Physics 202, Lecture 4

Today’s Topics

- More on Gauss’s Law
- Electric Potential (Ch. 23)
  - Electric Potential Energy and Electric Potential
  - Electric Potential and Electric Field

Gauss’s Law: Review

\[ \Phi_E = \oint E \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0} \]

Use it to obtain E field for highly symmetric charge distributions.

- **spherical**
  - (point charge, uniform sphere, spherical shell, ...)
  
- **cylindrical**
  - (infinite uniform line of charge or cylinder, ...)

- **planar**
  - (infinite uniform sheet of charge, ...)

Method: evaluate flux over carefully chosen “Gaussian surface”
**Spherical Symmetry: Examples**

Spherical symmetry: choose a spherical Gaussian surface, radius \( r \)

Then:\[ \oint \mathbf{E} \cdot d\mathbf{A} = E 4\pi r^2 = \frac{q_{\text{encl}}}{\varepsilon_0} \]

\[ E = \frac{q_{\text{encl}}}{4\pi\varepsilon_0 r^2} \]

**Example 1** (last lecture): uniformly charged sphere (radius \( a \), charge \( Q \))

\[ E_{r>a} = \frac{Q}{4\pi\varepsilon_0 r^2} \hat{r} \]

\[ E_{r<a} = \frac{Qr}{4\pi\varepsilon_0 a^3} \hat{r} \]

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**Example 2: Thin Spherical Shell**

Find \( E \) field inside/outside a uniformly charged thin spherical shell, surface charge density \( \sigma \), total charge \( q \)

\[ q_{\text{encl}} = q \]

\[ E_{r>a} = \frac{q}{4\pi\varepsilon_0 r^2} \]

\[ q_{\text{encl}} = 0 \]

\[ E_{r<a} = 0 \]

Trickier examples: set up (holes, conductors, …) 22.29-31, 22.54 (board)
Cylindrical symmetry: Examples

**Cylindrical symmetry.**  \( E \) indep of \( z, \theta \), in radial direction

**Gaussian surface:** cylinder of length \( L \)

Then:

\[
\oint \vec{E} \cdot d\vec{A} = E(r)2\pi r L = \frac{q_{encl}}{\varepsilon_0}
\]

\[
E = \frac{q_{encl}}{2\pi r L \varepsilon_0}
\]

Example: infinite uniform line of charge

\[
\vec{E}(r) = \frac{\lambda}{2\pi \varepsilon_0 r} \hat{r}
\]

Similar examples: infinite uniform cylinder, cylindrical shell

Text examples (board): 22.34-35

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Planar Symmetry: Examples

**Planar symmetry.**  Example: infinite uniform sheet of charge.

\( E \) indep of \( x, y \), in \( z \) direction

**Gaussian surface:** pillbox, area of faces = \( A \)

\[
\oint \vec{E} \cdot d\vec{A} = 2EA = \frac{q_{encl}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}
\]

\[
\vec{E} = \frac{\sigma}{2\varepsilon_0} \hat{z}
\]

Examples: multiple charged sheets, infinite slab (hmwk)

Text example (board): 22.24 (parallel plate)
Electric Potential Energy and Electric Potential

Review: Conservation of Energy (particle)

**Kinetic Energy (K).** Potential Energy $U$: for conservative forces (can be defined since work done by $F$ is path-independent)

$$K = \frac{1}{2}mv^2 \quad U(x, y, z)$$

If only conservative forces present in system, conservation of mechanical energy: $K + U = \text{constant}$

**Conservative forces:**
- Springs: elastic potential energy  
  $$U = \frac{1}{2}k_{\text{spring}}x^2$$
- Gravity: gravitational potential energy
- Electrostatic: electric potential energy (analogy with gravity)

**Nonconservative forces**
- Friction, viscous damping (terminal velocity)

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Electric Potential Energy

Given two positive charges $q$ and $q_0$: initially very far apart, choose $U_i = 0$

To push particles together requires work (they want to repel). Final potential energy will increase!

$$\Delta U = U_f - U_i = \Delta W$$

If $q$ fixed, what is work needed to move $q_0$ a distance $r$ from $q$?

$$\Delta W = \int_{\infty}^{r} \vec{F}_{\text{us}} \cdot d\vec{l} = -\int_{\infty}^{r} \vec{F}_e \cdot d\vec{l} = -\int_{\infty}^{r} \frac{kq q_0}{r^2} \, dr' = \frac{kq q_0}{r}$$

Note: if $q$ negative, final potential energy negative

Particles will move to minimize their final potential energy!
Electric Potential Energy

Electric potential energy between two point charges:

\[ U(r) = \frac{kq_0q}{r} \]

- \( U \) is a scalar quantity, can be + or -
- convenient choice: \( U=0 \) at \( r=\infty \)
- SI unit: Joule (J)

Electric potential energy for system of multiple charges: sum over pairs:

\[ U(r) = \sum_{i<j} \frac{kq_iq_j}{r_{ij}} \]

This is the work required to assemble charges.

Text example: 23.54

Electric Potential

Charge \( q_0 \) is subject to Coulomb force in electric field \( \vec{E} \).

Work done by electric force:

\[ W = \int \vec{F} \cdot d\vec{l} = q_0 \int \vec{E} \cdot d\vec{l} = -\Delta U \]

Electric Potential Difference:

\[ \Delta V \equiv \frac{\Delta U}{q_0} = -\int_{A}^{B} \vec{E} \cdot d\vec{l} = V_B - V_A \]

Units: Volts

(1 V = 1 J/C)

Often called potential \( V \), but meaningful only as potential difference

Customary to choose reference point \( V=0 \) at \( r = \infty \)
(OK for localized charge distribution)
Electric Potential and Point Charges

For point charge $q$ shown below, what is $V_B - V_A$?

$$V_B - V_A = -\int_A^B E(r)dr = -kq \int_A^B \frac{dr}{r^2} = kq \left( \frac{1}{r_B} - \frac{1}{r_A} \right)$$

independent of path b/w A and B!

Potential of point charge:

$$V(r) = \frac{kr}{r}$$

Many point charges: superposition

$$V(r) = k \sum q_i \frac{1}{r_i}$$

Equipotentials: lines of constant potential

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Electric Potential: Continuous Distributions

Two methods for calculating $V$:

1. Brute force integration (next lecture)

$$dV = k \frac{dq}{r}, V = \int dV$$

2. Obtain from Gauss’s law and definition of $V$:

$$V(r) = -\int_{ref}^{r} \vec{E} \cdot d\vec{l}$$

Examples (today or next lecture): 23.19, parallel plate
Obtaining the Electric Field From the Electric Potential

Three ways to calculate the electric field:
- Coulomb’s Law
- Gauss’s Law
- Derive from electric potential

Formalism

$$\Delta V = -\int_{A}^{B} \vec{E} \cdot d\vec{l}$$

$$dV = -\vec{E} \cdot d\vec{l} = -E_x dx - E_y dy - E_z dz$$

$$E_x = -\frac{\partial V}{\partial x}, \ E_y = -\frac{\partial V}{\partial y}, \ E_z = -\frac{\partial V}{\partial z} \quad \text{or} \quad \vec{E} = -\nabla V$$