Print your name and section clearly above. If you do not know your section number, write your TA’s name.

Your final answer must be placed in the box provided. **You must show all your work to receive full credit.** If you only provide your final answer (in the box), and do not show your work, you will not receive very many points.

Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units, and also the direction of vectors.

You are allowed one 8½ x 11” sheet of notes and no other references. The exam lasts exactly 90 minutes.

### Speed of light in vacuum:
\[ c = 3 \times 10^8 \text{ m/s} \]

### Permittivity of free space
\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2 \]

### Coulomb constant
\[ k = \frac{1}{4\pi \varepsilon_0} = 9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \]

### Magnitude of electron charge
\[ e = 1.6 \times 10^{-19} \text{ C} \]

### Permeability of free space
\[ \mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m}/\text{A} \]

### Electron mass
\[ m_e = 9.1 \times 10^{-31} \text{ kg} \]

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**Problem 1:** _______ / 20  
**Problem 2:** _______ / 15  
**Problem 3:** _______ / 20  
**Problem 4:** _______ / 20  
**Problem 5:** _______ / 20  

**TOTAL:** _______ / 95
1) [20 points, 4 points each]. Circle the correct answer, AND write some sentences or show a calculation explaining how you obtained the answer.

i) The electric field in the figure at right is \( \vec{E} = 400 \, \text{N/C} \, \hat{x} \). What is the minimum work you must do to move a charge \( q = -10 \, \mu \text{C} \) from point \( A = (0, 1) \) m to point \( B = (2, 0) \) m? Note that the charge is negative.

a) 0.2J
b) 0.004J
c) -0.004J
d) 0.008J
e) -0.008J

Explanation/work:

ii) A parallel-plate capacitor is connected to a battery and fully charged. While the battery is still connected, the plates are pulled apart so that the separation between them is doubled. The energy stored in the capacitor behaves as

a. \( U_{\text{after}} = U_{\text{before}} \)
b. \( U_{\text{after}} = 2U_{\text{before}} \)
c. \( U_{\text{after}} = 4U_{\text{before}} \)
d. \( U_{\text{after}} = (1/2)U_{\text{before}} \)
e. \( U_{\text{after}} = (1/4)U_{\text{before}} \)

Explanation/Work:

iii) An electron is moving at \( 10^4 \, \text{m/s} \) in the \( \hat{x} \) direction. It is in a uniform magnetic field of 2 T that points in the \( \hat{y} \) direction. Calculate the magnitude of the force on the electron from the magnetic field.

Explanation/work:

<table>
<thead>
<tr>
<th>Force</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
iv) An 0.5µF (1µF=10^{-6}F) capacitor is arranged so that a constant (does not depend on time) current of 1 µA (1µA=10^{-6}A) flows in its wire. How long will it take to charge the capacitor to 10V?

a. 0.1s
b. 0.2s
c. 0.5s
d. 1s
e. 2s
f. 5s
g. 10s

Explanation/Work

v) Calculate the power dissipated by the 60Ω resistor.

a) 0.2W
b) 0.6W
c) 1.0W
d) 1.95W
e) 2.4W
f) 3.6W
g) 5.4W

Explanation/work:
2) [15 points total] Below are four equally-spaced infinite sheets of charge. Graph the potential $V$ and x-component of the electric field $E_x$ as a function of distance along the x-axis. Plot both between and outside the sheets. For full credit, you need both for the correct sign and the correct x-dependence.

a) [5 points] E-field plot
($E_x>0$ means field points to right)

b) [5 points] Potential plot.
(leftmost plate defined to be $V=0$)
c) [5 pts] Calculate $V_b - V_a$, the electric potential difference between points $a$ and $b$, as shown in the figure above part a).

Use $\sigma = 2 \times 10^{-9} \text{C/m}^2$ to get the charge density of each sheet. Note that the sheets do not all have the same sign of charge density. Make sure you get these sign corrects, and also the sign of the potential!

$V_b - V_a =$

<table>
<thead>
<tr>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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</table>
3) [20 points total]
All parts of this problem ask about the circuit below:

![Circuit Diagram]

V_{RI} = V_a - V_b
V_{C} = V_e - V_f
V_{R2} = V_c - V_d

a) [10 points] The switch in the circuit above has been closed only a short time, so that
the capacitor is not fully charged. The table below lists pairs or groups of quantities
identified in the figure above. On the line to the right, fill in the relationship between
them from device rules, Kirchhoff's Rules, etc.
(The arrow in the figure shows the direction of current when it is positive)

<table>
<thead>
<tr>
<th>Quantities</th>
<th>Relationship (2pts each)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) I_{R1}, V_{R1}</td>
<td>V_{R1} = I_{R1}R_{1}</td>
</tr>
<tr>
<td>(b) V_{R2}, V_{C}</td>
<td></td>
</tr>
<tr>
<td>(c) I_{R1}, I_{R2}, I_{C}</td>
<td></td>
</tr>
<tr>
<td>(d) Q, V_{C}</td>
<td></td>
</tr>
<tr>
<td>(e) I_{C}, Q</td>
<td></td>
</tr>
<tr>
<td>(f) V_{batt}, V_{R1}, V_{R2}</td>
<td></td>
</tr>
</tbody>
</table>
For parts b) and c) use

\[ V_{\text{batt}} = 9 \text{ V} \]
\[ R_1 = 100 \Omega \]
\[ R_2 = 200 \Omega \]
\[ C = 2 \mu \text{F} \]

b) [5 points] In this part the switch has been closed long enough that the currents are steady, and do not change in time. The switch is then suddenly opened. What is the current \( I_C \) immediately after the switch is opened?
(The arrow in the figure shows the direction of current when it is positive)

\[ I_C = \]

<table>
<thead>
<tr>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
</table>

\[ I_C = \]

<table>
<thead>
<tr>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
</table>

c) [5 points] In part b), the charge on the capacitor changes in time after the switch is opened. Just as the capacitor charge reaches 10\( \mu \text{C} \), you now close the switch again. What is the current \( I_C \) immediately after the switch is closed?
(The arrow in the figure shows the direction of current when it is positive)

\[ I_C = \]

<table>
<thead>
<tr>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
</table>

\[ I_C = \]

<table>
<thead>
<tr>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
</table>
4) **[20 points total, 5 points each]**

a) **[5 points]** Calculate \( V_b - V_a \), the electric potential difference between points \( b \) and \( a \).

\[ \Delta V = V_b - V_a \]

<table>
<thead>
<tr>
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</tr>
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<tbody>
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</table>

b) **[5 points]** The resistivity of salt water depends on the salt content. You are told that the resistivity of saltwater appropriate for your aquarium is 0.5 \( \Omega \)-m. You measure by filling the container shown with saltwater, and passing 50 mA of current from the top to the bottom. What voltage should you measure for 0.5 \( \Omega \)-m saltwater?

\[ \text{Voltage} = \]
c) **[5 points]** A square ring of uniform positive charge density in the y-z plane as shown. Sketch a graph of \( V(x) \) vs \( x \), the electric potential on the x-axis as a function of the position \( x \) along the x-axis.

![Graph of V(x) vs x](image)

\[
V(x) = \begin{cases} 
\text{constant} & \text{for } x < 0 \\
0 & \text{for } 0 \leq x < 5 \\
\text{constant} & \text{for } x \geq 5 
\end{cases}
\]

\( V(x) \) is constant for \( x < 0 \) and \( x \geq 5 \), and zero for \( 0 \leq x < 5 \).

d) **[5 points]** Two capacitors are charged separately as shown below, with
\[
V_{C1} = V_a - V_b = +10V \\
V_{C2} = V_c - V_d = -5V
\]

Note that these voltages have opposite sign.

What is the voltage across the capacitors \( V_a - V_b \) after they are connected as shown?

\[
\begin{array}{l}
V_{C1} = V_a - V_b = +10V \\
b \end{array} \quad \begin{array}{l}
V_{C2} = V_c - V_d = -5V \\
d \end{array}
\]

\( C_1 = 1 \mu F \)
\( C_2 = 2 \mu F \)

\[
\begin{array}{|c|c|}
\hline
\text{Value} & \text{Units} \\
\hline
\hline
\end{array}
\]

\( V_a - V_b \) after connecting =
5) **[20 pts]** Short questions regarding magnetic fields and forces.

a) **[5 pts]** An infinite wire along the x-axis carries a current of 10A in the x-direction. Calculate the magnitude and direction of the magnetic field 2 cm from the wire on the x axis.

\[ B = \frac{\mu_0 I}{2\pi r} \]

where \( \mu_0 \) is the permeability of free space, I is the current, and r is the distance from the wire.

<table>
<thead>
<tr>
<th>Value</th>
<th>Units</th>
<th>Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I ( = 10A )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>r ( = 2 \text{ cm} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Magnetic field** =

\[ B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \text{T} \cdot \text{m/A} 	imes 10 \text{A}}{2\pi \times 0.02 \text{ m}} = 1 \text{T} \]

b) **[5 pts]** A circular loop of wire carries a current of \( I = 10A \) clockwise as shown. The magnetic field is zero for all \( x < 0 \), and nonzero for all \( x > 0 \). When you let go of the loop what direction does it move? EXPLAIN

**Motion direction**

\[ x < 0 \]

**EXPLANATION:**

The force on the current-carrying loop is given by the formula:

\[ F = BIL \sin \theta \]

where B is the magnetic field, I is the current, L is the length of the loop, and \( \theta \) is the angle between the current and the magnetic field. When the magnetic field is nonzero, the force is directed towards the positive x-axis, causing the loop to move to the right if \( x > 0 \).
c) [5 pts] A bent wire loop is entirely in the x-y plane. It carries 100A through all its sections in a clockwise direction. It is in a magnetic field of 1T that points in the x-direction. Calculate the magnitude of the total net force on the bent wire loop.

\[
\text{Force} = \begin{array}{c|c}
\text{Magnitude} & \text{Units} \\
\hline
\end{array}
\]

b) [5 pts] Below is a square loop with a clockwise current of 10 A in all of its sides. What is the direction of the magnetic field at the point indicated? You cannot make an airtight argument without a full calculation, but discuss in terms of the contributions of all the currents to the field.

\[
\text{Field direction} = \begin{array}{c}
\end{array}
\]

Explanation: