Physics 207, Lecture 29, Dec 14 ‘09

TODAY: Review!

ASSIGNMENTS:
● Practice Problems on MP

FINAL EXAM:
● Bring calculator, equation sheet (8.5” x 11”, 2 sides)
● Covers entire course
    - (but skip 17.6 ‘calorimetry’ and 17.8 ‘heat transfer’)
● Consultation hours continue through Tues Dec 22
Topics

Foundation: {Newton's Laws

Prob solving technique: Conservation

"Many particle behavior":

Chap 1: Concepts of motion
2 Kinematics in 1D
3 Vectors and coord sys.
4 Kinematics in 2D
5 Force and Motion
6 Dynamics in 1D
7 Newton's 3rd law
8 Dynamics in 2D

Chap 9: Impulse & Momentum
Chap 10: Energy
Chap 11: Work
Chap 12: Rotation of a rigid body

Chap 14: Oscillations
Chap 15: Fluids + Elasticity
Chap 20: Travelling waves
Chap 21: Superposition

Chap 17-19: Thermo
Strategy

Typical Problem Solving: a) Translate words into variables physics → b) Identify useful mathematical relationships c) Solve for the variables of interest

Typical error in student thinking: looking for the "right formula" Physics is not about cataloging all possible formulae on a cheat-sheet and looking for the right formula to plug numbers into in a single step. Instead, it is about using simple principles to derive mathematical relationships among physical quantities.
a.) A uniform rod of length 2.8 m and mass 3.9 kg is free to rotate on a frictionless pin through one end. The rod is released from rest in the horizontal position. What is the torque about the pin at the moment the rod is released? (13 pts)

b.) Same situation as "a". What is the angular velocity of the rod at its lowest position? (12 pts)
a.) A uniform rod of length 2.8 m and mass 3.9 kg is free to rotate on a frictionless pin through one end. The rod is released from rest in the horizontal position. What is the torque about the pin at the moment the rod is released? (13 pts)

\[ T = \frac{1}{2} mg \]  
\[ = \frac{2.8}{2} \times 3.9 \times 9.8 = 53. \]

53 N-m

b.) Same situation as "a". What is the angular velocity of the rod at its lowest position? (12 pts)

\[ \text{change in } PE = \text{change in } KE \]

\[ mg \frac{1}{2} L = \frac{1}{2} I \omega^2 \]

\[ I = \frac{1}{3} mL^2 \]

\[ \omega = \sqrt{\frac{3g}{L}} = \sqrt{\frac{3 \times 9.8}{2.8}} = 3.2 \frac{\text{rad}}{\text{sec}} \]
g. A block of mass $m_b$ rests on a horizontal surface and is accelerated by means of a horizontal cord that passes over a frictionless peg to a hanging weight of mass $m_W$. The coefficient of kinetic friction between the block and the horizontal surface is $\mu$. If you are given that the mass $m_W$ is accelerating downward, the acceleration is $\text{(3 pt)}$.
g. A block of mass \( m_b \) rests on a horizontal surface and is accelerated by means of a horizontal cord that passes over a frictionless peg to a hanging weight of mass \( m_w \). The coefficient of kinetic friction between the block and the horizontal surface is \( \mu \). If you are given that the mass \( m_w \) is accelerating downward, the acceleration is (3 pt)

a) \( \frac{m_w - \mu m_b}{m_b + m_w} \)  
b) \( \frac{m_b - \mu m_w}{m_b + m_w} \)  
c) \( \frac{m_w(1-\mu) - m_b}{m_b + m_w} \)  
d) \( \frac{m_b - m_w}{\mu m_b + m_w} \)  
e) none of the above

\[
\begin{align*}
    m_b a &= T - \mu m_b g \\
    m_w a &= m_w g - T \\
    (m_b + m_w) a &= (m_w - \mu m_b) g \\
    a &= \frac{(m_w - \mu m_b) g}{m_b + m_w}
\end{align*}
\]
On a stringed instrument w/ string of length L having a wave speed \( v_w \).

A) The allowed wavelength \( \lambda_n \) (\( n=1,2,3,\ldots \)) for standing waves on the string are which?

B) A standing wave of wavelength \( \lambda_n \) (on the string) has a frequency of oscillation \( f_n \) of what?
On a stringed instrument with string of length \( L \) having a wave speed \( \nu \).

A) The allowed wavelength \( \lambda_n \) (\( n = 1, 2, 3, \ldots \)) for standing waves on the string are which?

**Ans**

\[
\frac{A}{2} \left( \sin(kx - \omega t) + \sin(kx + \omega t) \right) = A \sin kx \cos \omega t
\]

\[
kL = n\pi
\]

\[
\frac{2\pi}{\lambda_n} L = n\pi \Rightarrow \lambda_n = \frac{2L}{n}
\]

B) A standing wave of wavelength \( \lambda_n \) (on the string) has a frequency of oscillation \( f_n \) of what?

\[
\lambda_n f_n = \nu \Rightarrow f_n = \frac{\nu}{2n}
\]
b. An ideal, incompressible fluid flows through a conical nozzle out into free air. (The fluid flows from the thick end of the nozzle to the thin end.) Circle the graph that best illustrates how the gauge pressure varies along the length of the nozzle. The points labeled A and B on the figure are shown on each graph. Assume that the viscosity of the fluid is negligible. (3 pts)
b. An ideal, incompressible fluid flows through a conical nozzle out into free air. (The fluid flows from the thick end of the nozzle to the thin end.) Circle the graph that best illustrates how the gauge pressure varies along the length of the nozzle. The points labeled A and B on the figure are shown on each graph. Assume that the viscosity of the fluid is negligible. (3 pts)

\[ \frac{1}{2} \rho v^2 + \rho g y + p \approx \text{const}, \]

\[ A \propto (c-x)^2 \]

\[ A_A v_A = A_B v_B \Rightarrow v \propto \frac{1}{(c-x)^2} \]

\[ p \approx \text{const} - \frac{\text{const}_2}{(c-x)^4} \]
A pendulum has length $l$ (the string is "massless"). The bob has a mass of $m$. We release the bob with zero speed when the string makes an angle $\theta = 90^\circ$ with the vertical. Friction of any kind can be ignored. The gravitational acceleration is $g$. Express your answers in terms of $l$, $m$, and $g$.

What is the tension in the string when $\theta = 0^\circ$?
A pendulum has length $l$ (the string is “massless”). The bob has a mass of $m$. We release the bob with zero speed when the string makes an angle $\theta = 90^\circ$ with the vertical. Friction of any kind can be ignored. The gravitational acceleration is $g$. Express your answers in terms of $l$, $m$, and $g$.

What is the tension in the string when $\theta = 0^\circ$?

\[ \frac{1}{2} m v^2 = m g l \]

\[ v = \sqrt{2 g l} \]

\[ T - m g = \frac{m v^2}{l} = \frac{m}{l} 2 g l \]

\[ \Rightarrow T = 3 m g \]
Mean and Standard Deviation of ‘Random’ or ‘Statistical’ Errors

What is the period of this pendulum?

<table>
<thead>
<tr>
<th>Trial number, $i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured Value, $x_i$ (seconds)</td>
<td>3.9</td>
<td>3.5</td>
<td>3.7</td>
<td>3.4</td>
<td>3.5</td>
</tr>
<tr>
<td>Deviation, $d_i = x_i - \bar{x}$ (seconds)</td>
<td>0.3</td>
<td>-0.1</td>
<td>0.1</td>
<td>-0.2</td>
<td>-0.1</td>
</tr>
</tbody>
</table>

Mean =

$$\bar{x} = \frac{1}{N} \sum x_i = \frac{3.9+3.5+3.7+3.4+3.5}{5} \text{ s} = 3.60 \text{ s}$$

Standard Deviation $\sigma = 

$$\sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2} = \sqrt{\frac{0.09+0.01+0.01+0.04+0.01}{4}} \text{ s} = 0.2 \text{ s}$$

Standard deviation of the mean
Standard error
Error in the mean

$$\left\{ \sigma_{\mu} = \frac{\sigma}{\sqrt{n}} = 0.09 \text{ s} \right\}$$

Our best estimate of the period is 3.60 +/- 0.09 s
Normal or Gaussian Distribution

- For random processes
- Suppose we make a total of 10 measurements of x, the period of the pendulum:
  - 3.9, 3.5, 3.7, 3.4, 3.5, 3.6, 3.7, 3.6, 3.8, 3.6
- We plot a histogram of the results:

\[ 2\sigma = 0.28 \]
Error Propagation:

If the errors are independent and random….

- addition and subtraction:
  
  \[ z = x + y \]
  
  \[ \Delta z = \sqrt{\Delta x^2 + \Delta y^2} \]

- multiplication and division:
  
  \[ z = \frac{x^n y^m}{q^k r^l} \ldots \]
  
  \[ \frac{\Delta z}{z} = \sqrt{\left(\frac{n \Delta x}{x}\right)^2 + \left(\frac{m \Delta y}{y}\right)^2 + \left(\frac{k \Delta q}{q}\right)^2 + \left(\frac{l \Delta r}{r}\right)^2} \]

‘relative error’

‘absolute error’
Error propagation

You are trying to compute the density of a spherical kidney stone. You measure the mass to be 8.0 g and the radius to be 2.1 mm. Your uncertainty in the measurements is 0.1 g and 0.1 mm, respectively. What is your best estimate of the density of the stone?

\[
\rho = \frac{m}{V} = \frac{m}{4\pi r^3/3} = 0.206
\]

\[
\frac{\Delta \rho}{\rho} = \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(3\frac{\Delta r}{r}\right)^2} = \sqrt{\left(0.1/8.0\right)^2 + \left(3\frac{0.1}{2.1}\right)^2} = 0.143
\]

only good to 1 sig fig.

\[
\Delta \rho = 0.143 \times 0.206 = 0.0295 = 0.03
\]

one sig. figure

\[
\rho = 0.21 \pm 0.03 \frac{g}{\text{mm}^3}
\]

adjust significant figures

only good to 2 sig figs