Physics 207, Lecture 9, Oct 5 ‘09

TODAY
- Circular motion: 4.5, 4.6, 8.2, 8.3
- Momentum: 9.1, 9.2
- Exams to be returned in discussion this week

Assignments:
- This week’s lab:
  - Equilibrium Force (M-2) & Circular Motion (M-6)
- HW 5 due Friday, Oct 9, 6 pm
Circular Motion

- Car driving around curved track
- Planets orbiting Sun (actually elliptical motion)
- Electrons orbiting nucleus (simplest “Bohr model” for atom)
Prof. Timbie whirls a tennis ball around his head on a string. If the string breaks, the ball:

A. Continues to follow a circular path
B. Follows a straight line
C. Follows a slightly curved path
\[ |\vec{v}| \equiv \text{speed} = \text{scalar} \quad \text{(distinct from } \vec{v} = \text{velocity}) = \text{vector} \]

**Example**

The figure shows the position of the moon at two times, about 7 days apart. Which vector best represents the change in the moon's velocity in this time interval?

Suppose \( |\vec{v}_1| = |\vec{v}_2| \)

**Hence, velocity change is large.**

**How about speed change?**
Question

Suppose the position of a car driving around a circular track is given by

\[ \vec{r}(t) = (R)(\hat{i}\cos(\omega t) + \hat{j}\sin(\omega t)) \]

where \( R \) and \( \omega \) are constants.

a) Between \( t=0 \) and \( t=T \), how much does the speed of the car vary?

b) Express the acceleration as a function of time.
\textbf{Answer} \quad \vec{F}(t) = R \left( \hat{i} \cos(\omega t) + \hat{j} \sin(\omega t) \right) \\
\text{(a)} \quad \vec{v} = \frac{d\vec{r}}{dt} = R \omega (\hat{i} \sin(\omega t) + \hat{j} \cos(\omega t)) \\
\text{speed} = |\vec{v}| = \sqrt{(-R \omega \sin(\omega t))^2 + (R \omega \cos(\omega t))^2} \\
= R \omega \sqrt{\sin^2(\omega t) + \cos^2(\omega t)} \\
= R \omega \\
\text{Hence speed is constant and there is no variation} \\
\text{(b)} \quad \vec{a} = \frac{d\vec{v}}{dt} = -R \omega^2 (\hat{i} \cos(\omega t) + \hat{j} \sin(\omega t)) \\
\& \text{ Note it points inward!}
Circular motion

\[ \vec{a}_c \]

\[ \vec{a}_t \]

Kinematics (i.e., the part)

Centripetal acc

\[ = \frac{v^2}{r} \]

tangential acc

\[ = \frac{dv}{dt} \]
Problem

A rock tied to a string with tension $T(t)$ is swung in a vertical circle (radius $R$) what is the tension at the top of the circle where the speed is $v$?
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**Ans**

\[ T \downarrow \downarrow mg \]

\[ \vec{F}_c = m \vec{a}_c \]

\[ T + mg = \frac{mv^2}{R} \]

\[ T = m \left( \frac{v^2}{R} - g \right) \]
M-6 Uniform Circular Motion

OBJECTIVE: To measure the centripetal force, $F_c$, and compare to

$$F_c = \frac{mv^2}{r} = m\omega^2 r$$

Test this relationship

APPARATUS:

Fig. 1 is a schematic of the equipment. The bobs and springs are removable for weighing. Not shown are table clamp and pulley, slotted masses and weight hanger.

Figure 1: The UCM apparatus.
Example: **Circular Motion Lab**

1. **Measure** $F_s$ (spring force) to stretch spring with uncertainty $\delta F_s \leq \text{error in the mean}$

2. **Compute** $F_c$ (centripetal force) = $m a$ with uncertainty $\delta F_c \leq \text{Error Propagation}$

Are they equal?
Error Analysis (continued)

Recall:

1. Gaussian (normal) Probability Distribution:

\[ f(x) = \frac{1}{\sqrt{2\pi} \sigma_x} e^{-\frac{(x-\bar{x})^2}{2\sigma_x^2}} \]

2. Mean:

\[ \bar{x}_m = \frac{1}{N} \sum_{i=1}^{N} x_i \]

3. Variance:

\[ \sigma_x^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x}_m)^2 \]

4. Error in the mean:

\[ \sigma_{\bar{x}_m}^2 = \frac{\sigma_x^2}{N} \Rightarrow \sigma_{\bar{x}_m} = \frac{\sigma_x}{\sqrt{N}} \]
What makes a rocket take off?

A. Newton’s third law  
B. Conservation of Momentum  
C. Hot gases push against air
"Professor Goddard does not know the relation between action and reaction and the need to have something better than a vacuum against which to react. He seems to lack the basic knowledge ladled out daily in high schools."

New York Times editorial, 1921, about Robert Goddard's revolutionary rocket work.

"Correction: It is now definitely established that a rocket can function in a vacuum. The 'Times' regrets the error."

Imagine a collision between 2 objects

\[ \overrightarrow{F}_{A\rightarrow B} = -\overrightarrow{F}_{B\rightarrow A} \]

\( F(t) \) may be complicated!

But,

\[ \overrightarrow{F}(t) = m \overrightarrow{a} = m \frac{d\overrightarrow{v}}{dt} \]

\[ \int_{t_i}^{t_f} \overrightarrow{F}(t) \, dt = m \int_{v_i}^{v_f} \overrightarrow{v} \, dv = m \overrightarrow{v}_f - m \overrightarrow{v}_i \]