A cell membrane can be treated as a parallel-plate capacitor where the two plates are separated by 8 nm (1 nm = $10^{-9}$ m). This is because the plasma membrane is an insulator, and the conducting fluids inside and outside the cell behave as metals that can have different charges, and be at different electric potentials.

a) If you made a scale model of the cell as a sphere, big enough so that the plasma membrane were 5mm thick, about what would be the diameter of the spherical cell?

b) In the space below, sketch a cross-section of the cell wall in your scale model (full size) showing the plasma membrane spacer, and the two conductors on either side. Show the curvature of the cell wall as accurately as you can. Explain if you have any difficulties.

c) Referring to your drawing, explain why it is a good approximation to model the cell membrane as a parallel plate capacitor.
d) Modeling the cell wall as a parallel-plate capacitor, find the capacitance of the cell wall of a 100µm diameter (50µm radius) spherical cell. Model the cell wall as a parallel plate capacitor, and assume that the plasma membrane acts only as a spacer, so that the space between the capacitor plates is empty.

e) The plasma membrane acts as a dielectric, increasing the wall capacitance to about 350pF for this size cell. Calculate the potential difference $V_{in} - V_{out}$ between the cell interior and exterior considering that outside the cell there are $N=150$ million more singly ionized sodium ions ($\text{Na}^+$, charge $= 1.6 \times 10^{-19}$ C) than inside the cell. This potential difference is called the resting potential of the cell.

f) The cell wall capacitor can change its potential difference by transferring charge from one side to the other through (ion) channels in the cell membrane. An experimenter measures a current of $+1.2$ pA ($1 \text{ pA} = 10^{-12}$ A) through a single ion channel in a cell membrane. The current flows from outside the cell to the inside. This is usually called depolarization, since it tends initially to decrease the charge separation on the capacitor.

Calculate the change in potential across the cell membrane that arises from the current. Again using 350 pF for the cell wall capacitance There are 1.0 channel/µm² ($10^6$ channels/cm²) and the current lasts for 1.0 milli-second.