambda
  \]

  Bright spots

- **Destructive:** $\Delta \phi = \pi, 3\pi, 5\pi, \ldots$, or $(2m+1)\pi$, $m=0,1,2\ldots$

  \[
  \frac{2\pi d}{\lambda} \sin \theta = 2(m+1)\pi \quad \rightarrow \quad d \sin \theta = (m + \frac{1}{2})\lambda
  \]

  Dark spots
Multi-Slit Interference

- \# secondary maxima = N - 2
- Higher N \rightarrow more suppression on secondary minima
  (Grating: N>1000, highly sensitive to \lambda, good for measuring \lambda.
Extra: Lenz’s Law and Jumping Ring

- Lenz’s law in plain words: the induced emf always tends to work against the original cause of flux change.