Physics 208 Group Problem
Superposition is one of the most useful ideas in physics. Here you apply superposition to electromagnetic (EM) waves, and prepare for the polarization lab this week.

An EM wave’s polarization is defined by the direction of its electric field vector. The plots at right show the electric field of two EM waves at a single position as a function of time. The electric field of the first oscillates along the y-axis, the second along the x-axis. Both of the waves propagate in the z-direction (not shown).

1) We can “add” these two EM waves at any time by adding the electric field vectors. This is the superposition principle. You do this in part b.
   a) First draw the electric field vectors of both waves at each instant of time.

   ![Diagram of electric field vectors at different times](image)

   b) On the plot above, draw the vector sum (superposition) of the two waves at each instant of time by adding as vectors the electric fields of the two waves.

   c) Describe the polarization of the resulting EM wave.
2) Here are two different EM waves. Again only the electric field vector is shown. They are linearly polarized with their plane of polarizations at 90°, as in part 1.

But the waves have a relative phase shift of 90° (1/4 wavelength) – when one electric field vector is at its maximum value, the other is zero. \( T \) is the period of the wave.

a) On the axes below, draw the electric fields of the two EM waves at the indicated time, and then draw their vector sum.

b) Describe the polarization of the resulting EM wave. This is circular polarization.
3) To make circularly polarized light, you need two EM waves with a relative delay of $\frac{1}{4}$ wavelength.

There is an easy way to do this. Some materials are optically anisotropic, meaning that light waves polarized along different directions travel at different speeds. This means that the slower one is delayed relative to the faster one. The thickness of the plate is such that the time delay is $\frac{1}{4}$ of the wave oscillation period $T$ (equivalent to a $\frac{1}{4}$ wavelength delay, or 90° in phase). These are called $\frac{1}{4}$-wave plates.

Suppose you have such a plate that gives exactly $\frac{1}{4}$ wavelength delay between the fast and slow axes for 540 nm wavelength light. The index of refraction along the slow axis is $n_{\text{slow}}=1.5$, and along the fast axis is $n_{\text{fast}}=1.4955$.

a) What is the propagation speed?
   i) along the slow axis?
   ii) along the fast axis?

b) How thick must the plate be so that the wave along the slow axis takes $T/4$ seconds longer to make it through the waveplate than the wave along the fast axis? Here $T$ is the oscillation period of 540 nm light in vacuum.

[Hint: if the waveplate thickness is $L$, how many seconds does it take for the slow wave to travel through the waveplate? The fast wave?]