Physics 202, Lecture 7

Today’s Topics

- Capacitance (Ch. 24-II)
- Review
- Energy storage in capacitors
- Dielectric materials, electric dipoles
- Dielectrics and Capacitance

- New plan for next 3 lectures:
  - Next lecture (2/14): Exam 1 info and review session
  - Next week: Ch. 26 (Thurs, 2/21) Ch. 25 (Tues, 2/19)
    (note: modified reading assignments -- see syllabus)
  - HW #4 (5 problems) due 2/20 at 11 PM

Capacitors: Summary

- Definition:
  \[
  C \equiv \frac{Q}{\Delta V}
  \]

- Capacitance depends on geometry:

  ![Diagram of Parallel Plates, Cylindrical, and Spherical Capacitors]

  \[
  C_{\text{Parallel Plates}} = \frac{\varepsilon_o A}{d}
  \]
  \[
  C_{\text{Cylindrical}} = \frac{2\pi \varepsilon_o L}{\ln \left( \frac{b}{a} \right)}
  \]
  \[
  C_{\text{Spherical}} = 4\pi \varepsilon_o \frac{ab}{b-a}
  \]

  \(C\) has units of “Farads” or F (1F = 1C/V)
  \(\varepsilon_o\) has units of F/m
Capacitors in Series and Parallel

Parallel:
\[ C_p = C_1 + C_2 \]
\[ \Delta V_1 = \Delta V_2 = \Delta V \]

Series:
\[ C_s = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} \]
\[ Q_1 = Q_2 = Q \]
\[ \Delta V = \Delta V_1 + \Delta V_2 \]

Example: Ch. 24 #29 (board)

Energy of a Capacitor

- How much energy is stored in a charged capacitor?
  - Calculate the work provided (usually by a battery) to charge a capacitor to +/- \( Q \):

  Incremental work \( dW \) needed to add charge \( dq \) to capacitor at voltage \( V \):

  \[ dW = V(q) \cdot dq = \left( \frac{q}{C} \right) \cdot dq \]

  - The total work \( W \) to charge to \( Q \) is then given by:

    \[ W = \frac{1}{C} \int q dq = \frac{1}{2} \frac{Q^2}{C} \]

    Two ways to write \( W \):

    - In terms of the voltage \( V \):
      \[ W = \frac{1}{2} CV^2 \]
Capacitor Variables

- The total work to charge capacitor to \( Q \) equals the energy \( U \) stored in the capacitor:
  \[
  U = \frac{1}{C} \int_0^Q q dq = \frac{1}{2} \frac{Q^2}{C}
  \]

- In terms of the voltage \( V \):
  \[
  U = \frac{1}{2} CV^2
  \]

You can do one of two things to a capacitor:

- hook it up to a battery \( \rightarrow \) specify \( V \) and \( Q \) follow
- put some charge on it \( \rightarrow \) specify \( Q \) and \( V \) follow

\[
\begin{align*}
Q &= CV \\
V &= \frac{Q}{C}
\end{align*}
\]

Example (I)

- Suppose the capacitor shown here is charged to \( Q \). The battery is then disconnected.
- Now suppose the plates are pulled further apart to a final separation \( d_f \).
- How do the quantities \( Q, C, E, V, U \) change?
  - \( Q \): remains the same.. no way for charge to leave.
  - \( C \): decreases.. capacitance depends on geometry
  - \( E \): remains the same... depends only on charge density
  - \( V \): increases.. since \( C \downarrow \), but \( Q \) remains same (or \( d \uparrow \) but \( E \) the same)
  - \( U \): increases.. add energy to system by separating

\[
\begin{align*}
C_i &= \frac{d_i}{d_f} C \\
V_i &= \frac{d_i}{d} V \\
U_i &= \frac{d_i}{d} U
\end{align*}
\]
Example (II)

- Suppose the battery \((V)\) is kept attached to the capacitor.
- Again pull the plates apart from \(d\) to \(d_1\).
- Now what changes?
  - \(C\): decreases (capacitance depends only on geometry)
  - \(V\): must stay the same - the battery forces it to be \(V\)
  - \(Q\): must decrease, \(Q=CV\) charge flows off the plate
  - \(E\): must decrease \((E=\frac{V}{D}, E=\frac{\sigma}{\epsilon_0})\)
  - \(U\): must decrease \((U=\frac{1}{2}CV^2)\)

\[
\begin{align*}
C_1 &= \frac{d}{d_1} C \\
E_1 &= \frac{d}{d_1} E \\
U_1 &= \frac{d}{d_1} U
\end{align*}
\]

Where is the Energy stored?

- Claim: energy is stored in the electric field itself.
- Consider the example of a constant field generated by a parallel plate capacitor:

\[
U = \frac{1}{2} Q^2 \frac{1}{C} = \frac{1}{2} \left(\frac{Q^2}{A \epsilon_0 / d}\right)
\]

- The electric field is given by:

\[
E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \quad \Rightarrow \quad U = \frac{1}{2} \epsilon_0 E^2 Ad
\]

- The energy density \(u\) in the field is given by:

\[
\begin{align*}
\frac{U}{\text{volume}} &= \frac{U}{Ad} = \frac{1}{2} \epsilon_0 E^2 \\
\text{Units:} \quad \frac{J}{m^3}
\end{align*}
\]

Example: 24.51
Dielectrics

- Empirical observation:
  Inserting a non-conducting material (dielectric) between the plates of a capacitor changes the VALUE of the capacitance.

- Definition:
  The dielectric constant \( \kappa \) of a material is the ratio of the capacitance when filled with the dielectric to that without it:
  \[
  \kappa = \frac{C}{C_0}
  \]
  \[\text{permittivity: } \varepsilon = \kappa \varepsilon_0\]
  \( \kappa \) values are always > 1 (e.g., glass = 5.6; water = 80)
  Dielectrics INCREASE the capacitance of a capacitor
  More energy can be stored on a capacitor at fixed voltage:
  \[
  U' = \frac{CV^2}{2} = \frac{\kappa C_0 V^2}{2} = \kappa U
  \]

Dielectric Materials

Dielectrics are electric insulators:
- Charges are not freely movable, but can still have small displacements in an external electric field
- Atomic view: composed of permanent (or induced) electric dipoles:
  (see Sec 21.11)

![Permanent dipole](image)

![Induced dipole](image)
Electric Dipole in External E Field

**Electric dipole moment** \( \mathbf{p} \).

\[
\mathbf{p} = q \mathbf{d}
\]

Dipole in constant E field:

- **Net Force**
  \[ \sum \mathbf{F} = 0 \]
- **Net Torque**
  \[ \mathbf{\tau} = \mathbf{p} \times \mathbf{E} \]
- **Potential energy**
  \[ U = -\mathbf{p} \cdot \mathbf{E} \]

Dielectrics In External Field

Alignment of permanent dipoles in external field (or alignment of non-permanent dipoles)

- **Zero external field**
- **Applying external E field**
  \[ \mathbf{E} = \frac{\sigma}{\varepsilon_0} \]
- **Equilibrium**
  \[ \frac{\varepsilon_0}{\kappa} \]

\[ \mathbf{E} = \mathbf{E}_0 - \mathbf{E}_{ind} = \frac{\varepsilon_0}{\kappa} \]

\[ \mathbf{E}_{ind} = \frac{\sigma_{ind}}{\varepsilon_0} \]

\[ \sigma_{ind} = \frac{\sigma - \varepsilon_0}{\kappa} \]

Note: induced field always opposite to the external field \( \mathbf{E}_0 \).
Parallel Plate Example (I)

- Deposit a charge $Q$ on parallel plates filled with vacuum (air)—capacitance $C_0$.
- Disconnect from battery.
- The potential difference is $V_0 = Q / C_0$.

Now insert material with dielectric constant $\kappa$.

Charge $Q$ remains constant.
Capacitance increases $C = \kappa C_0$.
Voltage decreases from $V_0$ to:

$$V = \frac{Q}{C} = \frac{Q}{\kappa C_0} = \frac{V}{\kappa}$$

Electric field decreases also:

$$E = \frac{V}{d} = \frac{V_0}{d\kappa} = \frac{E_0}{\kappa}$$

Note: The field only decreases when the charge is held constant!

Parallel Plate Example (II)

- Deposit a charge $Q_0$ on parallel plates filled with vacuum (air)—capacitance $C_0$.
- The potential difference is $V = Q / C_0$.
- Leave battery connected.

Now insert material with dielectric constant $\kappa$.

Voltage $V$ remains constant.
Capacitance increases $C = \kappa C_0$.
Charge increases from $Q_0$ to:

$$Q = CV = \kappa C_0 V = \kappa Q_0$$

Net Electric field stays same ($V$, $d$ constant)

Examples: text example 24-11, problems 24.55, 24.59, 24.60