TODAY: Thermodynamics

- Ideal Gas Law & Temperature, $T$
- Equipartition Theorem
- Heat, $Q$, and 1st Law of Thermodynamics: $\Delta E_{\text{th}} = W + Q$
- Heat Capacity
- Ideal Gas: PV Diagrams and Energy Conservation
  - isochoric ($V = \text{constant}$)
  - isobaric ($P = \text{constant}$)
  - isothermal ($T = \text{constant}$)
  - adiabatic ($Q = 0$)
- 2nd Law of Thermodynamics
- Heat Engines! - the Otto cycle
- Course Evals

ASSIGNMENTS:
- HW 11 due Saturday
- Practice Problems will appear on MP on Saturday
An ideal gas is characterized by \([P, V, T]\) (‘state variables’).

Since \(PV = Nk_B T\), \(T\) is not independent of \(P\) and \(V\). We can then pictorially represent the state of the system by a \(P-V\) plot.

Suppose a gas in a cylinder pushes against a piston and expands the volume.

(constant \(T\))
What is the work done by the gas to produce this change?

\[ W_{\text{by gas}} = \int_{v_1}^{v_2} dV P(V) \]

Pressure as a function of \( V \).

The work done on the gas is

\[ W = -\int_{v_1}^{v_2} dV P(V) \]

\[ \Delta E_{\text{th}} = W + Q \]
**Isothermal Process**

\[ PV = nRT \]

\[ \Delta E_{th} = W + Q = -P\Delta V + Q \]

\[ \Delta T = 0 \]

\[ \Delta P < 0 \]

\[ \Delta E_{th} = 0 \]

All the heat, \( Q \), goes into work, \( W \), and none goes into the thermal energy of the gas.

\[ P = \frac{(nRT)}{V} \]

\[ \Delta E_{th} = 0 = -\int_{V_i}^{V_2} PdV + Q \]

\[ W = Q = nRT \int_{V_i}^{V_2} \frac{dV}{V} \]

\[ = nRT \ln\left(\frac{V_2}{V_1}\right) \]
Isochoric Process

\[ PV = nRT \]

\[ \Delta E_{th} = W + Q = -P \Delta V + Q \]

\[ \Delta V = 0 \]

\[ \Delta E_{th} = -\int_{V_1}^{V_2} PdV + Q \]

\[ \Delta P = \left( \frac{nR}{V} \right) \Delta T \]

All the heat, \( Q \), goes into changing the thermal energy, \( E_{th} \), of the gas and none goes into work, \( W \).
Isobaric Process

\[ PV = nRT \]
\[ \Delta E_{th} = W + Q = -P\Delta V + Q \]

\[ \Delta E_{th} = -\int_{V_1}^{V_2} PdV + Q \]
\[ = -P(V_2 - V_1) + Q \]
\[ \Delta V = V_2 - V_1 = \left( \frac{nR}{P} \right) \Delta T \]

\[ \Delta P = 0 \]
\[ Q = nC_P \Delta T \]
\[ \Delta T > 0 \]
\[ \Delta V > 0 \]
\[ \Delta E_{th} > 0 \]

The heat goes into both the thermal energy of the gas and into work.
Adiabatic Process

\[ PV = nRT \]

\[ \Delta E_{th} = W + Q = -P \Delta V + Q \]

Q = 0
Adiabatic Expansion
(Compression)

\[ \Delta E_{th} = W = -P \Delta V \]

\[ \Delta V > 0 \]

\[ \Delta E_{th} < 0 \]

\[ \Delta T < 0 \]

The thermal energy goes into work.

Adiabatic processes obey

\[ PV^\gamma = \text{constant} \]

where \[ \gamma = \frac{C_p}{C_v} = \begin{cases} 
\frac{5}{3} & \text{monatomic gas} \\
\frac{7}{5} & \text{diatomic gas} 
\end{cases} \]

(See pg. 528 - 529)
Relationship between $C_p$ and $C_v$

\[ \Delta T = T_2 - T_1 \]

**A: isochoric**  \[ w = 0 \Rightarrow \Delta E_{th_A} = 0 + q |_v = n C_v \Delta T \]

**B: isobaric**  \[ w = -p \Delta V \Rightarrow \Delta E_{th_B} = -p_1 \Delta V + q |_p = -p_1 \Delta V + n C_p \Delta T \]

Since \[ PV = nRT, \quad \Delta V = \frac{nR \Delta T}{P_1} \]

\[ \therefore \Delta E_{th_B} = -R \frac{nR}{P_1} \Delta T + n C_p \Delta T \]

\[ = n (C_p - R) \Delta T \]

Since \[ \Delta E_{th_A} = \Delta E_{th_B} \]

\[ n C_v \Delta T = n (C_p - R) \Delta T \]

\[ \therefore C_v + R = C_p \]
Which is true?

A. $Q = 0$    B. $Q > 0$    C. $Q < 0$
Which is true?

A. $Q = 0$  
B. $Q > 0$  
C. $Q < 0$
The Second Law of Thermodynamics

No thermodynamic process has the sole effect of extracting heat from a colder reservoir and delivering it to a hotter reservoir.

Entropy

For heating processes at constant temperature, \( \Delta S = \frac{Q}{T} \)

Entropy characterizes the possible microstates of a system.

Goal: 1) Build heat engines for application (engineering)
2) Build refrigerators for application.
What is a Heat Engine anyway…?

A heat engine transforms heat into work and returns to its initial state (closed-cycle device)

\[
(\Delta E_{\text{th}})_{\text{net over a cycle}} = 0
\]

Energy conservation \((\omega / \Delta E_{\text{mech}} = 0)\)

\[
\Delta E_{\text{th}} = Q + W = 0
\]

\(W_s = \text{Work done on external material (e.g. piston)}\)

\[
-W \Rightarrow W_s = Q = Q_H - Q_c = W_{\text{out}}
\]

Hot reservoir at temperature \(T_h\)

Engine

Cold reservoir at temperature \(T_c\)
The Otto Cycle
(no - not ‘auto’)

Exhaust valve open
Gas vapor and air mixture
Intake valve open
A mixture of gasoline vapor and air enters the combustion chamber as the piston moves down.

Intake stroke

Exhaust stroke
The piston moves up again to exhaust the burned gases.

Both valves closed
The expanding gas moves the piston down, a stage called the power stroke.

Both valves closed
When the gas ignites, it expands.

Compression stroke

Figure 20-2 Internal-combus
Example of a heat engine: the Otto cycle

Nikolaus Otto 1867

Ignition (isochoric)

Power stroke (adiabatic expansion)

Compression stroke (adiabatic compression)

Exhaust (isochoric)