Final EXAM

Please print your name and section number (or TA’s name) clearly on the first page. Show all your work in the space immediately below each problem. Your final answer must be placed in the boxes, when provided. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. Each problem is worth 20 points. Try to be neat! Check your answers to see that they have the correct dimensions (units) and are the right order of magnitude. You are allowed two sheets of notes (2 sheets of 8.5" x 11", both sides), a calculator, and the constants in this exam booklet. The exam lasts exactly two hours.

Constants:

Moment of inertia about the axis of a solid disk of mass M and radius R: \( I_{\text{Disk}} = MR^2/2 \)

Acceleration due to gravity at the earth’s surface: \( g = 9.81 \text{ m/s}^2 \)

SCORE:

Problem 1: ______
Problem 2: ______
Problem 3: ______
Problem 4: ______
Problem 5: ______
TOTAL: ______

“...each of us must work for his own improvement, and ...share a general responsibility for all humanity, ...to aid those to whom we think we can be most useful.” Marie Curie

Please don't open the exam until you are instructed to start.
Part 1

A 100 g rubber ball is thrown horizontally with a speed of 5 m/s toward a wall. It is initially traveling to the left. It rebounds with no loss of speed. The collision force is shown in the top graph below.

a.) What is the value of the maximum force $F_{\text{max}}$? (4 pts)

$$F_{\text{max}} = \frac{\Delta p}{\Delta t} = \frac{2 \cdot m \cdot v}{\Delta t} = \frac{2 \times 0.1 \times 5}{0.010}$$

$$100 \text{ N}$$

b.) Draw an acceleration-versus-time graph for the collision on the middle set of axes below. Make your graph align vertically with the force graph, and provide an appropriate numerical scale on the vertical axis. (4 pts)

c.) Draw a velocity-versus-time graph for the collision on the bottom set of axes below. Make your graph align vertically with the acceleration graph, and provide an appropriate numerical scale on the vertical axis. (4 pts)
A 0.5 kg cart and a 2.0 kg cart are attached and are rolling forward with a speed of 2.0 m/s. Suddenly a spring-loaded plunger pops out and blows the two carts apart from each other. The smaller mass cart shoots backward at 2.0 m/s.

\[ \vec{v}_2 = \vec{v}_1 = \vec{v}_{2f} \]

\[ 2(m_1 + m_2) = -2m_1 + \frac{v_{2f}^2}{2} m_2 \]

\[ v_{2f} = \frac{4m_1 + 2m_2}{m_2} \frac{x_{0.5} + 2x_{2.0}}{2.0} = +3 \text{ m/s} \]

\[ +3 \text{ m/s} \]

d. What are the speed and direction of the 2 kg cart? (4 pts)

e. If the spring constant of the plunger is 25,000 N/m, by how much was the spring initially compressed? Assume there is no friction. (4 pts)

\[ \frac{1}{2} k \Delta x^2 = \Delta K = K_f - K_i = \frac{1}{2} m_2 v_{2f}^2 + \frac{1}{2} m_1 v_{1f}^2 - \frac{1}{2} (m_1 + m_2) v_i^2 \]

\[ \Delta x = \frac{1}{k} \left( 2 \times 3^2 + 0.5 \times 2^2 - 2.5 \times 2^2 \right) \]

\[ \Delta x = \sqrt{\frac{10}{25,000}} = \sqrt{4 \times 10^{-4}} = 0.02 \text{ m} \]
Part 2

a.) A 12-cm-diameter, 2.0 kg uniform circular disk, which is initially at rest, experiences the net torque shown in the figure below. What is the disk's angular velocity at $t = 12$ s? The disk rotates about an axis perpendicular to the plane of the disk and through its center. (Note: $I_{\text{disk}} = MR^2/2$) (10 pts)

\[
\tau = I\alpha = I \frac{d\omega}{dt} \Rightarrow \frac{d\omega}{dt} = \frac{\tau}{I}
\]

\[
\omega = \frac{1}{I} \int_0^t \tau \, dt
\]

\[
= \text{area under } \tau-t \text{ curve}
\]

\[
= \frac{1}{I} \left( \frac{1}{2} \times 0.2 - \frac{1}{2} \times 4 \times 0.1 \right)
\]

\[
= \frac{k}{2.0 \times 0.06^2} \times \frac{1}{2} (1.6 - 0.4)
\]

\[
= 170 \text{ rad/s}
\]
b.) A U-shaped tube is open to the air at both ends and is partially filled with Mercury (density = 13,600 kg/m$^3$). Water (density = 1000 kg/m$^3$) is poured into the left arm until the water is 10.0 cm deep. How far upward from its initial position does the mercury rise on the right side? (10 pts)

Answer:

Water and mercury are incompressible and immiscible liquids.

Visualize:

The water in the left arm floats on top of the mercury and presses the mercury down from its initial level. Because points 1 and 2 are level with each other and the fluid is in static equilibrium, the pressure at these two points must be equal. If the pressures were not equal, the pressure difference would cause the fluid to flow, violating the assumption of static equilibrium.

Solve: The pressure at point 1 is due to water of depth $d_w = 10$ cm:

$$p_1 = p_{atmos} + \rho_w g d_w$$

Because mercury is incompressible, the mercury in the left arm goes down a distance $h$ while the mercury in the right arm goes up a distance $h$. Thus, the pressure at point 2 is due to mercury of depth $d_{mg} = 2h$:

$$p_2 = p_{atmos} + \rho_{mg} g d_{mg} = p_{atmos} + 2 \rho_{mg} g h$$

Equating $p_1$ and $p_2$ gives

$$p_{atmos} + \rho_w g d_w = p_{atmos} + 2 \rho_{mg} g h \Rightarrow h = \frac{1}{2} \frac{\rho_w g d_w}{\rho_{mg} g} = \frac{1}{2} \frac{1000 \text{ kg/m}^3}{13,600 \text{ kg/m}^3} (10 \text{ cm}) = 3.7 \text{ mm}$$

The mercury in the right arm rises 3.7 mm above its initial level.
Consider the situation depicted below where the base of the triangular inclined plane structure has length L.

There is no friction between the mass $m_1$ and the inclined plane. The coefficient of kinetic friction between the inclined plane (with mass $m_2$) and the ground is $\mu_k$.

a) What is the magnitude of the force $F$ necessary to have the mass $m_1$ stationary with respect to the moving inclined plane?

Vertical:
$$m_1 g = N \cos \theta \quad \text{since the mass is neither moving up nor down.}$$

$$\therefore N = \frac{m_1 g}{\cos \theta}$$

Horizontal:
$$m_1 a = N \sin \theta \quad \Rightarrow a = g \tan \theta$$

Newton's 3rd law:
$$-N_2 \mu_k + F - N_2 \sin \theta = m_2 a$$

$$N_2 = N \cos \theta + m_2 g$$

$$F = m_2 a + N \sin \theta + N_2 \mu_k$$

$$= m_2 g \tan \theta + N \sin \theta (N \cos \theta + m_2 g) \mu_k$$

$$= m_2 g (\mu_k + \tan \theta) + N (\sin \theta + \mu_k \cos \theta)$$

$$= m_2 g (\mu_k + \tan \theta) + m_1 g (\mu_k + \tan \theta)$$

$$= (m_1 + m_2) g (\mu_k + \tan \theta)$$
b.) A skier, whose mass is 70 kg, stands at the top of a 10° slope on her new frictionless skis. A strong horizontal wind blows against her with a force of 50 N. Using the concepts of energy conservation, find the skier’s speed after traveling 100 meters down (along) the slope. (10 pts)

\[ K_i + U_i + W_{wind} = K_f + U_f + \Delta E_m \]

\[ \frac{1}{2} m v_i^2 + 0 + \Delta E_m = \frac{1}{2} m v_f^2 + 0 \]

\[ \phi + mg(100 \sin 10°) - 50 \cdot 60 \cdot 10 \cdot 100 = \frac{1}{2} m v_f^2 + \phi \]

\[ v_f^2 = 100 \left( 2g \sin 10° - \frac{2 \times 50 \times 60 \cdot 10}{m} \right) \]

\[ v_f = \sqrt{100 \left( 2 \times 9.8 \times \sin 10° - \frac{2 \times 50 \times 60 \cdot 10}{70} \right)} \approx 14 \text{ m/s} \]
Consider the frictionless piston system below where gravity is pointing downwards and outside of the piston-gas system is vacuum.

The piston mass is $M_p$ and its cross sectional area is $A$. The gas is ideal.

a) Suppose the piston is initially held by hand at a certain initial height, and the initial pressure of the gas at that point is $P_1$. Subsequently, the piston is released and allowed to fall. Write down an inequality expression involving $M_p$, $A$, $P_1$, and $g$ which is necessary for the piston to fall. Express your answer in the form “$P_1 < \ldots$”.

$$P_1 < \frac{M_p g}{A}$$

b) Suppose during the fall depicted in part a), heat is drawn out of the gas such that the gas maintains a constant temperature as the piston falls. If the initial height of the piston is $h$, what is the final height of the piston? [Express your final answer in terms of $M_p$, $A$, $P_1$, $g$, and $h$.]

Final equilibrium cond.: $M_p g = P_2 A$

Ideal gas: $P_2 V_2 = N k_B T$

$$P_2 A h_2 = N k_B T$$

$$h_2 = \frac{N k_B T}{P_2 A} = \frac{N k_B T}{M_p g}$$

Ideal gas: $N k_B T = P_1 A h$

$$\Rightarrow h_2 = \frac{P_1 A}{M_p g} h$$
Here’s a new problem:
c.) A tube, open at both ends, is filled with an unknown gas. The tube is 190 cm in length and 3.0 cm in diameter. By using different tuning forks, it is found that resonances can be excited at frequencies of 315 Hz, 420 Hz, and 525 Hz, and at no frequencies in between these.
What is the speed of sound in this gas? (10 pts)

Answer:
Standing waves in a structure that has the same boundary conditions at both ends (string fixed at each end or pipe open or closed at each end) will have resonances at:

\[ \lambda_1 = 2L, \quad \lambda_2 = L, \quad \ldots \quad \lambda_n = \frac{2L}{n} \]

or

\[ f_n = nf_1 = \frac{nv}{\lambda_1} = \frac{nv}{2L} \]

so \( \Delta f = f_n - f_{n-1} = \frac{v}{2L} \) or \( v = 2L\Delta f \)

In this problem \( \Delta f = 105 \text{ Hz} \) so \( v = 2 \times 1.90 \times 105 = 400 \text{ m/s} \)
i) A window washer of mass $M$ is sitting on a negligible mass platform suspended by a system of cables and pulleys with negligible mass as shown (only object that has appreciable mass is the washer):

![Diagram of window washer](image)

What is the approximate magnitude of the force $F$ that the washer must exert to pull himself (including the platform system) up at constant speed?

a) $(3/2)Mg$  b) $(2/3)Mg$  c) $3Mg$  d) $Mg/3$  e) $Mg$

Your answer: d

ii) A 2 kg mass is hung at the end of a 1 m long string to form a pendulum. Suppose the pendulum is released from rest with an initial angle of the string with respect to the vertical being 0.1 radian. How long does the pendulum take to reach -0.05 radians?

a) 0.7 s  b) 0.9 s  c) 0.4 s  d) 0.1 s  e) 0.2 s

Your answer: a

\[
\begin{align*}
\theta &= \theta_0 \cos(\omega t + \phi) \\
\theta(t=0) &= \theta_0 \cos(\phi) \\
\theta(t=0) &= 0.1 \\
\Rightarrow \quad \phi &= 0 \\
\therefore \quad \theta_0 &= 0.1 \\
\theta &= \theta_0 \cos(\omega t) = -0.05 \\
\therefore \quad \omega t &= \arccos\left(\frac{-0.05}{0.1}\right) \\
\quad &< \pi - \frac{\pi}{3} \\
\quad &= \frac{2\pi}{3} \\
\therefore \quad t &= \frac{2\pi}{3} \sqrt{\frac{l}{g}} = \boxed{0.67 \text{ s}}
\end{align*}
\]
iii) Consider the roller coaster ride segment shown below where the radius of the circle is 20.0 m:

\[
\begin{align*}
\vec{F} = mg & \\
\Rightarrow \quad v_{\min} &= \sqrt{\frac{mgR}{m}} = \sqrt{gR} = 14 \, \text{m/s}
\end{align*}
\]

What is the minimum speed of the roller coaster car at point A which would guarantee that the roller coaster car will not leave the track assuming that the roller coaster car is upside down at point A (neglect wind resistance and other similar complications)? (4 pts)

a) 7 m/s  b) 14 m/s  c) 20 m/s  d) 100 m/s  e) 200 m/s  

your answer: \( \square \)  

iv.) A 0.20 kg plastic cart and a 2.0 kg lead cart can both slide without friction along a horizontal track. Equal forces (same direction and magnitude) are used to push each cart forward for 1 second, starting from rest. After the force is removed at \( t = 1 \) s, which of the following statements is true? (4 pts)

a. The momentum of the plastic cart is less than the momentum of the lead cart.

b. The momentum of the plastic cart is equal to the momentum of the lead cart.

c. The momentum of the plastic cart is greater than the momentum of the lead cart.

d. There is not enough information to compare the momenta of the two carts.

your answer: \( \square \)  

v.) A 0.20 kg plastic cart and a 2.0 kg lead cart can both glide without friction along a horizontal track. Equal forces (same direction and magnitude) are used to push each cart forward for 1 m, starting from rest. After the force is removed at \( x = 1 \) m, which of the following statements is true? (4 pts)

a. The kinetic energy of the plastic cart is less than the kinetic energy of the lead cart.

b. The kinetic energy of the plastic cart is equal to the kinetic energy of the lead cart.

c. The kinetic energy of the plastic cart is greater than the kinetic energy of the lead cart.

d. There is not enough information to compare the kinetic energies of the two carts.

your answer: \( \square \)