13. From the Lorentz transformations

\[ \Delta t' = \gamma [\Delta t - v\Delta x/c^2] \]

But \( \Delta t' = 0 \) in this case, so solving for \( v \) we find

\[ v = c^2 \Delta t / \Delta x \]

Inserting the values \( \Delta t = t_2 - t_1 = -a/2c \) and \( \Delta x = x_2 - x_1 = a \),

\[ v = \frac{c^2 (-a/2c)}{a} = -c/2 \]

We conclude that the frame \( K' \) travels at a speed \( c/2 \) in the \(-x\)-direction. Note that there is no motion in the transverse direction.

19. With a contraction of 1%, \( L/L_0 = 0.99 = \sqrt{1 - \beta^2/c^2} \). Thus

\[ 1 - \beta^2 = (0.99)^2 = 0.9801 \]

Solving for \( \beta \), we find \( \beta = 0.14 \) or \( v = 0.14c \).
33. Conversion: 110 km/h = 30.556 m/s and 140 km/h = 38.889 m/s. Let \( u_x = 30.556 \text{ m/s} \) and \( v = -38.889 \text{ m/s}. \) Our premise is that \( c = 100 \text{ m/s}. \) Then by velocity addition

\[
u_x' = \frac{u_x - v}{1 - vu_x/c^2} = \frac{30.556 \text{ m/s} - (-38.889 \text{ m/s})}{1 - (-38.889 \text{ m/s})(30.556 \text{ m/s})/(100 \text{ m/s})^2} = 62.1 \text{ m/s}
\]

By symmetry each observer sees the other one traveling at the same speed.

* 51.

\[
f = f_0 \sqrt{\frac{1 - \beta}{1 + \beta}} = (400 \text{ Hz}) \sqrt{\frac{1 - 0.92}{1 + 0.92}} = 82 \text{ Hz}
\]
56. The magnitude of the centripetal force is

\[ \gamma ma = \gamma m \frac{v^2}{r} \]

for circular motion. For a charged particle \( F = qvB \), so

\[ qvB = \gamma m \frac{v^2}{r} \]

or, rearranging

\[ qBr = \gamma mv = p \]

\[ r = \frac{p}{qB} \]

When the speed increases the momentum increases, and thus for a given value of \( B \) the radius must increase.

57

61. 230-MeV protons have \( K = 230 \text{ MeV} \) and \( E = K + E_0 = 1168 \text{ MeV} \). Then

\[ p = \frac{\sqrt{E^2 - E_0^2}}{c} = 696.0 \text{ MeV}/c \]

Converting to SI units

\[ p = 696.0 \text{ MeV}/c \left( \frac{1.60 \times 10^{-13} \text{ J}}{\text{MeV}} \right) \left( \frac{c}{3.00 \times 10^8 \text{ m/s}} \right) = 3.71 \times 10^{-19} \text{ kg} \cdot \text{m/s} \]

From Problem 56

\[ B = \frac{p}{q\gamma r} = \frac{3.71 \times 10^{-19} \text{ kg} \cdot \text{m/s}}{(1.60 \times 10^{-19} \text{ C})(15 \text{ m})} = 0.155 \text{ T} \]