Physics 202, Lecture 3

Today’s Topics

- Electric Field (Review)
- Motion of charged particles in external E field
- Conductors in Electrostatic Equilibrium (Ch. 21.9)
- Gauss’s Law (Ch. 22)

Reminder: HW #1 due Wed 1/30 11 PM.

HW #2 on MasteringPhysics: due Wed 2/6 11 PM.

The Electric Field

1. Charges are a source of electric fields.

   Single point charge: \( \vec{E}(\vec{r}) = k \frac{q}{r^2} \hat{r} \)

   Visualization: field lines

   Multiple charges: \( \vec{E} = \sum_i \vec{E}_i = k \sum_i \frac{q_i}{r_i^2} \hat{r}_i \)

   Distributions: \( d\vec{E} = k \frac{dq}{r^2} \hat{r} \quad \vec{E} = \int d\vec{E} \)

Examples: charged line, ring, disk
The Electric Field (II)

2. Charges respond to electric fields.

\[ \vec{F} = q\vec{E} \]
\[ \vec{a} = \frac{F}{m} = \frac{q\vec{E}}{m} \]

Uniform $E$ field: uniformly accelerated motion!

$v = v_0 + at$
$s = s_0 + v_0t + \frac{1}{2}at^2$
$v^2 = v_0^2 + 2as$

Examples: 21.56, 2d example (text ex. 21-16)

Conductors And Electrostatic Equilibrium (21.9)

Conductors: charges (electrons) able to move freely $\Rightarrow$
Charges redistribute when subject to $E$ field.
Charge redistribution $\Rightarrow$ electrostatic equilibrium.

Initial $\Rightarrow$ transient, $<10^{-16}$s $\Rightarrow$ equilibrium
(right after $E$ applied)
Conductors And Electrostatic Equilibrium

1. E field is always zero inside the conductor.

2. All charges reside on the surface of conductor.

3. E field outside conductor is perpendicular to the surface, has magnitude \( E = \sigma / \varepsilon_0 \)
   (show later today using Gauss’s Law)

E field is also zero inside any cavity within the conductor.

The above properties are valid regardless of the shape of and the total charge on the conductor!

Introducing Gauss’s Law

An equivalent statement of Coulomb’s law:
(a actually more general, true in all situations)
\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_0} \]
Electric flux

Karl Friedrich Gauss

one of four fundamental equations of electromagnetism (Maxwell’s equations)

a powerful calculational tool for finding E when the charge distribution exhibits a high degree of symmetry.
Electric Flux

Electric flux $\Phi_E$ through a surface:
(component of E-field $\perp$ surface) $\times$ (surface area)

Flux proportional to 
# E-field lines 
penetrating surface

Why Perpendicular Component?

Normal vector to surface makes angle $\theta$ with respect to field:

$\vec{E} = E_{||}\hat{s} + E_{\perp}\hat{n}$

Component $||$ surface
Component $\perp$ surface

Only $\perp$ component 'goes through' surface

- $\Phi_E = EA \cos \theta$
- $\Phi_E = 0$ if E parallel A
- $\Phi_E = EA$ (max) if E $\perp$ A
- Flux SI units are N·m²/C
Total flux

E not constant:
add up small areas
where it is constant

Surface not flat
add up small areas
where it is ~ flat

\[ \delta \Phi_E^i = E_i \delta A_i \cos \theta = \vec{E}_i \cdot \vec{\delta A}_i \]

Add them all up:
\[ \Phi_E = \int_{\text{surface}} \vec{E} \cdot d\vec{A} \]

Flux through Closed Surfaces

Compare fluxes through closed surfaces \( s_1, s_2, s_3 \):

\[ \Phi_{s_1} = \Phi_{s_2} = \Phi_{s_3} \]

(# field lines same through all 3 surfaces)

Note: if no charge inside surface, \( \Phi_{s_1} = \Phi_{s_2} = \Phi_{s_3} = 0 \)

(# field lines going in = # field lines going out)
Gauss’s Law

Net electric flux through any closed surface (“Gaussian surface”) equals the total charge enclosed inside the closed surface divided by the permittivity of free space.

\[
\Phi_E = \oint E \cdot d\mathbf{A} = \sum q_{\text{encl}} / \varepsilon_0
\]

\(\varepsilon_0\): permittivity constant \((4\pi\varepsilon_0)^{-1} = k\)

Using Gauss’ Law

Choose a closed (Gaussian) surface such that the surface integral is trivial.

Use symmetry arguments:

1. **Direction.** Choose a Gaussian surface such that \(E\) is clearly either parallel or perpendicular to each piece of surface

2. **Magnitude.** Choose a surface such that \(E\) is known to have the same value at all points on the surface

Then:

\[
\oint E \cdot d\mathbf{A} = \oint E dA = EA = \frac{q_{\text{encl}}}{\varepsilon_0}
\]

Given \(q_{\text{encl}}\), can solve for \(E\) (at surface), and vice versa
Conductors and Gauss’s Law

Use Gauss’s Law to understand properties of conductors in electrostatic equilibrium:

1. Choose a Gaussian surface inside (as close to the surface as desired)

   No net flux through the Gaussian surface \( (E_{\text{in}}=0) \). Any net charge must reside on the surface (cannot be inside!)

2. \( \mathbf{E} \) field always perp to surface: choose Gaussian surface (“pillbox”)

\[
\Phi_E = EA = \frac{q_{\text{encl}}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} \quad E = \frac{\sigma}{\varepsilon_0}
\]

More examples: three general cases

Method: evaluate flux over carefully chosen “Gaussian surface”:

Works for highly symmetric charge distributions:

- **spherical**  
  (point charge, uniform sphere, spherical shell,...)
- **cylindrical**  
  (infinite uniform line of charge or cylinder,...)
- **planar**  
  (infinite uniform sheet of charge or slab,...)
Spherical Symmetry: Examples

Spherical symmetry: choose a spherical Gaussian surface, radius \( r \)

E.g. uniformly charged sphere radius \( a \) and total charge \( Q \),

\[
E = qencl
\frac{1}{4\pi\varepsilon_0 r^2}
\]

Example: Uniform Charge Sphere. Solution:

(see board)

Note: same form as a point charge
How not to apply Gauss’s Law

Two charges $+2Q$ and $-Q$ are placed at locations shown. Find the electric field at point P.

1. Draw a Gaussian surface passing through P

2. Apply Gauss’s law:

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{encl}}{\varepsilon_0}$$

$$q_{encl} = 2Q + (-Q) = Q$$

3. Surface integral:

$$\oint \mathbf{E} \cdot d\mathbf{A} = 4\pi r^2 E$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Is this correct? No! Which step is wrong? Last step

Example: Thin Spherical Shell

Find $E$ field inside/outside a uniformly charged thin spherical shell, surface charge density $\sigma$, total charge $q$

Gaussian Surface for $E$(outside)

$$q_{encl} = q$$

$$E_{r>a} = \frac{q}{4\pi\varepsilon_0 r^2}$$

Gaussian Surface for $E$(inside)

$$q_{encl} = 0$$

$$E_{r<a} = 0$$

Trickier examples: 22.29-31 (board: today or next lecture)
Gauss’s Law: Examples

Cylindrical symmetry.

Example: infinite uniform line of charge.

Symmetry: $E$ indep of $z$, $\theta$, in radial direction

Gaussian surface: cylinder of length $L$

$$\oint \vec{E} \cdot d\vec{A} = E(r)2\pi rL = \frac{q_{enc}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

$$\vec{E}(r) = \frac{\lambda}{2\pi \varepsilon_0 r} \hat{r}$$

Similar examples: infinite uniform cylinder, cylindrical shell

Text examples (board: today or next lecture): 22.34, 22.35

---

Gauss’s Law: Examples

Planar symmetry.

Example: infinite uniform sheet of charge.

Symmetry: $E$ indep of $x, y$, in $z$ direction

Gaussian surface: pillbox, area of faces=$A$

$$\oint \vec{E} \cdot d\vec{A} = 2EA = \frac{q_{enc}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} \quad \Rightarrow \quad \vec{E} = \frac{\sigma}{2\varepsilon_0} \hat{z}$$

Exercise: similar to $E$ field just outside of a conductor

Text example (today or next time): 22.24,…