FINAL EXAM

Please print your name and section number (or TA’s name) clearly on the first page. Show all your work in the space immediately below each problem. Your final answer must be placed in the boxes, when provided. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, the direction of vectors, and the correct number of significant figures. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitude. You are allowed 2 sides of 1 sheet of paper for notes (8.5” x 11”), a calculator, and the constants in this exam booklet. Try to be neat! The exam lasts 2 hours. The total number of points is 100.

Constants and Conversion Factors:

Acceleration due to gravity at Earth’s surface: \( g = 9.81 \text{ m/s}^2 \)
1.0 km = 0.62 miles
1 atm = 101,000 Pa
1 hp = 746 W
moment of inertia, \( I \), for a solid disk of mass \( M \) and radius \( R \) = \( MR^2/2 \)
Boltzmann’s constant, \( k_B = 1.38 \times 10^{-23} \text{ J/K} \)

SCORE:

Problem 1: ________
Problem 2: ________
Problem 3: ________
Problem 4: ________
Problem 5: ________
Problem 6: ________
Problem 7: ________

TOTAL: ________

“You can do what you have to do, and sometimes you can do it even better than you think you can.” Jimmy Carter

Please don't open the exam until you are instructed to start.
Part 1

A. A rocket, initially at rest, is fired vertically with a net upward acceleration of 14 m/s². At an altitude of 0.45 km, the engine of the rocket cuts off. What is the maximum altitude it achieves? (5 pts)

\[ v_{\text{off}}^2 = v_0^2 + 2a_d \quad \therefore \quad d = \frac{v_{\text{off}}^2}{2a} \]

\[ h_{\text{max}} = d + h = 450 + \frac{14 \times 450}{9.8} = 1100 \text{ m} \]

B. A 2.9-kg object has a velocity of 3.3 j m/s at \( t = 0 \). A constant resultant force of \( (2.3 \hat{i} + 4.6 \hat{j}) \) N then acts on the object for 3.6 s. What is the magnitude of the object's velocity at the end of the 3.6-s interval? (5 pts)

\[ \vec{v} = \vec{v}_0 + \vec{a} \Delta t \quad \vec{a} = \frac{\vec{F}}{m} \quad \vec{a} \text{ is constant} \]

\[ = (0 \hat{i} + 3.3 \hat{j}) + \frac{1}{2.9}(2.3 \hat{i} + 4.6 \hat{j}) \cdot 3.6 \]

\[ = 2.86 \hat{i} + 9.01 \hat{j} \quad |\vec{v}| = 9.5 \text{ m/s} \]

C. The sled dog in the figure drags sleds A and B, carrying emergency exam booklets, across the snow. The coefficient of friction between the sleds and the snow is 0.10. If the tension in rope 1 is 150 N, what is the tension in rope 2? (5 pts)

\[ T_2 - \text{mg} - T_1 = ma \]

\[ \Rightarrow a = \frac{150 - 100 \times 9.8 \times 0.1}{100} = 0.52 \text{ m/s}^2 \]

Plug into B:

\[ T_2 = 80 \times 8.8 \times 0.1 + 150 + 80 \times 0.52 = 270 \text{ N} \]
Part 2

A. A 200 g, 40-cm-diameter turntable rotates on frictionless bearings at 60 rpm. A 20 g block sits at the center of the turntable. A compressed spring shoots the block radially outward along a frictionless groove in the surface of the turntable. What is the turntable's rotation angular velocity when the block reaches the outer edge? (5 pts)

\[ \dot{\theta} = \frac{I_i \cdot \omega_i}{I_f} \]

\[ I_i \cdot \omega_i = I_f \cdot \omega_f \]

\[ I_i = \frac{1}{2} MR^2 \]

\[ \omega_i = \frac{60 \times 2\pi}{60} = 2\pi \]

\[ I_f = \frac{1}{2} MR^2 + mR^2 \]

\[ \omega_f = \frac{2 \pi}{1 + \frac{2m/M}{1 + 2x_0^2}} \]

5.2 rad/s

B. This history graph shows the displacement of a traveling wave measured at position x = 0. The wave is traveling in the positive x-direction with a velocity of 4.0 m/s.

i. What is the wavelength? (3 pts)

\[ \lambda = \frac{2\pi}{k} = 4.0 \times 10^{-2} = 0.8 \text{ m} \]

ii. Write the equation for D(x,t) for this wave. (7 pts)

\[ \omega = \frac{2\pi}{T} = \frac{2\pi}{0.2} \]

\[ k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.8} \]

\[ A = 2 \times 10^{-3} \]

\[ D(x,t) = -2 \times 10^{-3} \cos \left( \frac{2\pi}{0.2} x - \frac{2\pi}{0.2} t \right) \text{ m} \]
Part 3

A. A hockey puck slides on ice with an initial speed of 3.20 m/s. It travels a distance of 92.5 meters before its speed is reduced to half of its initial speed. Assuming that the net horizontal force on the puck is due to friction only, find the coefficient of kinetic friction, $\mu_k$, between the puck and the ice. (5 pts)

\[
\Delta KE = -\mu_k mg \Delta x
\]

\[
\mu_k = \frac{(K E - K^*E)}{mg \Delta x} = \frac{-\frac{1}{2} m (v_i^2 - v_f^2)}{\int_{v_i}^{v_f} g \Delta x}
\]

\[
\mu_k = 0.00432
\]

B. A sign is supported from a wall at two points, P and Q, as shown. If the sign is a uniform square 0.4 m on a side and its mass is 4.0 kg, what is the magnitude of the horizontal force that P experiences? Assume the length of the supports at P and Q is negligibly small. (10 pts)

\[
\Sigma z = 0
\]

\[
\Sigma \text{ moments at Q}
\]

\[
+ \frac{mgL}{2} - P_x L = 0
\]

\[
P_x = \frac{mg}{2} = \frac{4.0 \times 9.8}{2} = 19.6
\]

\[
= 20.0 \text{ N}
\]
Part 4

A. In the Simple Harmonic Motion and Resonance Lab you measured the mass, \( m \), and the effective spring constant, \( k_{eff} \), for a mass attached to a pair of springs. Suppose you determined that \( m = 0.21 \text{ kg} \) and \( k_{eff} = 1.3 \text{ N/m} \). At what angular frequency, \( \omega \), do you expect the mass and spring to oscillate and what is the uncertainty, \( \Delta \omega \), in this prediction? (5 pts)

\[
\omega = \sqrt{\frac{k_{eff}}{m}} = 2.49
\]

\[
\frac{\Delta \omega}{\omega} = \sqrt{\left(\frac{1}{2} \frac{\Delta k_{eff}}{k_{eff}}\right)^2 + \left(\frac{1}{2} \frac{\Delta m}{m}\right)^2} = \sqrt{\left(\frac{1}{2} \frac{\Delta k_{eff}}{k_{eff}}\right)^2 + \left(\frac{1}{2} \frac{\Delta m}{m}\right)^2} \approx 0.045 \Rightarrow 0.05 \text{ rad/s}
\]

\[
\Delta \omega = 0.045 \times 2.49 = 0.12 \Rightarrow 0.1 \text{ rad/s}
\]

\[
\omega = 2.5 \pm 0.1 \text{ rad/s}
\]

B. A 1 kg block is attached to a spring with spring constant 16 N/m. While the block is at rest a student hits it with a hammer and almost instantaneously gives it a velocity of 40 cm/s. Assume there is no friction or damping in the system.

i. What is the amplitude, \( A \), of the subsequent oscillations? (5 pts)

\[
PE_{max} = KE_{max}
\]

\[
\frac{1}{2} k A^2 = \frac{1}{2} m v_{max}^2
\]

\[
A = \left( m v_{max} \right)^{\frac{1}{2}} = \left( 0.1 \times 0.4 \right)^{\frac{1}{2}} = 0.1 \text{ m}
\]

ii. What is the block's speed at the point where \( x = A/2 \)? (5 pts)

\[
\frac{1}{2} k A^2 = \frac{1}{2} m v^2 + \frac{1}{2} k \left( \frac{A}{2} \right)^2
\]

\[
v = A \sqrt{\frac{k}{m} \left( 1 - \frac{1}{4} \right)}
\]

\[
v = 0.3 \text{ m/s}
\]
Part 5

A. Each second 5.0 x 10^{23} nitrogen molecules (molecular mass m = 4.65 x 10^{-23} g) collide with a wall with area 10 cm^2. Assume that the molecules all travel with a common speed of 290 m/s and strike the wall head on. What is the pressure on the wall? (5 pts)

\[ P = \frac{F}{A} = \frac{1}{A} \frac{\Delta P}{\Delta t} = \frac{1}{A} \frac{2 m v}{13} = \frac{2 \times 4.65 \times 10^{-23} \text{ g}}{(0.1)^2} \times 290 \times 5 \times 10^{23} \]

= 1300 Pa

B. At what temperature would the average thermal speed of oxygen molecules be 13.0 m/s? Oxygen is assumed to approximate an ideal gas. The mass of one O_2 molecule is 5.31 x 10^{-26} kg. (5 pts)

\[ \frac{3}{2} \frac{k}{m} = \frac{3}{2} m \left( \frac{v}{2} \right)^2 \]

\[ T = \frac{1}{k_B} m \left( \frac{v}{2} \right)^2 = \frac{5.31 \times 10^{-26} \text{ kg}}{1.38 \times 10^{-23}} \times \left( \frac{13}{2} \right)^2 \]

= 0.650 K
Part 6

A. A gas fills a chamber that has a low-friction piston that can move, thereby increasing its volume. The piston is held in place primarily by gravity. The chamber is heated, and the piston moves accordingly. What kind of process is this? (i.e., what quantity is constant during this process?) (5 pts)

B. The figure shows a cycle for a heat engine for which \( Q_H = 35 \) J. What is the thermal efficiency? (10 pts)

\[ \eta = \frac{W_{\text{out}}}{Q_H} \]

The net work done by the heat engine gas in one cycle is the area enclosed on the \( P-V \) diagram.

\[ W_{\text{out}} = \frac{1}{2} (100 \times 10^{-6} \text{ m}^3) (200 \times 10^3 \text{ Pa}) \]

\[ = 10 \text{ J} \]

\[ \eta = \frac{10}{35} = 0.29 \]
Part 7

A. A 200.0 kg flat-bottomed boat floats in fresh water, which has a density of 1000.0 kg/m³. Assuming that the base of the boat is 1.42 m wide and 4.53 m long, how much of the boat is submerged (how far is the bottom below the surface of the water) when it carries three passengers whose total mass is 193 kg? (5 pts)

\[ F_B = \rho \cdot V \cdot g = \rho \cdot A \cdot \delta \cdot g \]
\[ \delta = \frac{M_{\text{total}}}{\rho \cdot A \cdot g} = \frac{200 + 193}{1000 \times 1.42 \times 4.53} = 0.0010 \]

0.0010 m

B. Air flows through this tube. Assume the air is an ideal fluid. (The density of air is 1.28 kg/m³. The density of mercury (Hg) in the tube is 13,600 kg/m³.)

i.) What is the difference in pressure between points 1 and 2 (i.e. what is \( P_2 - P_1 \)) (3 pts)?

\[ P_2 - P_1 = \rho \cdot g \cdot h = 13,000 \text{ Pa} \]

ii.) What are the air speeds \( v_1 \) and \( v_2 \) at points 1 and 2? (4 pts)

\[ P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2 \]
\[ v_1 = \left( \frac{1}{(0.2)} \right)^2 \cdot v_2 = 2.5 \cdot v_2 \]
\[ \frac{1}{2} \rho \left( (2.5v_2)^2 - v_2^2 \right) = P_2 - P_1 \Rightarrow v_2 = 5.8 \text{ m/s}, \quad v_1 = 14.0 \text{ m/s} \]

iii.) What is the volume flow rate of the air? (3 pts)

\[ I_v = v \cdot A = v_2 \cdot A_2 \]
\[ = 5.8 \cdot \frac{\pi}{4} \left( 10^{-2} \right)^2 \]
\[ = 4.5 \times 10^{-4} \text{ m}^3/\text{s} \]