Homework # 4 Solutions

1. (a) Given the radius a of the inner sphere, the radius b of the outer sphere, and the charge q on the sphere, we are asked to calculate ΔV. To do so, use the formula for the capacitance of two concentric spheres,

$$C = \frac{ab}{k(b-a)},$$

to obtain

$$\Delta V = \frac{q}{C} = kq\frac{b-a}{ab}.$$

(b) The energy U is

$$U = \frac{1}{2}q\Delta V = \frac{1}{2}\frac{kq^2(b-a)}{ab}.$$

2. We are told that the energy of the two capacitors C_1 and C_2 is n times greater when they are connected to a battery in parallel than when they are connected in series to the same battery, and asked to determine the ratio C_1/C_2. Given

$$U_p = \frac{1}{2}C_pV^2 = nU_s = n\left(\frac{1}{2}C_sV^2\right),$$

we see that

$$\frac{C_p}{C_s} = n. \tag{1}$$

Since

$$C_p = C_1 + C_2$$

and

$$C_s = \frac{C_1C_2}{C_1C_2},$$

Eq. (1) becomes

$$(C_1 + C_2)\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = n.$$

Therefore,

$$\frac{C_1}{C_2} + \frac{C_2}{C_1} = n - 2. \tag{2}$$

Defining $x \equiv C_1/C_2$, Eq. (2) becomes

$$x^2 - (n-2)x + 1 = 0,$$

which can be solved using the quadratic formula:

$$x = \frac{C_1}{C_2} = \frac{n-2}{2} \pm \sqrt{\frac{(n-2)^2}{4} - 1}.$$

3. (a) We are given the plate area A, spacing d, dielectric constant κ, and the electric field in the dielectric E_d. The surface charge density can be found from

$$E_d = \frac{\sigma}{\kappa\epsilon_0},$$

such that

$$\sigma = \kappa \epsilon_0 E_d.$$

(b) The charge density on the surface of the dielectric (the bound charge density) is

$$\sigma_1 = \frac{\kappa - 1}{\kappa} \sigma = (\kappa - 1) \epsilon_0 E_d.$$

(c) The total energy stored in the capacitor is the energy density ($u = \frac{1}{2}\kappa\epsilon_0 E_d^2$) multiplied by the volume (Ad):

$$U = \frac{1}{2} \kappa \epsilon_0 E_d^2 A d.$$

4. The energy stored in the capacitor with the mica dielectric ($\kappa = 7$) is given by

$$U = \frac{Q^2}{2C},$$

where

$$C = \kappa C_0 = \kappa \frac{\epsilon_0 A}{d}.$$

Therefore,

$$U = \frac{Q^2 d}{2\kappa \epsilon_0 A}.$$

5. (a) In this problem, a capacitor is connected to a battery of voltage V and charged, then the battery is disconnected. The charge on the capacitor is

$$Q_1 = C_1 V = \frac{\epsilon_0 A}{d_1} V.$$

(b) Since the battery is disconnected before the spacing is adjusted, the charge is constant:

$$Q_2 = Q_1.$$

(c) The capacitor spacing is adjusted to the value d_2, such that

$$C_2 = \frac{\epsilon_0 A}{d_2}.$$

The voltage is

$$V_2 = \frac{Q_2}{C_2} = \left(\frac{\epsilon_0 A}{d_1} V\right)\left(\frac{d_2}{\epsilon_0 A}\right) = \frac{d_2}{d_1} V.$$

(d) The work required to move the capacitor plates is

$$W = \Delta U = \frac{Q_2^2}{2C_2} - \frac{Q_1^2}{2C_1}.$$

From the results to (a)–(c), we see that this is given by

$$W = \frac{1}{2}\left(\frac{\epsilon_0 A V}{d_1}\right)^2 \left(\frac{d_2}{\epsilon_0 A} - \frac{d_1}{\epsilon_0 A}\right) = \frac{1}{2}\frac{\epsilon_0 A V^2}{d_1^2}(d_2 - d_1).$$