Physics 208 Exam 2
Nov. 1, 2006

Print your name and section clearly above. If you do not know your section number, write your TA’s name.
Your final answer must be placed in the box provided. **You must show all your work to receive full credit.** If you only provide your final answer (in the box), and do not show your work, you will receive very few points.
Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units, and also the direction of vectors.
You are allowed one 8½ x 11” sheet of notes and no other references. The exam lasts exactly 90 minutes.

Speed of light in vacuum:
\[ c = 3 \times 10^8 \text{ m/s} \]

Permittivity of free space
\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2 \]

Permeability of free space
\[ \mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m}/\text{A} \]

Problem 1: _______ / 20

Problem 2: _______ / 20

Problem 3: _______ / 20

Problem 4: _______ / 25

Problem 5: _______ / 20

TOTAL: _______ / 105
1) [20 pts, 4 pts each] Multiple choice/short answer.

i) A negatively charged particle is moving at speed $v$ to the right in a magnetic field directed into the page. The direction of the force on the particle is

- a) up
- b) down
- c) left
- d) right
- e) into page
- f) out of page
- g) none of these

ii) Current flows through all parts of the infinitely long wire as shown. What is the magnetic field strength at the center of the loop of radius $R$?

- a) $B = \mu_0 I / 4R$
- b) $B = \mu_0 I / 4R + \mu_0 I / 2\pi R$
- c) $B = \mu_0 I / 4R + 2\mu_0 I / 2\pi R$
- d) $B = -\mu_0 I / 4R + 2\mu_0 I / 2\pi R$
- e) $B = \mu_0 I / 2R$
- f) none of these

iv) Water (index of refraction=1.33) is inside a glass fishtank (index of refraction=1.5). A flashlight held underwater is pointed at the glass. The critical angle for total internal reflection $\theta_c$ satisfies

- a) $0^\circ < \theta_c \leq 30^\circ$
- b) $30^\circ < \theta_c \leq 45^\circ$
- c) $45^\circ < \theta_c \leq 60^\circ$
- d) $60^\circ < \theta_c < 90^\circ$
- e) none of the above

v) At right is an infinitely long coaxial cable. The inner and outer conductors uniformly carry equal currents in opposite directions as shown. The magnitude of the magnetic field outside the outer conductor and between the two conductors compare as

- a) $B_{\text{outside}} = B_{\text{between}}$
- b) $B_{\text{outside}} < B_{\text{between}}$
- c) $B_{\text{outside}} > B_{\text{between}}$
- d) none of these true for all points
2) [20 pts, 5 pts each] Your Badger radio network (WIBA) is 1310 kHz on the AM dial (this is the frequency in kHz).

a) What is the wavelength of this wave when it propagates in air (n=1.0)?

\[ \lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{ m/s}}{1310 \times 10^3 \text{ Hz}} = 229 \text{ m} \]

This is two and a half football fields!

b) You take your radio under water in a swimming pool to listen for a while. Calculate how fast the radio wave travels under the water (water index of refraction=1.33).

\[ v = \frac{c}{n} = \frac{3 \times 10^8 \text{ m/s}}{1.33} = 2.26 \times 10^8 \text{ m/s} \]

c) Find the frequency and wavelength under water.

Frequency doesn’t change,

\[ \frac{\lambda}{f} = \frac{v}{f} = \frac{3 \times 10^8 \text{ m/s/1.33}}{1310 \times 10^3 \text{ Hz}} = 172 \text{ m} \]

d) The WIBA transmission tower is south of the beltline on Fish Hatchery road, about 5 km from campus, and broadcasts 5 kW (5000 Watts) of EM radiation. Calculate the amplitude of the electric field on campus (no water!), assuming no absorption or reflection by buildings, trees, etc.

Spherical wave emitted from the broadcast tower – the power is spread out over the surface of a sphere, so that the power per unit area is \( P/4\pi R^2 \). The Poynting vector \( \mathbf{S} = \mathbf{E} \times \mathbf{B} / \mu_0 \), expresses the instantaneous power per unit area in terms of the fields.

In terms of the electric field alone, this is \( \frac{1}{2} c \varepsilon_0 E^2 \), so that

\[ \frac{1}{2} c \varepsilon_0 E^2 = \frac{P_{\text{radio}}}{4\pi R^2} \Rightarrow E = \left( \frac{2P_{\text{radio}}}{4\pi R^2 c \varepsilon_0} \right)^{1/2} = \left( \frac{2 \cdot 5000 \text{ W}}{4\pi(5000 \text{ m})^2 \left( 3 \times 10^8 \text{ m/s} \right) \left( 8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2 \right)} \right)^{1/2} \]

= 0.1095 V/m
3) [20 pts, 5 pts each] A loop of wire with dimensions as shown is oriented with plane of the loop parallel to a uniform magnetic field of strength $B=1 \ T$ in the $z$-direction. A current $I=1 \ A$ is flowing clockwise in the loop.

$L=2 \ m$, $W=1 \ m$, $B=1.0 \ T$, $I=1 \ A$.

a) Calculate the net force on the loop. **Explain.**

The net force is zero, because the force on the left and right segments is zero ($I \parallel B$), and the force on the top segment is equal and opposite to the force on the bottom segment.

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<thead>
<tr>
<th>Value</th>
<th>Units</th>
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<tbody>
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<td>$F$</td>
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b) Calculate the magnetic dipole moment of the loop.

The magnetic dipole moment has magnitude $IA$, and a direction such that the moment points in the direction of the field at the center of the loop. So the magnitude is $(1 \ A)(2m)(1m)=2 \ Am^2$, and the direction is into the page.

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<thead>
<tr>
<th>Magnitude</th>
<th>Units</th>
<th>Direction</th>
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c) Calculate the torque on the loop.

The torque on the loop is $\vec{\mu} \times \vec{B}$. Since $\vec{\mu} \perp \vec{B}$, the magnitude is $\mu B = (2Am^2)(1T) = 2N \cdot m$

The direction is perpendicular to both $\vec{\mu}$ and $\vec{B}$, and by the right hand rule is to the right.

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d) Calculate the amount of work **you** must do to rotate the top of the loop 90° into the page so that the plane of the loop is perpendicular with the magnetic field.

The work done is equal to the final potential energy minus the initial potential energy. The potential energy is $-\vec{\mu} \cdot \vec{B}$. In the initial configuration, $\vec{\mu} \perp \vec{B}$ and so the initial potential energy is zero. In the final configuration, $\vec{\mu}$ is antiparallel to $\vec{B}$, the high energy configuration of $+\mu B$. So the work I must do is $+\mu B = +\left(2Am^2\right)(1T) = +2J$
4) [25 pts, 5 pts each] Three linear polarizers are arranged as shown, with transmission axes indicated by the heavy line. The unpolarized incoming light is propagating in the z-direction.

![Diagram of polarizers](image)

a) Write down the space- and time-dependent electric field vector of the light after the first polarizer (region I), assuming its amplitude is $E_o$.

*Light transmitted through the first polarizer has an electric field vector along the transmission axis $\hat{y}$. The EM wave propagates in the $\hat{z}$ direction.*

$$E = E_o \cos(kz - \omega t) \hat{y}$$

b) What is the amplitude of the electric field vector of the EM wave after it passes through the second polarizer (region II in the diagram).

*The component of the electric field along the transmission axis is transmitted. This component is $E_o \cos 30$. This is the amplitude of the electric field in region II.*
c) Write down the space- and time-dependent electric field vector in region II as a sum of vector components in the x and y directions.

In region II, the E-field vector points along the transmission axis of polarizer II. This is 30° with respect to the vertical. So the x-component is the amplitude times sin30. The y-component is the amplitude times cos30. The amplitude is $E_o \cos \theta$.

$$E = E_o \cos(30) \sin(30) \hat{x} + E_o \cos(30) \cos(30) \hat{y}$$

d) Write down an expression for space- and time-dependent electric field vector in region III. Explain your reasoning.

The third polarizer has its transmission axis along the $\hat{x}$ direction. This polarizer will transmit the component of the field along the x-axis. From part c), the amplitude of this wave is $E_o \cos(30) \sin(30)$. The E-field vector is then

$$E = E_o \cos(30) \sin(30) \cos(kz - \omega t) \hat{x} = E_o \cos(30) \cos(60) \cos(kz - \omega t) \hat{x}$$

e) Find an expression for the intensity of the light in region III (after passing through the third polarizer) in terms of $E_o$ and fundamental constants.

The intensity is $\frac{1}{2} c \varepsilon_o E^2$, where $E$ is the amplitude of the electric field. Then the intensity is

$$\frac{c E_o}{2} E_o^2 \cos^2(30) \cos^2(60)$$
5) [20 pts, 5 pts each] A square loop lying flat on a table is pushed across a current-carrying wire at constant speed as shown. The field is largest closest to the wire, and is of opposite sign on either side of wire.

You don’t need to calculate any integrals for this problem! Just use physical reasoning and explanations in words, diagrams, or simple equations.

a) As the loop approaches the wire from the left, does the induced loop current flow clockwise or counterclockwise? Explain your reasoning.

As the loop approaches the wire, the magnetic field increases in magnitude. This means that the flux out of the page increases. The induced current produces a magnetic field to oppose this increase, hence produces a field into the page. By the right-hand-rule, this is a clockwise current.

b) What direction is the net force on the loop from the magnetic field as the loop approaches the wire from the left (as in part a)? Explain your reasoning.

The force on the loop from the field can be determined by adding the forces on the four sides of the loop. The forces on the top and bottom segments cancel, since they are equal and opposite. The forces on the left and right segments are in opposite direction, but they are not equal since the field is bigger on the right side of the loop. By the right-hand rule, the force on the right segment of the loop is to the left. Hence the net force on the loop is to the left.
c) Now suppose the loop has moved past the wire, and is moving at the same constant speed to the right of the wire. does the induced loop current flow clockwise or counterclockwise? **Explain your reasoning.**

*The field through the loop is into the page, and is decreasing in magnitude. The flux through the loop is decreasing. The induced current in the loop will oppose the change in flux by producing a field into the page. By the right-hand rule, this is a clockwise current.*

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d) What direction is the net force on the loop from the magnetic field now that it has moved past the wire? **Explain your reasoning.**

*The force on the loop from the field can be determined by adding the forces on the four sides of the loop. The forces on the top and bottom segments cancel, since they are equal and opposite. The forces on the left and right segments are in opposite direction, but they are not equal since the field is bigger on the left side of the loop. By the right-hand rule, the force on the left segment of the loop is to the left. Hence the net force on the loop is to the left.*