TODAY:

- Conservation of Angular momentum (Ch 12.11)
- Conservation of Rotational Energy
- Oscillations

ASSIGNMENTS:

- Read Chapter 14 ‘Oscillations’
- BoD meeting

"Science is a way to teach how something gets to be known, what is not known, to what extent things are known (for nothing is known absolutely), how to handle doubt and uncertainty, what the rules of evidence are, how to think about things so that judgments can be made, how to distinguish truth from fraud, and from show."

Richard Feynman
What does the area between the stretch and relax curves for a rubber band represent?
Force Exerted on Rubber band

Area = work done on rubber band to stretch it

\[ W = \int F(x) \, dx \]
Force Exerted on Rubber band

\[ W = \int F(x) \, dx \]

Area = work done by rubber band as it contracts
Force Exerted on Rubber band

Area = Mechanical energy lost during one cycle = energy released as heat
Bivalve Molluscs:
How do you open a shell from inside?

Shell is “spring-loaded”:
Hinge ligament is made of an elastic protein (*Abductin*).

Muscle closes shell against the force of this spring.
Torque and Angular Acceleration

\[ \vec{\tau} = \vec{r} \times \vec{F} \]
\[ \tau = r F \]
\[ = r (ma) \]  
\[ a = r \alpha \]
\[ = m r^2 \alpha \]

For solid body

\[ \vec{\tau} = \left( \sum_{i=1}^{n} \left( m_i \vec{r}_i \times \vec{a}_i \right) \right) \]
\[ = I \vec{\alpha} \]
\[ (F = m \vec{a}) \]
Example

\[
\hat{z} \begin{array}{c} \hat{y} \\ \hat{x} \end{array} \hat{y} T \quad \text{disk can rotate about center}
\]

(There are no other forces on this.)

1) What is the net force? 
   a) 2T  b) -2T  c) 2T \hat{y}  d) -2T \hat{y}  e) 0

2) Will the center of mass move? 
   a) yes  b) no

3) What is the net torque? 
   a) 0  b) 2TR \hat{z}  c) TR \hat{z}  d) -2TR \hat{z}

4) Will the disk be in static equilibrium? 
   a) yes  b) no
A rope is wrapped around a cylinder which is constrained to rotate about its long axis. The cylinder has moment of inertia, $I$, and radius, $R$. A mass, $M$, is attached to the rope and allowed to fall. What is the velocity of the mass after it falls a distance, $d$?
Angular Momentum

Recall that \( F = \frac{d\vec{p}}{dt} = m\vec{a} \)

and that \( \vec{L} = \vec{r} \times \vec{p} \)

Define angular momentum \( \vec{L} = \vec{r} \times \vec{p} \)

\[
\vec{L} = \sum m_i \hat{r}_i \times \vec{v}_i = \sum m_i \hat{r}_i \vec{v}_i \cdot \frac{A}{2} \\
= \sum m_i \hat{r}_i \vec{v}_i \cdot \hat{\omega} \\
= \sum m_i \hat{r}_i \vec{v}_i \omega \cdot \frac{A}{2} \\
= \sum m_i \hat{r}_i \vec{v}_i \omega \\
\]

\( \vec{L} = I \vec{\omega} \)

(let \( \vec{p} = m \vec{v} \))
Example: Rotating Table

A student sits on a rotating stool with his arms extended and a weight in each hand. The total moment of inertia is $I_j$, and he is rotating with angular speed $\omega_j$. He then pulls his hands in toward his body so that the moment of inertia reduces to $I_f$. What is his final angular speed $\omega_f$?
Angular Momentum

- A student sits on a freely turning stool and rotates with constant angular velocity $\omega_1$. She pulls her arms in, and due to angular momentum conservation her angular velocity increases to $\omega_2$. In doing this her kinetic energy:

(a) increases  (b) decreases  (c) stays the same
Oscillations

Oscillations are everywhere:

- speaker making sound
- heart beat
- seasons
- vibrating guitar string
- undulations of water in a pool
- pendulum of a clock

In fact almost all of modern physics that explains all physical phenomena is based on the physics of oscillations.

Next two lectures: familiarize you w/ the mathematics + physical examples containing oscillations.
The most natural oscillation problem in Newtonian physics is a mass attached to a spring:

\[ x_i \rightarrow \text{has mass } m \]

\[ \text{no friction} \]

\[ \text{equilibrium spring position} \]

Let \( x_i \) be the initial displacement. Suppose the mass is released from rest. What is the displacement from the equilibrium position as a function of time?
Method 1

\[ E = \frac{1}{2} k \frac{x^2}{x_0^2} + \frac{1}{2} m \left( \frac{dx}{dt} \right)^2 \]

\[ \pm \sqrt{\frac{2E}{m}} - \frac{k}{m} x^2 = \frac{dx}{dt} \]

\[ \int dt = \int \frac{dx}{\sqrt{\frac{2E}{m} - \frac{k}{m} x^2}} \]

\[ t + C = \arcsin \left( \frac{\sqrt{\frac{k}{m} x}}{x_0} \right) \]

\[ \sin \left( \sqrt{\frac{k}{m}} t + C \right) = \sqrt{\frac{k}{2E}} x \]

Since \( E = \frac{1}{2} k \frac{x^2}{x_0^2} \) and \( x = x_i \) at \( t = 0 \),

\[ \sin (C) = \sqrt{\frac{k}{kx_i^2}} x_i = \frac{1}{2} \]

\[ C = \frac{\pi}{2} \text{ and } x = x_i \cos \left( \sqrt{\frac{k}{m}} t \right) \]

Method 2

\[ -kx = m \frac{d^2x}{dt^2} \]

Using the knowledge

\[ \frac{d^2 \cos(\omega t + \theta)}{dt^2} = -\omega^2 \cos(\omega t + \theta) \]

Guess \( x = A \cos(\omega t + \theta) \).

i.e. plug this into

\[ \frac{d^2x}{dt^2} = -\frac{k}{m} x \]

\[ A \omega^2 \cos(\omega t + \theta) = -\frac{k}{m} A \cos(\omega t + \theta) \]

\[ \therefore \omega = \sqrt{\frac{k}{m}} \]

Since \( x = x_i \) at \( t = 0 \) and

\[ \frac{dx}{dt} = 0 \text{ at } t = 0 \]

\[ x_i = A \cos \theta, \omega A \sin \theta = 0 \]

\[ \therefore \theta = 0 \text{ and } A = x_i \]

\[ x = x_i \cos \left( \sqrt{\frac{k}{m}} t \right) \]
Simple harmonic motion is a sinusoidal oscillation w/ a period $T$

$$x = x_i \cos (\omega t) \quad \omega = \sqrt{\frac{k}{m}}$$

$T = \text{period} = \frac{2\pi}{\omega}$ since $\cos$ is a function w/ period $2\pi$. 

$\omega = \text{angular frequency}$

$f = \text{frequency} = \frac{\omega}{2\pi} = \frac{1}{T}$

$x_i = \text{amplitude}$

If $x = x_i \cos(\omega t + \Theta)$

$\omega t + \Theta \equiv \text{phase}$

$\Theta \equiv \text{phase constant}$
\[ x = x_i \cos(\omega t) \quad \omega = \sqrt{\frac{k}{m}} \]

**Kinetic**
\[ v = \frac{dx}{dt} = -\omega x_i \sin(\omega t) \]
\[ K = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 x_i^2 \sin^2(\omega t) \]

**Potential**
\[ U = \frac{1}{2} k x_i^2 \]
\[ = \frac{1}{2} k x_i^2 \cos^2(\omega t) \]

**Total energy:**
\[ K + U = \frac{1}{2} m \omega^2 x_i^2 \sin^2(\omega t) + \frac{1}{2} k x_i^2 \cos^2(\omega t) \]
\[ = \frac{1}{2} m \frac{k}{m} x_i^2 \sin^2(\omega t) + \frac{1}{2} k x_i^2 \cos^2(\omega t) \]
\[ = \frac{1}{2} k x_i^2 (\sin^2(\omega t) + \cos^2(\omega t)) \]
\[ = \frac{1}{2} k x_i^2 \]

The total energy remains constant at \( \frac{1}{2} k x_i^2 \)
The instantaneous speed of a mass undergoing simple harmonic motion on the end of a spring depends on

A) the amplitude of oscillation.
B) the frequency of oscillation.
C) the period of oscillation.
D) the time at which the speed is measured.
E) all of these.

The force constant of a massless spring is 25.0 N/m. A mass of 0.45 kg is oscillating in simple harmonic motion at the end of the spring with an amplitude of 0.32 m. The maximum speed of the mass is

A) 5.7 m/s  B) 56 m/s  C) 7.4 m/s  D) 2.4 m/s  E) 10 m/s